

# **PCS5024 - Statistical Machine Learning**

**Escola Politecnica, USP**

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# Exercicio de Reinforcement Learning

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**Exercicio 1:** Indicar uma história com 12 passos e sua respectiva sequência de comandos (uma só) e respectiva probabilidade:

In [ ]: Resp: Utilizando a mesma sequência de comandos contida nos slides "[U ,U, R, R, R]" é impossível executar 12 passos. Tendo o mesmo "Trasition Model" também será impossível obter a mesma probabilidade obtidas no exemplo dado em classe. Porém, Tendo uma sequência de no mínimo 12 paços, sim é possível chegar no "goal" (12 passos):

S: (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)

A: [L, R, U, D]

h(x): [L, L, L, L, L, L, L, U, U, R, R, R]

p(x):  $(0,8)^{12} = 0.068719476736$

V(x):  $(-0.04 \cdot 11) + 1 = 0.56$

**Exercicio 2:** Resolver usando o algorithm "valion interation":

- a) Quantas interações ?
- b) Quais os valores ?

In [ ]: *### Informarion:*  
Gama = 1/2  
A: red, green  
S: s1, s2  
|A|= Duas ações and |S|= Dois estados  
V0(s1)= 0 and V0(s2)= 0

### Calculation:

-----  
1.A)

$$V1(s1) = r(s1) + \max(a) (\text{gama}*(p(s2|s1,\text{red})*V0(s2) + p(s1|s1,\text{red})*V0(s1)), \\ (\text{gama}*(p(s2|s1,\text{green})*V0(s2))))$$

$$V1(s1) = 3 + \max(a) ( (0.5*(0.5*0 + 0.5*0)), (0.5*(1*0)) )$$

$$V1(s1) = 3 + \max(a) ( (0.5*(0 + 0)), (0.5*(0)) )$$

$$V1(s1) = 3 + \max(a) ( (0), (0)) )$$

$$V1(s1) = 3 + 0$$

$$V1(s1) = 3$$

1.B)

$$V1(s2) = r(s2) + \max(a) (\text{gama}*(p(s2|s2,\text{red})*V0(s2)), \\ (\text{gama}*(p(s1|s2,\text{green})*V0(s1))))$$

$$V1(s2) = -1 + \max(a) ( (0.5*(0.5*0)), (0.5*(1*0)) )$$

$$V1(s2) = -1 + \max(a) ( (0.5*(0)), (0.5*(0)) )$$

$$V1(s2) = -1 + \max(a) ( (0), (0)) )$$

$$V1(s2) = -1 + 0$$

$$V1(s2) = -1$$

-----  
2.A)

$$V2(s1) = r(s1) + \max(a) (\text{gama}*(p(s2|s1,\text{red})*V1(s2) + p(s1|s1,\text{red})*V1(s1)), \\ (\text{gama}*(p(s2|s1,\text{green})*V1(s2))))$$

$$V2(s1) = 3 + \max(a) ( (0.5*(0.5*3 + 0.5*(-1))), (0.5*(1*(-1))) )$$

$$V2(s1) = 3 + \max(a) ( (0.5*(1.5 + (-0.5))), (0.5*(-1)) )$$

$$V2(s1) = 3 + \max(a) ( (0.5*(1), (-0.5)) )$$

$$V2(s1) = 3 + \max(a) ( (0.5), (-0.5)) )$$

$$V2(s1) = 3 + 0.5$$

$$V2(s1) = 3.5$$

2.B)

$$V2(s2) = r(s2) + \max(a) (\text{gama}*(p(s2|s2,\text{red})*V1(s2)), \\ (\text{gama}*(p(s1|s2,\text{green})*V1(s1))))$$

$$V2(s2) = -1 + \max(a) ( (0.5*(0.5*(-1))), (0.5*(1*3)) )$$

$$V2(s2) = -1 + \max(a) ( (0.5*(-0.5)), (0.5*3)) )$$

$$V2(s2) = -1 + \max(a) ( (0.25), (1.5)) )$$

$$V2(s2) = -1 + 1.5$$

$$V2(s2) = 0.5$$

-----

3.A)

$$V3(s1) = r(s1) + \max(a) ( \text{gama} * (p(s2|s1, \text{red}) * V2(s2) + p(s1|s1, \text{red}) * V2(s1)), \\ ( \text{gama} * (p(s2|s1, \text{green}) * V2(s2))) )$$

$$V3(s1) = 3 + \max(a) ( (0.5 * (0.5 * 0.5 + 0.5 * 3.5), (0.5 * (1 * 0.5)) )$$

$$V3(s1) = 3 + \max(a) ( (0.5 * (0.25 + 1.75), (0.5 * 0.5)) )$$

$$V3(s1) = 3 + \max(a) ( (0.5 * 2), (0.25)) )$$

$$V3(s1) = 3 + \max(a) ( (1), (0.25)) )$$

$$V3(s1) = 3 + 1$$

$$V3(s1) = 4$$

3.B)

$$V3(s2) = r(s2) + \max(a) ( \text{gama} * (p(s2|s2, \text{red}) * V2(s2)), \\ ( \text{gama} * p(s1|s2, \text{green}) * V2(s1))) )$$

$$V3(s2) = -1 + \max(a) ( (0.5 * (0.5 * 0.5)), (0.5 * (1 * 3.5)) )$$

$$V3(s2) = -1 + \max(a) ( (0.5 * 0.25), (0.5 * 3.5)) )$$

$$V3(s2) = -1 + \max(a) ( (0.125), (1.75)) )$$

$$V3(s2) = -1 + 1.75$$

$$V3(s2) = 0.75$$

-----  
4.A)

$$V4(s1) = r(s1) + \max(a) ( \text{gama} * (p(s2|s1, \text{red}) * V3(s2) + p(s1|s1, \text{red}) * V3(s1)), \\ ( \text{gama} * (p(s2|s1, \text{green}) * V3(s2))) )$$

$$V4(s1) = 3 + \max(a) ( (0.5 * (0.5 * 0.75 + 0.5 * 4), (0.5 * (1 * 0.75)) )$$

$$V4(s1) = 3 + \max(a) ( (0.5 * (0.375 + 2), (0.5 * 0.75)) )$$

$$V4(s1) = 3 + \max(a) ( (0.5 * (2.375)), (0.5 * 0.75)) )$$

$$V4(s1) = 3 + \max(a) ( (1.1875), (0.375)) )$$

$$V4(s1) = 3 + 1.1875$$

$$V4(s1) = 4.1875 \sim 4.2$$

4.B)

$$V4(s2) = r(s2) + \max(a) ( \text{gama} * (p(s2|s2, \text{red}) * V3(s2)), \\ ( \text{gama} * p(s1|s2, \text{green}) * V3(s1))) )$$

$$V4(s2) = -1 + \max(a) ( (0.5 * (0.5 * 0.75)), (0.5 * (1 * 4)) )$$

$$V4(s2) = -1 + \max(a) ( (0.5 * 0.375), (0.5 * 4)) )$$

$$V4(s2) = -1 + \max(a) ( (0.1875), (2)) )$$

$$V4(s2) = -1 + 2$$

$$V4(s2) = 1$$

-----

5.A)

$$V5(s1) = r(s1) + \max(a) (\text{gama} * (p(s2|s1, \text{red}) * V4(s2) + p(s1|s1, \text{red}) * V4(s1)), \\ (\text{gama} * (p(s2|s1, \text{green}) * V4(s2))))$$

$$V5(s1) = 3 + \max(a) ((0.5 * (0.5 * 1 + 0.5 * 4.2), (0.5 * (1 * 1)) )$$

$$V5(s1) = 3 + \max(a) ((0.5 * (0.5 + 2.1), (0.5 * 1) )$$

$$V5(s1) = 3 + \max(a) ((0.5 * 2.6), (0.5) )$$

$$V5(s1) = 3 + \max(a) ((1.3), (0.5) )$$

$$V5(s1) = 3 + 1.3$$

$$V5(s1) = 4.3$$

5.B)

$$V5(s2) = r(s2) + \max(a) (\text{gama} * (p(s2|s2, \text{red}) * V4(s2)), \\ (\text{gama} * p(s1|s2, \text{green}) * V4(s1))))$$

$$V5(s2) = -1 + \max(a) ( (0.5 * (0.5 * 1)), (0.5 * (1 * 4.2)) )$$

$$V5(s2) = -1 + \max(a) ( (0.25), (2.1) )$$

$$V5(s2) = -1 + 2.1$$

$$V5(s2) = 1.1$$

-----  
6.A)

$$V6(s1) = r(s1) + \max(a) (\text{gama} * (p(s2|s1, \text{red}) * V5(s2) + p(s1|s1, \text{red}) * V5(s1)), \\ (\text{gama} * (p(s2|s1, \text{green}) * V5(s2))))$$

$$V6(s1) = 3 + \max(a) ((0.5 * (0.5 * 1.1 + 0.5 * 4.3)), (0.5 * (1 * 1.1)) )$$

$$V6(s1) = 3 + \max(a) ((0.5 * (0.55 + 2.15)), (0.5 * 1.1) )$$

$$V6(s1) = 3 + \max(a) ((0.5 * 2.7), (0.55) )$$

$$V6(s1) = 3 + \max(a) ((1.35), (0.55) )$$

$$V6(s1) = 3 + 1.35$$

$$V6(s1) = 4.35 \sim 4.4$$

6.B)

$$V6(s2) = r(s2) + \max(a) (\text{gama} * (p(s2|s2, \text{red}) * V5(s2)), \\ (\text{gama} * p(s1|s2, \text{green}) * V5(s1))))$$

$$V6(s2) = -1 + \max(a) ( (0.5 * (0.5 * 1.1)), (0.5 * (1 * 4.3)) )$$

$$V6(s2) = -1 + \max(a) ( (0.5 * 0.55), (0.5 * 4.3) )$$

$$V6(s2) = -1 + \max(a) ( (0.275), (2, 15) )$$

$$V6(s2) = -1 + 2, 15$$

$$V6(s2) = 1, 15 \sim 1.2$$

-----

7.A)

$$V7(s1) = r(s1) + \max(a) (gama * (p(s2|s1, red) * V6(s2) + p(s1|s1, red) * V6(s1)), \\ (gama * (p(s2|s1, green) * V6(s2))))$$

$$V7(s1) = 3 + \max(a) ((0.5 * (0.5 * 1.2 + 0.5 * 4.4)), (0.5 * (1 * 1.2) )$$

$$V7(s1) = 3 + \max(a) ((0.5 * (0.6 + 2.2)), (0.5 * 1.2) )$$

$$V7(s1) = 3 + \max(a) ((0.5 * 2.8), (0.6) )$$

$$V7(s1) = 3 + \max(a) ((1.4), (0.6) )$$

$$V7(s1) = 3 + 1.4$$

$$V7(s1) = 4.4$$

7.B)

$$V7(s2) = r(s2) + \max(a) (gama * (p(s2|s2, red) * V6(s2)), \\ (gama * p(s1|s2, green) * V6(s1))))$$

$$V7(s2) = -1 + \max(a) ( (0.5 * (0.5 * 1.2)), (0.5 * (1 * 4.4)) )$$

$$V7(s2) = -1 + \max(a) ( (0.5 * 0.6), (0.5 * 4.4) )$$

$$V7(s2) = -1 + \max(a) ( (0.3), (2.2) )$$

$$V7(s2) = -1 + 2.2$$

$$V7(s2) = 1.2$$

-----  
Critério de parada:  $V_i(s) = V_{i+1}(s)$

$$V6(s1) = V7(s1)$$

$$V6(s2) = V7(s2)$$

a) Número de interações: 14

b) Valores:

v(s1) v(s2)	
0   0	
3   -1	
3.5   0.5	
4.0   0.75	
4.2   1.0	
4.3   1.1	
4.1   1.2	

**Exercício 3:** Mostrar a melhor função de valor possível ( $V^*(s)$ ) e a melhor política possível ( $p^*(s)$ ) utilizando o algoritmo de Value Iteration:

**Referência:**

	1	2	3	4
3	(3,1)	(3,2)	(3,3)	(3,4)
2	(2,1)		(2,3)	(2,4)
1	(1,1)	(1,2)	(1,3)	(1,4)

**Funcao de Valor:**

0)

	1	2	3	4
3	0	0	0	100
2	0		0	-100
1	0	0	0	0

1)

	1	2	3	4
3	-1.00	-1.00	71.00	100
2	-1.00		-10.00	-100
1	-1.00	-1.00	-1.00	-1.00

2)

	1	2	3	4
3	-1.90	49.94	76.49	100
2	-1.90		40.22	-100
1	-1.90	-1.90	-1.90	-1.90

3)

	1	2	3	4
3	34.61	63.06	81.50	100
2	-2.71		48.69	-100
1	-2.71	-2.71	27.62	-2.71

4)

	1	2	3	4
3	47.28	69.03	82.72	100
2	23.43		53.07	-100
1	-3.44	18.40	33.57	9.64

5)

	1	2	3	4
3	55.07	70.98	83.22	100
2	37.26		54.33	-100
1	17.22	26.48	39.73	15.04

6)

	1	2	3	4
3	58.42	71.70	83.38	100
2	45.36		54.81	-100
1	29.76	32.37	41.86	19.96

7)

	1	2	3	4
3	59.96	71.94	83.44	100
2	49.22		54.97	-100
1	37.25	34.96	43.17	21.93

8)

	1	2	3	4
3	60.62	72.02	83.46	100
2	51.03		55.02	-100
1	40.94	36.38	43.70	23.06

9)

	1	2	3	4
3	60.91	72.05	83.46	100
2	51.83		55.04	-100
1	42.70	37.01	43.96	23.54

10)

	1	2	3	4
3	61.02	72.06	83.47	100
2	52.18		55.05	-100
1	43.49	37.32	44.08	23.77

11)

	1	2	3	4
3	61.07	72.07	83.47	100
2	52.33		55.05	-100
1	43.84	37.45	44.13	23.88

12)

	1	2	3	4
3	61.09	72.07	83.47	100
2	52.39		55.05	-100
1	43.99	37.52	44.16	23.92

13)

	1	2	3	4
3	61.10	72.07	83.47	100
2	52.42		55.05	-100
1	44.06	37.54	44.17	23.94

14)

	1	2	3	4
3	61.11	72.07	83.47	100
2	52.43		55.05	-100
1	44.09	37.56	44.17	23.95

15)

	1	2	3	4
3	61.11	72.07	83.47	100
2	52.43		55.05	-100
1	44.10	37.56	44.17	23.96



16)

	1	2	3	4
3	61.11	72.07	83.47	100
2	52.44		55.05	-100
1	44.10	37.57	44.17	23.96

17)

	1	2	3	4
3	61.11	72.07	83.47	100
2	52.44		55.05	-100
1	44.10	37.57	44.17	23.96

Política:

→	→	→	100
↑		↑	-100
↑	→	↑	←