Ratio of uniforms and beyond

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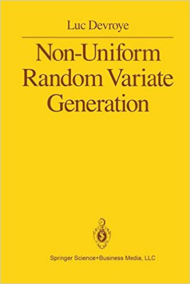


Ratio of uniforms: what?

Consider the set A of (u, v)'s in $\mathbb{R}^+ \times \mathcal{X}$ such that

$$0 \leqslant u^2 \leqslant f(v/u)$$

Then a uniform distribution on A induces the distribution with density proportional to f on V/U.

[Kinderman and Monahan's (1977)]

Ratio of uniforms: why?

Consider the change of variables from (u, v) to $(u, w = v/u)$ with Jacobian u, then (u, w) has the density

$$u\,\mathbb{I}_{(0,f(w)^{1/2})}(u)$$

Integrating out u leads to

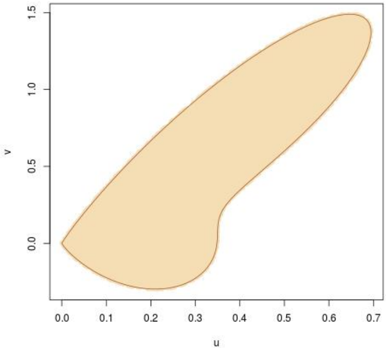
$$\int_0^{f(w)^{1/2}} u\,\mathrm{d}u = f(w)^{1/2\times 2} = f(w)$$

as proportional to the density of V/U

Ratio of uniforms: how?

Simulating a uniform distribution on A means identifying the region within a simple box $\mathfrak{B}$
Boundaries of A given by (?)

$$A^b = \{(u(x) = f(x)^{1/2}, v(x) = xf(x)^{1/2});\ x \in \mathcal{X}\}$$



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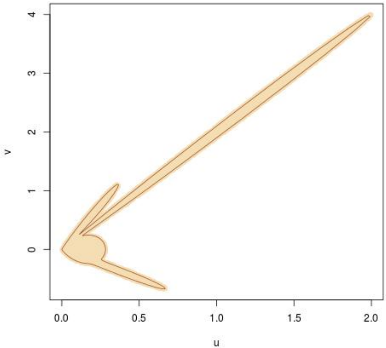
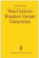




Ratio of uniforms: why?

There exists a compact box $\mathfrak{B}$ containing A iff

$$0 \leqslant f(x) \leqslant \bar{f} \qquad 0 \leqslant xf(x)^{1/2} \leqslant \tilde{f}$$

Applications to standard distributions like Student's t

[Devroye, 1986, Section 3.7]

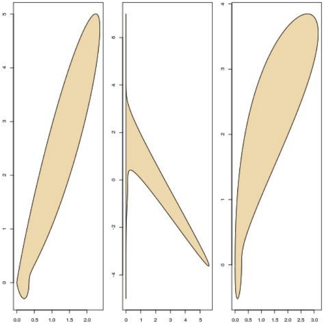
Ratio of uniforms: where?

Principle that can be generalised to a monotone transform of f, $h(f)$, and the set

$$\mathfrak{H} = \{(u,v);\ 0 \leqslant u \leqslant h(f(v/g(u)))\}$$

which still produces a distribution with density proportional to f when

$$g(x) = {}^{\mathrm{d}G}/_{\mathrm{d}x}(x) \qquad G(x) = h^{-1}(x)$$



Ratio of uniforms: when?

- choice of transform f most adequate for a given f
- slice sampler deduced from this construct
- case of an unbounded density f