Introduction

This paper contains the computation of the motive of the irreducible $SL_4(k)$ -character variety of torus knots for any algebraically closed field k. The calculation is based on the methods introduced in the paper [1].

The notations used in this paper are the following:

- X_4^{irr} is the irreducible $\mathrm{SL}_4(k)$ -representation variety of torus knots, that is, the variety of irreducible representations $\rho:\Gamma\to\mathrm{SL}_4(k)$ where $\Gamma=\Gamma_{n,m}$ is the fundamental group of the complement of the (n,m)-torus knot (see section 4 of [1]).
- $\mathfrak{M}_4^{\text{irr}} = X_4^{\text{irr}} /\!\!/ \operatorname{SL}_4(k)$ is the irreducible $\operatorname{SL}_4(k)$ -character variety of torus knots, that is, the moduli space of representations (see section 4 of [1]).
- $\kappa = (\epsilon, \epsilon)$ is a configuration of eigenvalues, that is a collection of possible eigenvalues for the matrices A and B of a torus knot representation $\rho = (A, B)$ (see section 4 of [1]).
- τ is the type of a semi-simple filtration of a torus knot representation (see section 2.1 of [1]).
- ξ is the shape of the type τ , that is the collection of dimensions and multiplicities of each isotypic component (see section 2.1 of [1]).
- σ_A are the collections of eigenvalues of A for each isotypic component of a torus knot representation $\rho = (A, B)$ (see section 7.1 of [1]).
- σ_B are the collections of eigenvalues of B for each isotypic component of a torus knot representation $\rho = (A, B)$ (see section 7.1 of [1]).
- \mathcal{M}_{τ} is the space parametrizing possible completions of a semi-simple representation to a general representation of type τ (see section 4 of [1])
- \mathcal{G}_{τ} is the gauge group acting on $\mathcal{M}_{\tau} \times \mathrm{SL}_4(k)$ that identifies isomorphic completions (see section 4 of [1]).
- $\mathfrak{M}_{\tau}^{irr}$ is the variety of possible semi-simplifications of a representation of type τ (see section 4 of [1]).
- $X(\tau)$ is the variety of representations of type τ .
- $m_{\kappa}(\tau)$ is the multiplicity of the type τ , that is the number of isomorphic components $X(\tau')$ of types τ' with the same shape as τ but whose eigenvalues are given by a permutation of the ones of τ that preserves their multiplicity (see section 5 of [1]).
- $C_{\pi,\pi'}$ are the number of isomorphic components given by configurations of eigenvalues with the same structure of repeated eigenvalues (see Section 6 of [1]). Here, π,π' are two partitions of 4 that determine the number of repeated eigenvalues of the matrices A and B of a representation $\rho = (A,B)$. If $\pi = \{1^{e_1},\ldots,4^{e_4}\}$ and $\pi' = \{1^{e'_1},\ldots,4^{e'_4}\}$ we have the following characterization in terms of multinomial numbers (Theorem 6.8 of [1])

$$C_{\pi,\pi'} = \frac{4}{nm} \begin{pmatrix} n \\ e_1, \dots, e_4 \end{pmatrix} \begin{pmatrix} m \\ e'_1, \dots, e'_4 \end{pmatrix}.$$

Combinatorial formulas for the motives $[\mathcal{M}_{\tau}]$, $[\mathcal{G}_{\tau}]$ and $[\mathfrak{M}_{\tau}^{irr}]$ are described in section 5 of [1] in terms of the structure of the type τ .

The structure of the paper is as follows. Each section describes the count of the motive $[\mathfrak{M}_{\kappa}]$ for a possible configuration of eigenvalues κ . For that purpose, we analyze all the types τ compatible with κ and compute the motives $[X(\tau)]$. A configuration of eigenvalues κ not appearing as a section of the paper means that $X_{\kappa}^{\text{irr}} = \emptyset$ (see Remark 4.5 and Proposition 8.1 of [1]). In the final section of this paper, we summarize the results for each configuration κ and we provide the final result depending on the combinatorial coefficients $C_{\pi,\pi'}$.

Warning: The script generating this paper is only valid for rank ≤ 4 . The result for higher rank may not be correct.

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1. Configuration $\epsilon = (\epsilon_1, \epsilon_1, \epsilon_1, \epsilon_2)$ and $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_4,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_4,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_1, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + 3q^{14} + 5q^{13} + 5q^{12} + 2q^{11} 2q^{10} 5q^9 5q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_4,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 q^2$.

- $[\mathcal{G}_{\tau}] = (q-1)^2 q^2$. $[\mathfrak{M}_{\tau}^{irr}] = q^2 3 q + 3$. $[X(\tau)] = q^{17} 3 q^{16} + 2 q^{15} + 2 q^{14} q^{13} + q^{12} 5 q^{11} + q^{10} + 2 q^8 + 3 q^7 3 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_4,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{15} q^{14} 2q^{13} q^{12} + 3q^{10} + 2q^9 + q^8 q^7 2q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_4,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{13} + 2q^{12} + 2q^{11} + q^{10} q^9 2q^8 2q^7 q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^4 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_4,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 q)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_4,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 q)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_4,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^2 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{17} 3q^{16} + 2q^{15} + 2q^{14} q^{13} + q^{12} 5q^{11} + q^{10} + 2q^{8} + 3q^{7} 3q^{6}$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_4,\epsilon_1,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\pi}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_4,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 q)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^3$.
- $[\mathfrak{M}_{\tau}^{\text{irr}}] = q 2$. $[X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_4,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 12$.

Total count of $\kappa = ((\epsilon_1, \epsilon_1, \epsilon_1, \epsilon_2), (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4))$

 $[X_{\kappa}^{\rm red}] = 8\,q^{17} + 4\,q^{16} + 16\,q^{15} + 22\,q^{14} + 16\,q^{13} + 20\,q^{12} + 2\,q^{11} + 6\,q^{10} + 6\,q^7 - 4\,q^6,$ $[X_{\kappa}^{\text{irr}}] = q^{18} - 4\,q^{17} + 5\,q^{16} - q^{15} - 3\,q^{14} + 3\,q^{13} - 5\,q^{12} + 7\,q^{11} - 2\,q^{10} + q^9 - 6\,q^7 + 4\,q^6,$ $[X_{\kappa}] = q^{18} + 4q^{17} + 9q^{16} + 15q^{15} + 19q^{14} + 19q^{13} + 15q^{12} + 9q^{11} + 4q^{10} + q^9,$ $[\mathfrak{M}_{\kappa}] = q^3 - 4q^2 + 6q - 4.$

2. Configuration $\epsilon=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2)$ and $\epsilon=(\varepsilon_1,\varepsilon_1,\varepsilon_2,\varepsilon_2)$

$$\begin{array}{l} \textbf{Total count of } \kappa = ((\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_2), (\varepsilon_1, \varepsilon_1, \varepsilon_2, \varepsilon_2)) \\ [X_\kappa^{\rm red}] = q^{16} + 2\,q^{15} + 5\,q^{14} + 6\,q^{13} + 8\,q^{12} + 6\,q^{11} + 5\,q^{10} + 2\,q^9 + q^8, \\ [X_\kappa^{\rm irr}] = 0, \\ [X_\kappa] = q^{16} + 2\,q^{15} + 5\,q^{14} + 6\,q^{13} + 8\,q^{12} + 6\,q^{11} + 5\,q^{10} + 2\,q^9 + q^8, \\ [\mathfrak{M}_\kappa] = 0. \end{array}$$

3. Configuration $\epsilon=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2)$ and $\epsilon=(\varepsilon_1,\varepsilon_1,\varepsilon_2,\varepsilon_3)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$. $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3$.
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 2$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{16} 3q^{15} + 3q^{13} + 2q^{12} + 3q^{11} 4q^{10} 3q^9 3q^8 + 4q^6$
- $m_{\kappa}(\tau) = 1$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_2, \epsilon_3, \epsilon_1, \epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 1$.

$$\xi = ((1,2),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{10} + 2q^9 + 3q^8 + 3q^7 + 2q^6 + q^5$
- $m_{\kappa}(\tau) = 2$.

$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^2 q.$
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $\bullet \ [X(\tau)] = q^{17} 3q^{16} + 2q^{15} + 2q^{14} q^{13} + q^{12} 5q^{11} + q^{10} + 2q^8 + 3q^7 3q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_2, \epsilon_1, \epsilon_3, \epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{15} q^{14} 2q^{13} q^{12} + 3q^{10} + 2q^9 + q^8 q^7 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^3 q.$
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ [X(\tau)] = q^{15} q^{14} 2\,q^{13} q^{12} + 3\,q^{10} + 2\,q^9 + q^8 q^7 2\,q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_2), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 q$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{12} + 2q^{11} + 2q^{10} + q^9 q^8 2q^7 2q^6 q^5$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ \ [X(\tau)] = q^{13} + 3\,q^{12} + 4\,q^{11} + 3\,q^{10} 3\,q^8 4\,q^7 3\,q^6 q^5$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{13} + 2q^{12} + 2q^{11} + q^{10} q^9 2q^8 2q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} 3q^9 4q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_2), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ [X(\tau)] = q^{14} + 2\,q^{13} + q^{12} q^{11} 3\,q^{10} 3\,q^9 q^8 + q^7 + 2\,q^6 + q^5$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1), (1,1), (1,2)), \quad \sigma_A = (\epsilon_1, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_2, \epsilon_3, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^4 q^3 q^2 + q$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^2 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $\bullet \ [X(\tau)] = q^{17} 4q^{16} + 3q^{15} + 3q^{14} q^{13} + q^{12} 7q^{11} + q^{10} + 3q^8 + 4q^7 4q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{15} q^{14} 2q^{13} q^{12} + 3q^{10} + 2q^9 + q^8 q^7 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_3,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = q^4 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 1$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{13} + q^{12} q^{10} 2q^9 q^8 + q^6 + q^5$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_3,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q^2 q)^2.$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)(q - 1)q$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2$$
.

$$\bullet \ [X(\tau)] = q^{16} - 2\,q^{15} - q^{14} + q^{13} + q^{12} + 3\,q^{11} - q^{10} - q^9 - 2\,q^8 - q^7 + 2\,q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

•
$$[\mathcal{M}_{\tau}] = (q-1)^2 q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{14} + q^{13} - q^{11} - 2q^{10} - q^9 + q^7 + q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)(q - 1)q$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$$

•
$$[X(\tau)] = q^{14} + q^{13} - q^{11} - 2q^{10} - q^9 + q^7 + q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_1, \epsilon_3, \epsilon_2, \epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)(q - 1)q$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{14} + q^{13} - q^{11} - 2q^{10} - q^9 + q^7 + q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_3,\epsilon_2).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)(q - 1)q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1.$$

•
$$[X(\tau)] = q^{15} + q^{14} - q^{12} - 2q^{11} - q^{10} + q^8 + q^7$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

$$\bullet \ [\mathcal{M}_{\tau}] = q^2 - q.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^2 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q^2 - 3q + 3.$$

$$\bullet [X(\tau)] = q^{17} - 3q^{16} + 2q^{15} + 2q^{14} - q^{13} + q^{12} - 5q^{11} + q^{10} + 2q^8 + 3q^7 - 3q^6$$

•
$$m_{\kappa}(\tau) = 2$$
.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + q^{14} q^{13} 2q^{12} 2q^{11} + 2q^9 + 2q^8 + q^7 q^6 q^5$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

Total count of $\kappa = ((\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_2), (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3))$

$$\begin{split} [X_\kappa^{\rm red}] &= 6\,q^{17} + 3\,q^{16} + 13\,q^{15} + 17\,q^{14} + 11\,q^{13} + 14\,q^{12} + 5\,q^{10} + q^9 + q^8 + 5\,q^7 - 4\,q^6, \\ [X_\kappa^{\rm irr}] &= q^{18} - 3\,q^{17} + 4\,q^{16} - 2\,q^{15} - 3\,q^{14} + 3\,q^{13} - 3\,q^{12} + 7\,q^{11} - 2\,q^{10} - q^8 - 5\,q^7 + 4\,q^6, \\ [X_\kappa] &= q^{18} + 3\,q^{17} + 7\,q^{16} + 11\,q^{15} + 14\,q^{14} + 14\,q^{13} + 11\,q^{12} + 7\,q^{11} + 3\,q^{10} + q^9, \\ [\mathfrak{M}_\kappa] &= q^3 - 3\,q^2 + 5\,q - 4. \end{split}$$

4. Configuration $\epsilon=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2)$ and $\boldsymbol{\varepsilon}=(\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$. $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3$.
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 8$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{16} 3q^{15} + 3q^{13} + 2q^{12} + 3q^{11} 4q^{10} 3q^9 3q^8 + 4q^6$
- $m_{\kappa}(\tau) = 3$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_1, \epsilon_3, \epsilon_2, \epsilon_4).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 q$.
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^2 q.$
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{18} 2q^{17} q^{16} + 4q^{15} + q^{14} 4q^{12} 4q^{11} + q^{10} + 2q^{9} + 5q^{8} 3q^{6}$
- $m_{\kappa}(\tau) = 8$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

$$\bullet \ [\mathcal{M}_{\tau}] = q^3 - q.$$

$$\bullet \ [\mathcal{G}_{\tau}] = (q-1)^4 q.$$

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} - 3q^9 - 4q^8 - 3q^7 - q^6$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$$

•
$$[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} - 4q^{12} - 6q^{11} - 4q^{10} + 3q^8 + 3q^7 + q^6$$

•
$$m_{\kappa}(\tau) = 12$$
.

 $\xi=((2,1),(2,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2),\quad \sigma_B=(\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = q^4 - q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^2 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{\text{irr}}] = q^2 - 4q + 4.$$

$$\bullet \ \ [X(\tau)] = q^{18} - 3\,q^{17} - q^{16} + 6\,q^{15} + 2\,q^{14} - 6\,q^{12} - 6\,q^{11} + q^{10} + 3\,q^9 + 7\,q^8 - 4\,q^6$$

•
$$m_{\kappa}(\tau) = 6$$
.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2), \quad \sigma_B = (\epsilon_1, \epsilon_3, \epsilon_2, \epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q - 2$$
.

•
$$[X(\tau)] = q^{16} - 2q^{15} - q^{14} + q^{13} + q^{12} + 3q^{11} - q^{10} - q^9 - 2q^8 - q^7 + 2q^6$$

•
$$m_{\kappa}(\tau) = 12$$
.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = q^4 - q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q - 2$$
.

•
$$[X(\tau)] = q^{16} - 3q^{14} - 3q^{13} - q^{12} + 3q^{11} + 5q^{10} + 3q^9 - 3q^7 - 2q^6$$

•
$$m_{\kappa}(\tau) = 12$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

$$\bullet \ [\mathcal{M}_{\tau}] = (q^2 - q)^2.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q^2$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{14} + q^{13} - q^{11} - 2q^{10} - q^9 + q^7 + q^6$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2),\quad \sigma_B=(\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + q^{14} q^{12} 2q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 q$.
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^2 q.$
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $\bullet \ [X(\tau)] = q^{18} 2\,q^{17} q^{16} + 4\,q^{15} + q^{14} 4\,q^{12} 4\,q^{11} + q^{10} + 2\,q^9 + 5\,q^8 3\,q^6$
- $m_{\kappa}(\tau) = 8$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1).$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^3 q.$
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet [X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + q^{15} q^{14} 2q^{13} 2q^{12} + 2q^{10} + 2q^9 + q^8 q^7 q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2),\quad \sigma_B=(\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q^2$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 48$.

Total count of $\kappa = ((\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_2), (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4))$

$$\begin{split} [X_\kappa^{\mathrm{red}}] &= 22\,q^{18} + 22\,q^{17} + q^{16} + 39\,q^{15} + 28\,q^{14} + 25\,q^{13} + 16\,q^{12} - 21\,q^{11} + 4\,q^{10} - 11\,q^9 + 19\,q^8 + 18\,q^7 - 18\,q^6, \\ [X_\kappa^{\mathrm{irr}}] &= q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^9 - 19\,q^8 - 18\,q^7 + 18\,q^6, \\ [X_\kappa] &= q^{20} + 4\,q^{19} + 10\,q^{18} + 18\,q^{17} + 25\,q^{16} + 28\,q^{15} + 25\,q^{14} + 18\,q^{13} + 10\,q^{12} + 4\,q^{11} + q^{10}, \\ [\mathfrak{M}_\kappa] &= q^5 + 4\,q^4 - 11\,q^3 + q^2 + 18\,q - 18. \end{split}$$

5. Configuration $\epsilon=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)$ and $\epsilon=(\varepsilon_1,\varepsilon_1,\varepsilon_2,\varepsilon_2)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$. $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3$.
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 2$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{16} 3q^{15} + 3q^{13} + 2q^{12} + 3q^{11} 4q^{10} 3q^9 3q^8 + 4q^6$
- $m_{\kappa}(\tau) = 1$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 1$.

$$\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_2).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{10} + 2q^9 + 3q^8 + 3q^7 + 2q^6 + q^5$
- $m_{\kappa}(\tau) = 2$.

$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2).$

•
$$[\mathcal{M}_{\tau}] = q^2 - 1$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2$$
.

•
$$[X(\tau)] = q^{16} - 3q^{14} - 3q^{13} - q^{12} + 3q^{11} + 5q^{10} + 3q^9 - 3q^7 - 2q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

$$\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_2).$$

•
$$[\mathcal{M}_{\tau}] = q^2 - 1.$$

•
$$[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{12} + 2q^{11} + 2q^{10} + q^9 - q^8 - 2q^7 - 2q^6 - q^5$$

•
$$m_{\kappa}(\tau) = 4$$
.

$$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

•
$$[\mathcal{M}_{\tau}] = q^2 - 1$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} - 3q^9 - 4q^8 - 3q^7 - q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

$$\bullet \ [\mathcal{M}_{\tau}] = q^2 - q.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^2 q$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^2 q$$
.
• $[\mathfrak{M}_{\tau}^{\text{irr}}] = q^2 - 3 q + 3$.

•
$$[X(\tau)] = q^{17} - 3q^{16} + 2q^{15} + 2q^{14} - q^{13} + q^{12} - 5q^{11} + q^{10} + 2q^8 + 3q^7 - 3q^6$$

•
$$m_{\kappa}(\tau) = 2$$
.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

$$\bullet \ [\mathcal{M}_{\tau}] = q^2 - q.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2.$$

•
$$[X(\tau)] = q^{15} - q^{14} - 2q^{13} - q^{12} + 3q^{10} + 2q^9 + q^8 - q^7 - 2q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

$$\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$$

$$\bullet \ [\mathcal{M}_{\tau}] = q^2 - q.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q - 2$$
.

•
$$[X(\tau)] = q^{15} - q^{14} - 2q^{13} - q^{12} + 3q^{10} + 2q^9 + q^8 - q^7 - 2q^6$$

•
$$m_{\kappa}(\tau) = 2$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{13} + 3q^{12} + 4q^{11} + 3q^{10} 3q^8 4q^7 3q^6 q^5$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = q^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{13} + 2q^{12} + 2q^{11} + q^{10} q^9 2q^8 2q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1), (1,1), (2,1)), \quad \sigma_A = (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_2, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $\bullet \ \ [X(\tau)] = q^{16} 3\,q^{14} 3\,q^{13} q^{12} + 3\,q^{11} + 5\,q^{10} + 3\,q^9 3\,q^7 2\,q^6$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + 2q^{13} + q^{12} q^{11} 3q^{10} 3q^9 q^8 + q^7 + 2q^6 + q^5$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^2 q.$
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4.$
- $[X(\tau)] = q^{17} 4q^{16} + 3q^{15} + 3q^{14} q^{13} + q^{12} 7q^{11} + q^{10} + 3q^{8} + 4q^{7} 4q^{6}$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^3 q.$
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{15} q^{14} 2q^{13} q^{12} + 3q^{10} + 2q^9 + q^8 q^7 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{13} + q^{12} q^{10} 2q^9 q^8 + q^6 + q^5$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,1), (1,1), (1,2)), \quad \sigma_A = (\epsilon_2, \epsilon_3, \epsilon_1), \quad \sigma_B = (\epsilon_2, \epsilon_2, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^4 q^3 q^2 + q$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + q^{14} q^{12} 2q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ \ [X(\tau)] = q^{16} 2\,q^{15} q^{14} + q^{13} + q^{12} + 3\,q^{11} q^{10} q^9 2\,q^8 q^7 + 2\,q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_1), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_2, \epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q-1)^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q-1)^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^2 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{17} 3q^{16} + 2q^{15} + 2q^{14} q^{13} + q^{12} 5q^{11} + q^{10} + 2q^{8} + 3q^{7} 3q^{6}$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_2,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = (q-1)^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2$$
.

•
$$[X(\tau)] = q^{16} - 2q^{15} - q^{14} + q^{13} + q^{12} + 3q^{11} - q^{10} - q^9 - 2q^8 - q^7 + 2q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

•
$$[\mathcal{M}_{\tau}] = (q-1)^2 q$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q - 2$$
.

•
$$[X(\tau)] = q^{16} - 2q^{15} - q^{14} + q^{13} + q^{12} + 3q^{11} - q^{10} - q^9 - 2q^8 - q^7 + 2q^6$$

•
$$m_{\kappa}(\tau) = 2$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)(q - 1)^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{15} + q^{14} - q^{13} - 2q^{12} - 2q^{11} + 2q^9 + 2q^8 + q^7 - q^6 - q^5$$

•
$$m_{\kappa}(\tau) = 2$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_2).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)(q - 1)^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{15} - q^{13} - q^{12} - q^{11} + q^{10} + q^9 + q^8 - q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

•
$$[\mathcal{M}_{\tau}] = (q-1)^2 q$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2$$
.

$$\bullet \ [X(\tau)] = q^{16} - 2q^{15} - q^{14} + q^{13} + q^{12} + 3q^{11} - q^{10} - q^9 - 2q^8 - q^7 + 2q^6$$

• $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_2).$

$$\bullet \ [\mathcal{M}_{\tau}] = (q-1)^3 q.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{15} - q^{13} - q^{12} - q^{11} + q^{10} + q^9 + q^8 - q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_2,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q-1)^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

Total count of $\kappa = ((\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3), (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_2))$

$$\begin{split} [X_\kappa^{\rm red}] &= 6\,q^{17} + 3\,q^{16} + 13\,q^{15} + 17\,q^{14} + 11\,q^{13} + 14\,q^{12} + 5\,q^{10} + q^9 + q^8 + 5\,q^7 - 4\,q^6, \\ [X_\kappa^{\rm irr}] &= q^{18} - 3\,q^{17} + 4\,q^{16} - 2\,q^{15} - 3\,q^{14} + 3\,q^{13} - 3\,q^{12} + 7\,q^{11} - 2\,q^{10} - q^8 - 5\,q^7 + 4\,q^6, \\ [X_\kappa] &= q^{18} + 3\,q^{17} + 7\,q^{16} + 11\,q^{15} + 14\,q^{14} + 14\,q^{13} + 11\,q^{12} + 7\,q^{11} + 3\,q^{10} + q^9, \\ [\mathfrak{M}_\kappa] &= q^3 - 3\,q^2 + 5\,q - 4. \end{split}$$

6. Configuration $\epsilon=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)$ and $\epsilon=(\varepsilon_1,\varepsilon_1,\varepsilon_2,\varepsilon_3)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 2$.

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^4 + 4q^3 9q^2 3q + 12.$
- $\bullet \ \ [X(\tau)] = q^{18} + 5\,q^{17} 5\,q^{16} 13\,q^{15} + 3\,q^{14} + 12\,q^{13} + 17\,q^{12} + 4\,q^{11} 16\,q^{10} 10\,q^{10} + 10\,q^{10$ $17q^9 - 12q^8 + 9q^7 + 12q^6$
- $m_{\kappa}(\tau) = 1$.

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{\text{irr}}] = q^2 3q + 3.$ $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 2$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{16} 3q^{15} + 3q^{13} + 2q^{12} + 3q^{11} 4q^{10} 3q^9 3q^8 + 4q^6$
- $m_{\kappa}(\tau) = 2$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 2$.

$$\xi = ((1,2),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^3 q$.

- $m_{\kappa}(\tau) = 1$.

$$\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 2$.

$$\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $\bullet \ [X(\tau)] = q^{10} + 2\,q^9 + 3\,q^8 + 3\,q^7 + 2\,q^6 + q^5$
- $m_{\kappa}(\tau) = 2$.

$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 1$.

$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = q^2 - 1$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^2$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q^2 - 3q + 3.$$

$$\bullet \ [X(\tau)] = q^{18} - 2q^{17} - q^{16} + 4q^{15} + q^{14} - 4q^{12} - 4q^{11} + q^{10} + 2q^9 + 5q^8 - 3q^6$$

•
$$m_{\kappa}(\tau) = 2$$
.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3).$

•
$$[\mathcal{M}_{\tau}] = q^3 - 1$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2.$$

$$\bullet \ [X(\tau)] = q^{17} - 2\,q^{15} - 4\,q^{14} - 3\,q^{13} + 2\,q^{12} + 5\,q^{11} + 6\,q^{10} + 2\,q^9 - 2\,q^8 - 3\,q^7 - 2\,q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_2, \epsilon_1, \epsilon_3, \epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = q^2 - 1.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^3$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2$$
.

•
$$[X(\tau)] = q^{16} - 3q^{14} - 3q^{13} - q^{12} + 3q^{11} + 5q^{10} + 3q^9 - 3q^7 - 2q^6$$

•
$$m_{\kappa}(\tau) = 4$$
.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = q^2 - 1$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2$$
.

•
$$[X(\tau)] = q^{16} - 3q^{14} - 3q^{13} - q^{12} + 3q^{11} + 5q^{10} + 3q^9 - 3q^7 - 2q^6$$

•
$$m_{\kappa}(\tau) = 2$$
.

 $\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3).$

•
$$[\mathcal{M}_{\tau}] = q^3 - 1$$
.

•
$$[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{13} + 2q^{12} + 3q^{11} + 2q^{10} - 2q^8 - 3q^7 - 2q^6 - q^5$$

• $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = q^2 - 1$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$$

•
$$[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} - 3q^9 - 4q^8 - 3q^7 - q^6$$

•
$$m_{\kappa}(\tau) = 2$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} 3q^9 4q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + 3q^{14} + 5q^{13} + 5q^{12} + 2q^{11} 2q^{10} 5q^9 5q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^2 q.$
- $\left[\mathfrak{M}_{\tau}^{\text{irr}}\right] = q^4 + 4q^3 9q^2 3q + 12.$
- $[X(\tau)] = q^{19} + 4q^{18} 10q^{17} 8q^{16} + 16q^{15} + 9q^{14} + 5q^{13} 13q^{12} 20q^{11} q^{10} + 5q^9 + 21q^8 + 3q^7 12q^6$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^2 q$.
- $[\mathfrak{M}_{\tau}^{\text{irr}}] = q^2 3q + 3.$
- $\bullet \ [X(\tau)] = q^{18} 2\,q^{17} q^{16} + 4\,q^{15} + q^{14} 4\,q^{12} 4\,q^{11} + q^{10} + 2\,q^9 + 5\,q^8 3\,q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{15} q^{14} 2q^{13} q^{12} + 3q^{10} + 2q^9 + q^8 q^7 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^3 q.$
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{15} q^{14} 2q^{13} q^{12} + 3q^{10} + 2q^9 + q^8 q^7 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $[X(\tau)] = q^{15} 3q^{13} 3q^{12} q^{11} + 3q^{10} + 5q^9 + 3q^8 3q^6 2q^5$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_1,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^3 q.$
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{15} q^{14} 2q^{13} q^{12} + 3q^{10} + 2q^9 + q^8 q^7 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ [X(\tau)] = q^{13} + 3 \, q^{12} + 4 \, q^{11} + 3 \, q^{10} 3 \, q^8 4 \, q^7 3 \, q^6 q^5$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_1), \quad \sigma_B = (\epsilon_2, \epsilon_1, \epsilon_1, \epsilon_3).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} 3q^9 4q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_1,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} 3q^9 4q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{13} + 2q^{12} + 2q^{11} + q^{10} q^9 2q^8 2q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((1,2),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = q^4 1$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{16} q^{15} q^{14} 2q^{13} 2q^{12} + 2q^{11} + 2q^{10} + 4q^9 + q^8 q^7 q^6 2q^5$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^3 1.$
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $\bullet \ \ [X(\tau)] = q^{17} 2\,q^{15} 4\,q^{14} 3\,q^{13} + 2\,q^{12} + 5\,q^{11} + 6\,q^{10} + 2\,q^9 2\,q^8 3\,q^7 2\,q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + 2q^{13} + q^{12} q^{11} 3q^{10} 3q^9 q^8 + q^7 + 2q^6 + q^5$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_2, \epsilon_3, \epsilon_1, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} 4q^{12} 6q^{11} 4q^{10} + 3q^{8} + 3q^{7} + q^{6}$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^2 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{18} 3q^{17} q^{16} + 6q^{15} + 2q^{14} 6q^{12} 6q^{11} + q^{10} + 3q^9 + 7q^8 4q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi=((2,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3),\quad \sigma_B=(\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1).$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^3 q.$
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_2, \epsilon_3, \epsilon_1, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- \bullet $[\mathfrak{M}^{irr}] = a 2$
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_3,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^4 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ \ [X(\tau)] = q^{17} 2\,q^{15} 4\,q^{14} 3\,q^{13} + 2\,q^{12} + 5\,q^{11} + 6\,q^{10} + 2\,q^9 2\,q^8 3\,q^7 2\,q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + 2q^{13} + q^{12} q^{11} 3q^{10} 3q^9 q^8 + q^7 + 2q^6 + q^5$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1).$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^4 q.$
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_3,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ [X(\tau)] = q^{15} + 2\,q^{14} + q^{13} q^{12} 3\,q^{11} 3\,q^{10} q^9 + q^8 + 2\,q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ \ [X(\tau)] = q^{17} q^{16} 3\,q^{15} + 2\,q^{13} + 4\,q^{12} + 2\,q^{11} 2\,q^{10} 3\,q^9 3\,q^8 + q^7 + 2\,q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1), (1,1), (1,2)), \quad \sigma_A = (\epsilon_2, \epsilon_3, \epsilon_1), \quad \sigma_B = (\epsilon_2, \epsilon_3, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^4 q^3 q^2 + q$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} 4q^{12} 6q^{11} 4q^{10} + 3q^{8} + 3q^{7} + q^{6}$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,2), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q+1)(q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + q^{14} q^{12} 2q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + 2q^{15} + q^{14} q^{13} 3q^{12} 3q^{11} q^{10} + q^9 + 2q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + q^{15} q^{13} 2q^{12} q^{11} + q^9 + q^8$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_2, \epsilon_1, \epsilon_3, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + q^{14} q^{12} 2q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + q^{14} q^{12} 2q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ \ [X(\tau)] = q^{15} + q^{14} q^{12} 2 \, q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3),\quad \sigma_B=(\epsilon_1,\epsilon_1,\epsilon_3,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} + q^{15} q^{13} 2q^{12} q^{11} + q^9 + q^8$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $\bullet \ [X(\tau)] = q^{17} q^{16} 3\,q^{15} + 2\,q^{13} + 4\,q^{12} + 2\,q^{11} 2\,q^{10} 3\,q^9 3\,q^8 + q^7 + 2\,q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_1), \quad \sigma_B = (\epsilon_2, \epsilon_1, \epsilon_1, \epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + q^{14} q^{12} 2q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1),\quad \sigma_B=(\epsilon_1,\epsilon_3,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + q^{14} q^{12} 2q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $\bullet \ [X(\tau)] = q^{15} + 2\,q^{14} + q^{13} q^{12} 3\,q^{11} 3\,q^{10} q^9 + q^8 + 2\,q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $\bullet \ [X(\tau)] = q^{16} + 2q^{15} + q^{14} q^{13} 3q^{12} 3q^{11} q^{10} + q^9 + 2q^8 + q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^2 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^2 q$.
- $[\mathfrak{M}_{\tau}^{\text{irr}}] = q^4 + 4q^3 9q^2 3q + 12.$ $[X(\tau)] = q^{19} + 4q^{18} 10q^{17} 8q^{16} + 16q^{15} + 9q^{14} + 5q^{13} 13q^{12} 20q^{11} 10q^{15} + 10q^$ $q^{10} + 5q^9 + 21q^8 + 3q^7 - 12q^6$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^2 q.$
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $\bullet \ [X(\tau)] = q^{18} 2\,q^{17} q^{16} + 4\,q^{15} + q^{14} 4\,q^{12} 4\,q^{11} + q^{10} + 2\,q^9 + 5\,q^8 3\,q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $\bullet \ \ [X(\tau)] = q^{18} 2\,q^{17} q^{16} + 4\,q^{15} + q^{14} 4\,q^{12} 4\,q^{11} + q^{10} + 2\,q^9 + 5\,q^8 3\,q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_2, \epsilon_1, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_3, \epsilon_1, \epsilon_2, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{17} 4q^{15} 3q^{14} + 2q^{13} + 6q^{12} + 6q^{11} 5q^9 6q^8 2q^7 + 3q^6 + 2q^5$
- $m_{\kappa}(\tau) = 1$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\pi}^{irr}] = q 2$
- $[X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + q^{14} q^{13} 2q^{12} 2q^{11} + 2q^9 + 2q^8 + q^7 q^6 q^5$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2 q$.
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^4 q.$
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + q^{15} q^{14} 2q^{13} 2q^{12} + 2q^{10} + 2q^9 + q^8 q^7 q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} + q^{15} q^{14} 2q^{13} 2q^{12} + 2q^{10} + 2q^9 + q^8 q^7 q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_2,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q-1)^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi=((1,1),(2,1),(1,1)),\quad \sigma_A=(\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 2$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1),\quad \sigma_B=(\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q-1)^3 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^4 q.$
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\overline{a}}^{irr}] = q 2$
- $\bullet [X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1), (1,1), (2,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_2, \epsilon_1, \epsilon_3, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q-1)^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3),\quad \sigma_B=(\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2.$
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q-1)^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ \ \dot{[X(\tau)]} = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $\bullet \ [X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ \ [X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 4$.

Total count of $\kappa = ((\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3), (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3))$

$$\begin{split} [X_\kappa^{\mathrm{red}}] &= 2\,q^{19} + 21\,q^{18} + 14\,q^{17} + 8\,q^{16} + 41\,q^{15} + 29\,q^{14} + 27\,q^{13} + 6\,q^{12} - 17\,q^{11} + q^{10} - 5\,q^9 + 22\,q^8 + 10\,q^7 - 15\,q^6, \\ [X_\kappa^{\mathrm{irr}}] &= q^{20} + 2\,q^{19} - 11\,q^{18} + 4\,q^{17} + 17\,q^{16} - 13\,q^{15} - 4\,q^{14} - 9\,q^{13} + 4\,q^{12} + 21\,q^{11} + 5\,q^9 - 22\,q^8 - 10\,q^7 + 15\,q^6, \\ [X_\kappa] &= q^{20} + 4\,q^{19} + 10\,q^{18} + 18\,q^{17} + 25\,q^{16} + 28\,q^{15} + 25\,q^{14} + 18\,q^{13} + 10\,q^{12} + 4\,q^{11} + q^{10}, \end{split}$$

 $[\mathfrak{M}_{\kappa}] = q^5 + 2q^4 - 10q^3 + 7q^2 + 10q - 15.$

7. Configuration $\epsilon=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)$ and $\boldsymbol{\varepsilon}=(\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 8$.

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $\left[\mathfrak{M}_{\tau}^{\text{irr}}\right] = q^4 + 4q^3 9q^2 3q + 12.$
- $[X(\tau)] = q^{18} + 5q^{17} 5q^{16} 13q^{15} + 3q^{14} + 12q^{13} + 17q^{12} + 4q^{11} 16q^{10} 17q^9 12q^8 + 9q^7 + 12q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{16} 3q^{15} + 3q^{13} + 2q^{12} + 3q^{11} 4q^{10} 3q^9 3q^8 + 4q^6$
- $m_{\kappa}(\tau) = 6$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 24$.

$$\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 6$.

$$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $\bullet \ [X(\tau)] = q^{19} 2q^{18} + q^{16} + 3q^{15} + 2q^{14} 5q^{13} 3q^{12} 4q^{11} + 3q^{10} + 5q^{9} + 2q^{8} 3q^{6}$
- $m_{\kappa}(\tau) = 8$.

 $\xi=((2,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3),\quad \sigma_B=(\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{17} 2q^{15} 4q^{14} 3q^{13} + 2q^{12} + 5q^{11} + 6q^{10} + 2q^9 2q^8 3q^7 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 1.$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ \ [X(\tau)] = q^{15} + 3\,q^{14} + 5\,q^{13} + 5\,q^{12} + 2\,q^{11} 2\,q^{10} 5\,q^9 5\,q^8 3\,q^7 q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^2 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^4 + 4q^3 9q^2 3q + 12.$
- $\bullet \ [X(\tau)] = q^{20} + 5\,q^{19} 6\,q^{18} 18\,q^{17} + 8\,q^{16} + 25\,q^{15} + 14\,q^{14} 8\,q^{13} 33\,q^{12} 18\,q^{14} + 14\,q^{14} 8\,q^{13} 18\,q^{14} 21q^{11} + 4q^{10} + 26q^9 + 24q^8 - 9q^7 - 12q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = q^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

$$\bullet \ [\mathcal{M}_{\tau}] = q^3 - q.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$$

•
$$[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} - 3q^9 - 4q^8 - 3q^7 - q^6$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = q^4 - 1.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^3$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2$$
.

$$\bullet \ [X(\tau)] = q^{18} - 2\,q^{16} - 3\,q^{15} - 4\,q^{14} + 4\,q^{12} + 6\,q^{11} + 5\,q^{10} - 2\,q^8 - 3\,q^7 - 2\,q^6$$

•
$$m_{\kappa}(\tau) = 6$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)^2$$
.

$$\bullet \ [\mathcal{G}_{\tau}] = (q-1)^4.$$

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} - 4q^{12} - 6q^{11} - 4q^{10} + 3q^8 + 3q^7 + q^6$$

•
$$m_{\kappa}(\tau) = 12$$
.

 $\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

$$\bullet \ [\mathcal{M}_{\tau}] = q^4 - q.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^2 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q^2 - 4q + 4$$

•
$$[\mathfrak{M}_{\tau}^{\text{irr}}] = q^2 - 4q + 4$$
.
• $[X(\tau)] = q^{19} - 3q^{18} + 2q^{16} + 5q^{15} + 3q^{14} - 7q^{13} - 5q^{12} - 6q^{11} + 4q^{10} + 7q^9 + 3q^8 - 4q^6$

•
$$m_{\kappa}(\tau) = 12$$
.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_1, \epsilon_3, \epsilon_2, \epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)(q^2 - 1).$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q - 2$$
.

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

$$\bullet \ [\mathcal{M}_{\tau}] = q^4 - q.$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = q - 2$$
.

$$\bullet \ [X(\tau)] = q^{17} - 2q^{15} - 4q^{14} - 3q^{13} + 2q^{12} + 5q^{11} + 6q^{10} + 2q^9 - 2q^8 - 3q^7 - 2q^6$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1).$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^4 q.$
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_4,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{18} 4q^{16} 3q^{15} + 2q^{14} + 6q^{13} + 6q^{12} 5q^{10} 6q^9 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_4,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} 4q^{12} 6q^{11} 4q^{10} + 3q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $\bullet \ [X(\tau)] = q^{17} + 2q^{16} + q^{15} q^{14} 3q^{13} 3q^{12} q^{11} + q^{10} + 2q^9 + q^8$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{18} 2\,q^{17} q^{16} + q^{15} + q^{14} + 3\,q^{13} q^{12} q^{11} 2\,q^{10} q^9 + 2\,q^8$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} + q^{15} q^{13} 2q^{12} q^{11} + q^9 + q^8$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)(q - 1)q^2$$
.

- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{18} q^{17} 3q^{16} + 2q^{14} + 4q^{13} + 2q^{12} 2q^{11} 3q^{10} 3q^9 + q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ \ [X(\tau)] = q^{16} + 2\,q^{15} + q^{14} q^{13} 3\,q^{12} 3\,q^{11} q^{10} + q^9 + 2\,q^8 + q^7$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^2 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^4 + 4q^3 9q^2 3q + 12.$
- $[X(\tau)] = q^{20} + 5q^{19} 6q^{18} 18q^{17} + 8q^{16} + 25q^{15} + 14q^{14} 8q^{13} 33q^{12} 21q^{11} + 4q^{10} + 26q^9 + 24q^8 9q^7 12q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi=((1,1),(2,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{19} 2q^{18} + q^{16} + 3q^{15} + 2q^{14} 5q^{13} 3q^{12} 4q^{11} + 3q^{10} + 5q^9 + 2q^8 3q^6$
- $m_{\kappa}(\tau) = 8$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_4,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{17} q^{16} 3 \, q^{15} + 2 \, q^{13} + 4 \, q^{12} + 2 \, q^{11} 2 \, q^{10} 3 \, q^9 3 \, q^8 + q^7 + 2 \, q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_2,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q-1)^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi=((1,1),(2,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $\bullet \ [X(\tau)] = q^{18} q^{17} 3q^{16} + 2q^{14} + 4q^{13} + 2q^{12} 2q^{11} 3q^{10} 3q^9 + q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi=((1,1),(2,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1),\quad \sigma_B=(\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

- $\bullet \ [\mathcal{M}_{\tau}] = \left(q^2 1\right)^2 q.$
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^3 q.$
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ [X(\tau)] = q^{18} 4q^{16} 3q^{15} + 2q^{14} + 6q^{13} + 6q^{12} 5q^{10} 6q^9 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + q^{15} q^{14} 2q^{13} 2q^{12} + 2q^{10} + 2q^9 + q^8 q^7 q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_2, \epsilon_1, \epsilon_1, \epsilon_3), \quad \sigma_B = (\epsilon_3, \epsilon_1, \epsilon_2, \epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{17} + q^{16} q^{15} 2q^{14} 2q^{13} + 2q^{11} + 2q^{10} + q^9 q^8 q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_4,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 q)(q^2 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3 q$.
- $[\mathfrak{M}_{\overline{a}}^{irr}] = q 2$
- $\bullet \ [X(\tau)] = q^{18} q^{17} 3q^{16} + 2q^{14} + 4q^{13} + 2q^{12} 2q^{11} 3q^{10} 3q^9 + q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_4,\epsilon_2).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)(q - 1)^2 q$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{16} - q^{14} - q^{13} - q^{12} + q^{11} + q^{10} + q^9 - q^7$$

•
$$m_{\kappa}(\tau) = 48$$
.

 $\xi=((1,1),(1,1),(2,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3),\quad \sigma_B=(\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)(q - 1)q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2.$$

•
$$[X(\tau)] = q^{18} - q^{17} - 3q^{16} + 2q^{14} + 4q^{13} + 2q^{12} - 2q^{11} - 3q^{10} - 3q^9 + q^8 + 2q^7$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3),\quad \sigma_B=(\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = (q-1)^3 q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$$

•
$$[X(\tau)] = q^{16} - q^{14} - q^{13} - q^{12} + q^{11} + q^{10} + q^9 - q^7$$

•
$$m_{\kappa}(\tau) = 48$$
.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_2,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - q)(q - 1)q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3 q$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q - 2.$$

$$\bullet \ [X(\tau)] = q^{18} - 2\,q^{17} - q^{16} + q^{15} + q^{14} + 3\,q^{13} - q^{12} - q^{11} - 2\,q^{10} - q^9 + 2\,q^8$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1),\quad \sigma_B=(\epsilon_3,\epsilon_4,\epsilon_1,\epsilon_2).$

•
$$[\mathcal{M}_{\tau}] = (q-1)^3 q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$$

•
$$[X(\tau)] = q^{17} - q^{15} - q^{14} - q^{13} + q^{12} + q^{11} + q^{10} - q^{8}$$

• $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_3), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_2,\epsilon_4).$

•
$$[\mathcal{M}_{\tau}] = (q-1)^3 q^3$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4 q$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$$

•
$$[X(\tau)] = q^{17} - q^{15} - q^{14} - q^{13} + q^{12} + q^{11} + q^{10} - q^{8}$$

•
$$m_{\kappa}(\tau) = 48$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_4,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{17} q^{15} q^{14} q^{13} + q^{12} + q^{11} + q^{10} q^{8}$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1), \quad \sigma_B = (\epsilon_3,\epsilon_1,\epsilon_4,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4 q$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{17} q^{15} q^{14} q^{13} + q^{12} + q^{11} + q^{10} q^{8}$
- $m_{\kappa}(\tau) = 48$.

Total count of $\kappa = ((\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3), (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4))$

$$\begin{split} [X_\kappa^{\mathrm{red}}] &= 8\,q^{20} + 68\,q^{19} + 56\,q^{18} - 4\,q^{17} + 2\,q^{16} + 78\,q^{15} + 102\,q^{14} + 46\,q^{13} - 20\,q^{12} - 92\,q^{11} - 38\,q^{10} + 30\,q^9 + 82\,q^8 + 18\,q^7 - 48\,q^6, \\ [X_\kappa^{\mathrm{irr}}] &= q^{22} + 5\,q^{21} + 6\,q^{20} - 40\,q^{19} - 13\,q^{18} + 57\,q^{17} + 51\,q^{16} - 35\,q^{15} - 74\,q^{14} - 32\,q^{13} + 25\,q^{12} + 93\,q^{11} + 38\,q^{10} - 30\,q^9 - 82\,q^8 - 18\,q^7 + 48\,q^6, \\ [X_\kappa] &= q^{22} + 5\,q^{21} + 14\,q^{20} + 28\,q^{19} + 43\,q^{18} + 53\,q^{17} + 53\,q^{16} + 43\,q^{15} + 28\,q^{14} + 14\,q^{13} + 5\,q^{12} + q^{11}, \\ [\mathfrak{M}_\kappa] &= q^7 + 5\,q^6 + 7\,q^5 - 34\,q^4 + 34\,q^2 + 18\,q - 48. \end{split}$$

8. Configuration $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$ and $\epsilon = (\epsilon_1, \epsilon_1, \epsilon_1, \epsilon_2)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 6$.

$$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_1,\epsilon_2).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = q 1$.
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^2.$
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{17} 3q^{16} + 2q^{15} + 2q^{14} q^{13} + q^{12} 5q^{11} + q^{10} + 2q^{8} + 3q^{7} 3q^{6}$
- $m_{\kappa}(\tau) = 4$.

$$\xi=((2,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = q 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{15} q^{14} 2q^{13} q^{12} + 3q^{10} + 2q^9 + q^8 q^7 2q^6$
- $m_{\kappa}(\tau) = 12$.

$$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_1,\epsilon_2).$$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + 3q^{14} + 5q^{13} + 5q^{12} + 2q^{11} 2q^{10} 5q^9 5q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{13} + 2q^{12} + 2q^{11} + q^{10} q^9 2q^8 2q^7 q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_1, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $[X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),\quad \sigma_B=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + 2q^{14} + q^{13} q^{12} 3q^{11} 3q^{10} q^9 + q^8 + 2q^7 + q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = q 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $\bullet \ [X(\tau)] = q^{17} 3\,q^{16} + 2\,q^{15} + 2\,q^{14} q^{13} + q^{12} 5\,q^{11} + q^{10} + 2\,q^8 + 3\,q^7 3\,q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\pi}^{irr}] = 1$.
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^{3}.$
- $[\mathfrak{M}_{\tau}^{\text{fir}}] = q 2$. $[X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 12$.

$$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = (q-1)^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 12$.

Total count of $\kappa = ((\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), (\epsilon_1, \epsilon_1, \epsilon_1, \epsilon_2))$

$$\begin{split} [X_\kappa^{\mathrm{red}}] &= 8\,q^{17} + 4\,q^{16} + 16\,q^{15} + 22\,q^{14} + 16\,q^{13} + 20\,q^{12} + 2\,q^{11} + 6\,q^{10} + 6\,q^7 - 4\,q^6, \\ [X_\kappa^{\mathrm{irr}}] &= q^{18} - 4\,q^{17} + 5\,q^{16} - q^{15} - 3\,q^{14} + 3\,q^{13} - 5\,q^{12} + 7\,q^{11} - 2\,q^{10} + q^9 - 6\,q^7 + 4\,q^6, \\ [X_\kappa] &= q^{18} + 4\,q^{17} + 9\,q^{16} + 15\,q^{15} + 19\,q^{14} + 19\,q^{13} + 15\,q^{12} + 9\,q^{11} + 4\,q^{10} + q^9, \end{split}$$
 $[\mathfrak{M}_{\kappa}] = q^3 - 4q^2 + 6q - 4.$

9. Configuration $\epsilon=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4)$ and $\boldsymbol{\varepsilon}=(\varepsilon_1,\varepsilon_1,\varepsilon_2,\varepsilon_2)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$. $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3$.
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 8$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{16} 3q^{15} + 3q^{13} + 2q^{12} + 3q^{11} 4q^{10} 3q^9 3q^8 + 4q^6$
- $m_{\kappa}(\tau) = 3$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 12$.

$\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),\quad \sigma_B=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 6$.

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{18} 2q^{17} q^{16} + 4q^{15} + q^{14} 4q^{12} 4q^{11} + q^{10} + 2q^{9} + 5q^{8} 3q^{6}$
- $m_{\kappa}(\tau) = 8$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} 3q^9 4q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 24$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{18} 3q^{17} q^{16} + 6q^{15} + 2q^{14} 6q^{12} 6q^{11} + q^{10} + 3q^9 + 7q^8 4q^6$
- $m_{\kappa}(\tau) = 6$.

$$\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_2, \epsilon_1).$$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{16} 2\,q^{15} q^{14} + q^{13} + q^{12} + 3\,q^{11} q^{10} q^9 2\,q^8 q^7 + 2\,q^6$
- $m_{\kappa}(\tau) = 12$.

$$\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{16} 3\,q^{14} 3\,q^{13} q^{12} + 3\,q^{11} + 5\,q^{10} + 3\,q^9 3\,q^7 2\,q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),\quad \sigma_B=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} 4q^{12} 6q^{11} 4q^{10} + 3q^{8} + 3q^{7} + q^{6}$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{14} + q^{13} q^{11} 2q^{10} q^9 + q^7 + q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_2, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} + q^{14} q^{12} 2q^{11} q^{10} + q^8 + q^7$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$. $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3$.
- $[X(\tau)] = q^{18} 2q^{17} q^{16} + 4q^{15} + q^{14} 4q^{12} 4q^{11} + q^{10} + 2q^{9} + 5q^{8} 3q^{6}$
- $m_{\kappa}(\tau) = 8$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $\bullet \ [X(\tau)] = q^{16} 2\,q^{15} q^{14} + q^{13} + q^{12} + 3\,q^{11} q^{10} q^9 2\,q^8 q^7 + 2\,q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{17} 2q^{16} q^{15} + q^{14} + q^{13} + 3q^{12} q^{11} q^{10} 2q^9 q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + q^{15} q^{14} 2q^{13} 2q^{12} + 2q^{10} + 2q^9 + q^8 q^7 q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_2,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ [X(\tau)] = q^{17} 2\,q^{16} q^{15} + q^{14} + q^{13} + 3\,q^{12} q^{11} q^{10} 2\,q^9 q^8 + 2\,q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 48$.

Total count of $\kappa = ((\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_2))$

 $[X_{\kappa}^{\rm red}] = 22\,q^{18} + 22\,q^{17} + q^{16} + 39\,q^{15} + 28\,q^{14} + 25\,q^{13} + 16\,q^{12} - 21\,q^{11} + 4\,q^{10} 11q^9 + 19q^8 + 18q^7 - 18q^6$,
$$\begin{split} &[X_{\kappa}^{irr}] = q^{20} + 4\,q^{19} - 18\,q^{7} + 18\,q^{6} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^{9} - 19\,q^{8} - 18\,q^{7} + 18\,q^{6}, \\ &[X_{\kappa}] = q^{20} + 4\,q^{19} + 10\,q^{18} + 18\,q^{17} + 25\,q^{16} + 28\,q^{15} + 25\,q^{14} + 18\,q^{13} + 10\,q^{12} + 4\,q^{11} + q^{10}, \end{split}$$

 $[\mathfrak{M}_{\kappa}] = q^5 + 4q^4 - 11q^3 + q^2 + 18q - 18.$

10. Configuration $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$ and $\epsilon = (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{16} 2q^{15} + 2q^{13} + q^{12} + 2q^{11} 3q^{10} 2q^9 2q^8 + 3q^6$
- $m_{\kappa}(\tau) = 8$.

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{\text{irr}}] = q^4 + 4 q^3 9 q^2 3 q + 12.$
- $[X(\tau)] = q^{18} + 5q^{17} 5q^{16} 13q^{15} + 3q^{14} + 12q^{13} + 17q^{12} + 4q^{11} 16q^{10} 17q^9 12q^8 + 9q^7 + 12q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{16} 3q^{15} + 3q^{13} + 2q^{12} + 3q^{11} 4q^{10} 3q^9 3q^8 + 4q^6$
- $m_{\kappa}(\tau) = 6$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 6$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 24$.

$$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{12} + 3q^{11} + 5q^{10} + 6q^9 + 5q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $\bullet \ [X(\tau)] = q^{19} 2q^{18} + q^{16} + 3q^{15} + 2q^{14} 5q^{13} 3q^{12} 4q^{11} + 3q^{10} + 5q^{9} + 2q^{8} 3q^{6}$
- $m_{\kappa}(\tau) = 8$.

 $\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $\left[\mathfrak{M}_{\tau}^{\text{irr}}\right] = q^4 + 4q^3 9q^2 3q + 12.$
- $[X(\tau)] = q^{20} + 5q^{19} 6q^{18} 18q^{17} + 8q^{16} + 25q^{15} + 14q^{14} 8q^{13} 33q^{12} 18q^{15} + 14q^{14} 8q^{15} + 14q^{14} + 14$ $21q^{11} + 4q^{10} + 26q^9 + 24q^8 - 9q^7 - 12q^6$
- $m_{\kappa}(\tau) = 4$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1.$
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{16} 3q^{14} 3q^{13} q^{12} + 3q^{11} + 5q^{10} + 3q^9 3q^7 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $\bullet \ [X(\tau)] = q^{17} 2\,q^{15} 4\,q^{14} 3\,q^{13} + 2\,q^{12} + 5\,q^{11} + 6\,q^{10} + 2\,q^9 2\,q^8 3\,q^7 2\,q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{15} + 3q^{14} + 5q^{13} + 5q^{12} + 2q^{11} 2q^{10} 5q^9 5q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{14} + 3q^{13} + 4q^{12} + 3q^{11} 3q^9 4q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $\begin{array}{l} \bullet \ \ [\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q^2 4\,q + 4\,. \\ \bullet \ \ [X(\tau)] = q^{19} 3\,q^{18} + 2\,q^{16} + 5\,q^{15} + 3\,q^{14} 7\,q^{13} 5\,q^{12} 6\,q^{11} + 4\,q^{10} + 7\,q^9 + 3\,q^8 4\,q^6 \end{array}$
- $m_{\kappa}(\tau) = 12$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{18} 4q^{16} 3q^{15} + 2q^{14} + 6q^{13} + 6q^{12} 5q^{10} 6q^9 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ [X(\tau)] = q^{17} q^{16} 3\,q^{15} + 2\,q^{13} + 4\,q^{12} + 2\,q^{11} 2\,q^{10} 3\,q^9 3\,q^8 + q^7 + 2\,q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = q^4 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{18} 2q^{16} 3q^{15} 4q^{14} + 4q^{12} + 6q^{11} + 5q^{10} 2q^8 3q^7 2q^6$
- $m_{\kappa}(\tau) = 6$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ \ [X(\tau)] = q^{17} 2\,q^{15} 4\,q^{14} 3\,q^{13} + 2\,q^{12} + 5\,q^{11} + 6\,q^{10} + 2\,q^9 2\,q^8 3\,q^7 2\,q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)^2$$
.

$$\bullet \ [\mathcal{G}_{\tau}] = (q-1)^4.$$

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} - 4q^{12} - 6q^{11} - 4q^{10} + 3q^8 + 3q^7 + q^6$$

•
$$m_{\kappa}(\tau) = 12$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4$$
.

•
$$[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$$

•
$$[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} - 4q^{12} - 6q^{11} - 4q^{10} + 3q^{8} + 3q^{7} + q^{6}$$

•
$$m_{\kappa}(\tau) = 12$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)(q - 1).$$

•
$$[\mathcal{G}_{\tau}] = (q-1)^4$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{15} + 2q^{14} + q^{13} - q^{12} - 3q^{11} - 3q^{10} - q^9 + q^8 + 2q^7 + q^6$$

•
$$m_{\kappa}(\tau) = 48$$
.

 $\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_1).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)(q - 1)q$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^3$$
.

•
$$\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q - 2.$$

$$\bullet \ [X(\tau)] = q^{18} - q^{17} - 3q^{16} + 2q^{14} + 4q^{13} + 2q^{12} - 2q^{11} - 3q^{10} - 3q^9 + q^8 + 2q^7$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

•
$$[\mathcal{M}_{\tau}] = (q-1)^2 q^2$$
.

$$\bullet \ [\mathcal{G}_{\tau}] = (q-1)^{3}.$$

•
$$[\mathfrak{M}_{\tau}^{irr}] = q - 2$$
.

$$\bullet \ [X(\tau)] = q^{18} - 2\,q^{17} - q^{16} + q^{15} + q^{14} + 3\,q^{13} - q^{12} - q^{11} - 2\,q^{10} - q^9 + 2\,q^8$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

•
$$[\mathcal{M}_{\tau}] = (q^2 - 1)(q - 1)q^2$$
.

•
$$[\mathcal{G}_{\tau}] = (q-1)^4$$
.

•
$$[\mathfrak{M}_{\tau}^{irr}] = 1$$
.

•
$$[X(\tau)] = q^{17} + 2q^{16} + q^{15} - q^{14} - 3q^{13} - 3q^{12} - q^{11} + q^{10} + 2q^9 + q^8$$

•
$$m_{\kappa}(\tau) = 24$$
.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + 2q^{15} + q^{14} q^{13} 3q^{12} 3q^{11} q^{10} + q^9 + 2q^8 + q^7$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_2, \epsilon_1, \epsilon_1, \epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} + q^{15} q^{13} 2q^{12} q^{11} + q^9 + q^8$
- $m_{\kappa}(\tau) = 48$.

$$\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = q^2 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{\text{irr}}] = q^4 + 4q^3 9q^2 3q + 12.$
- $[X(\tau)] = q^{20} + 5q^{19} 6q^{18} 18q^{17} + 8q^{16} + 25q^{15} + 14q^{14} 8q^{13} 33q^{12} 21q^{11} + 4q^{10} + 26q^9 + 24q^8 9q^7 12q^6$
- $m_{\kappa}(\tau) = 4$.

$$\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = q^3 1.$
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^2 3q + 3.$
- $[X(\tau)] = q^{19} 2q^{18} + q^{16} + 3q^{15} + 2q^{14} 5q^{13} 3q^{12} 4q^{11} + 3q^{10} + 5q^9 + 2q^8 3q^6$
- $m_{\kappa}(\tau) = 8$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ [X(\tau)] = q^{16} 2q^{15} q^{14} + q^{13} + q^{12} + 3q^{11} q^{10} q^9 2q^8 q^7 + 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\overline{a}}^{irr}] = q 2$
- $\bullet [X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 24$.

$$\xi=((1,1),(2,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),\quad \sigma_B=(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3).$$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{18} q^{17} 3q^{16} + 2q^{14} + 4q^{13} + 2q^{12} 2q^{11} 3q^{10} 3q^9 + q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

$$\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $\bullet \ [X(\tau)] = q^{18} q^{17} 3q^{16} + 2q^{14} + 4q^{13} + 2q^{12} 2q^{11} 3q^{10} 3q^9 + q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

$$\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{18} 4\,q^{16} 3\,q^{15} + 2\,q^{14} + 6\,q^{13} + 6\,q^{12} 5\,q^{10} 6\,q^9 2\,q^8 + 3\,q^7 + 2\,q^6$
- $m_{\kappa}(\tau) = 12$.

$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_3,\epsilon_1,\epsilon_1).$

- $\bullet \ [\mathcal{M}_{\tau}] = (q-1)^3 q.$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 48$.

$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\bar{z}}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + q^{15} q^{14} 2q^{13} 2q^{12} + 2q^{10} + 2q^{9} + q^{8} q^{7} q^{6}$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $\bullet \ \ [X(\tau)] = q^{17} + q^{16} q^{15} 2\,q^{14} 2\,q^{13} + 2\,q^{11} + 2\,q^{10} + q^9 q^8 q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi=((1,1),(1,1),(2,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $\left[\mathfrak{M}_{\tau}^{\mathrm{irr}}\right] = q 2.$
- $\bullet \ [X(\tau)] = q^{18} q^{17} 3q^{16} + 2q^{14} + 4q^{13} + 2q^{12} 2q^{11} 3q^{10} 3q^9 + q^8 + 2q^7$
- $m_{\kappa}(\tau) = 24$.

 $\xi=((1,1),(1,1),(2,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),\quad \sigma_B=(\epsilon_2,\epsilon_1,\epsilon_1,\epsilon_3).$

- $[\mathcal{M}_{\tau}] = (q-1)^2 q^2$.
- $\bullet \ [\mathcal{G}_{\tau}] = (q-1)^{3}.$
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $\bullet \ [X(\tau)] = q^{18} 2\,q^{17} q^{16} + q^{15} + q^{14} + 3\,q^{13} q^{12} q^{11} 2\,q^{10} q^9 + 2\,q^8$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_2, \epsilon_3, \epsilon_1, \epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{17} q^{15} q^{14} q^{13} + q^{12} + q^{11} + q^{10} q^{8}$
- $m_{\kappa}(\tau) = 24$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{16} q^{14} q^{13} q^{12} + q^{11} + q^{10} + q^9 q^7$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{17} q^{15} q^{14} q^{13} + q^{12} + q^{11} + q^{10} q^{8}$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_3,\epsilon_1,\epsilon_2).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{17} q^{15} q^{14} q^{13} + q^{12} + q^{11} + q^{10} q^{8}$
- $m_{\kappa}(\tau) = 48$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_2,\epsilon_1,\epsilon_3,\epsilon_1).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{17} q^{15} q^{14} q^{13} + q^{12} + q^{11} + q^{10} q^{8}$
- $m_{\kappa}(\tau) = 48$.

Total count of $\kappa = ((\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), (\epsilon_1, \epsilon_1, \epsilon_2, \epsilon_3))$

$$\begin{split} [X_{\kappa}^{\mathrm{red}}] &= 8\,q^{20} + 68\,q^{19} + 56\,q^{18} - 4\,q^{17} + 2\,q^{16} + 78\,q^{15} + 102\,q^{14} + 46\,q^{13} - 20\,q^{12} - 92\,q^{11} - 38\,q^{10} + 30\,q^9 + 82\,q^8 + 18\,q^7 - 48\,q^6, \\ [X_{\kappa}^{\mathrm{ir}}] &= q^{22} + 5\,q^{21} + 6\,q^{20} - 40\,q^{19} - 13\,q^{18} + 57\,q^{17} + 51\,q^{16} - 35\,q^{15} - 74\,q^{14} - 32\,q^{13} + 25\,q^{12} + 93\,q^{11} + 38\,q^{10} - 30\,q^9 - 82\,q^8 - 18\,q^7 + 48\,q^6, \\ [X_{\kappa}] &= q^{22} + 5\,q^{21} + 14\,q^{20} + 28\,q^{19} + 43\,q^{18} + 53\,q^{17} + 53\,q^{16} + 43\,q^{15} + 28\,q^{14} + 14\,q^{13} + 5\,q^{12} + q^{11}, \end{split}$$

 $[\mathfrak{M}_{\kappa}] = q^7 + 5 q^6 + 7 q^5 - 34 q^4 + 34 q^2 + 18 q - 48.$

11. Configuration $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$ and $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[\mathfrak{M}_{\tau}^{irr}] = q^4 + 4q^3 9q^2 3q + 12.$
- $\bullet \ [X(\tau)] = q^{18} + 5q^{17} 5q^{16} 13q^{15} + 3q^{14} + 12q^{13} + 17q^{12} + 4q^{11} 16q^{10} 17q^9 - 12q^8 + 9q^7 + 12q^6$
- $m_{\kappa}(\tau) = 16$.

$$\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$. $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$.
- $[X(\tau)] = q^{16} 3q^{15} + 3q^{13} + 2q^{12} + 3q^{11} 4q^{10} 3q^9 3q^8 + 4q^6$
- $m_{\kappa}(\tau) = 18$.

$$\xi = ((2,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $[X(\tau)] = q^{14} 2q^{12} 3q^{11} 3q^{10} + 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 72$.

$$\xi=((1,1),(1,1),(1,1),(1,1)),\quad \sigma_A=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),\quad \sigma_B=(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $\bullet \ [X(\tau)] = q^{12} + 3 q^{11} + 5 q^{10} + 6 q^9 + 5 q^8 + 3 q^7 + q^6$
- $m_{\kappa}(\tau) = 24$.

$$\xi = ((3,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = q^3 1.$
- $[\mathcal{G}_{\tau}] = (q-1)^2$. $[\mathfrak{M}_{\tau}^{irr}] = q^4 + 4q^3 9q^2 3q + 12$.
- $\bullet \ [X(\tau)] = q^{21} + 5 q^{20} 5 q^{19} 14 q^{18} 2 q^{17} + 17 q^{16} + 30 q^{15} + q^{14} 28 q^{13} 14 q^{18} 2 q^{17} + 17 q^{16} + 30 q^{15} + q^{14} 28 q^{13} 12 q^{16} + 10 q$ $34q^{12} - 16q^{11} + 25q^{10} + 29q^9 + 12q^8 - 9q^7 - 12q^6$
- $m_{\kappa}(\tau) = 16$.

$$\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = q^3 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $[X(\tau)] = q^{17} 2q^{15} 4q^{14} 3q^{13} + 2q^{12} + 5q^{11} + 6q^{10} + 2q^9 2q^8 3q^7 2q^6$

• $m_{\kappa}(\tau) = 144$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^3 1.$
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $\bullet \ [X(\tau)] = q^{15} + 3q^{14} + 5q^{13} + 5q^{12} + 2q^{11} 2q^{10} 5q^9 5q^8 3q^7 q^6$
- $m_{\kappa}(\tau) = 96$.

 $\xi = ((2,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^4 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^2$.
- $[g_{\tau}] = (q-1)$. $[\mathfrak{M}_{\tau}^{irr}] = q^2 4q + 4$. $[X(\tau)] = q^{20} 3q^{19} + 3q^{17} + q^{16} + 6q^{15} 4q^{14} 6q^{13} 5q^{12} 3q^{11} + q^{16} + 6q^{16} + 6q^{16$ $8q^{10} + 3q^9 + 3q^8 - 4q^6$
- $m_{\kappa}(\tau) = 36$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $\bullet \ [X(\tau)] = q^{18} 4q^{16} 3q^{15} + 2q^{14} + 6q^{13} + 6q^{12} 5q^{10} 6q^9 2q^8 + 3q^7 + 2q^6$
- $m_{\kappa}(\tau) = 72$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = q^4 1$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ [X(\tau)] = q^{18} 2\,q^{16} 3\,q^{15} 4\,q^{14} + 4\,q^{12} + 6\,q^{11} + 5\,q^{10} 2\,q^8 3\,q^7 2\,q^6$
- $m_{\kappa}(\tau) = 72$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{16} + 3q^{15} + 3q^{14} 4q^{12} 6q^{11} 4q^{10} + 3q^8 + 3q^7 + q^6$
- $m_{\kappa}(\tau) = 144$.

 $\xi = ((2,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $\bullet \ \ [X(\tau)] = q^{19} q^{18} 3\,q^{17} + 2\,q^{15} + 4\,q^{14} + 2\,q^{13} 2\,q^{12} 3\,q^{11} 3\,q^{10} + q^9 + 2\,q^8$
- $m_{\kappa}(\tau) = 144$.

 $\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{17} + 2q^{16} + q^{15} q^{14} 3q^{13} 3q^{12} q^{11} + q^{10} + 2q^9 + q^8$
- $m_{\kappa}(\tau) = 288$.

$$\xi = ((1,1),(3,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = q^3 1.$

- $\begin{array}{l} \bullet \ [\mathcal{G}_{\tau}] = (q-1)^2. \\ \bullet \ [\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q^4 + 4\,q^3 9\,q^2 3\,q + 12. \\ \bullet \ [X(\tau)] = q^{21} + 5\,q^{20} 5\,q^{19} 14\,q^{18} 2\,q^{17} + 17\,q^{16} + 30\,q^{15} + q^{14} 28\,q^{13} q^{16} + 30\,q^{16} + q^{16} +$ $34q^{12} - 16q^{11} + 25q^{10} + 29q^9 + 12q^8 - 9q^7 - 12q^6$
- $m_{\kappa}(\tau) = 16$.

$$\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1).$
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{irr}] = q 2$.
- $[X(\tau)] = q^{17} q^{16} 3q^{15} + 2q^{13} + 4q^{12} + 2q^{11} 2q^{10} 3q^9 3q^8 + q^7 + 2q^6$
- $m_{\kappa}(\tau) = 144$.

$\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q-1)^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{15} q^{13} q^{12} q^{11} + q^{10} + q^9 + q^8 q^6$
- $m_{\kappa}(\tau) = 96$.

$$\xi = ((1,1),(2,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$$

- $[\mathcal{M}_{\tau}] = (q^2 1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2$.
- $\bullet \ [X(\tau)] = q^{19} 4\,q^{17} 3\,q^{16} + 2\,q^{15} + 6\,q^{14} + 6\,q^{13} 5\,q^{11} 6\,q^{10} 2\,q^9 + 3\,q^8 + 2\,q^7$
- $m_{\kappa}(\tau) = 144$.

$\xi = ((1,1), (1,1), (1,1), (1,1)), \quad \sigma_A = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), \quad \sigma_B = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)^2 q$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1$.
- $[X(\tau)] = q^{17} + q^{16} q^{15} 2q^{14} 2q^{13} + 2q^{11} + 2q^{10} + q^9 q^8 q^7$
- $m_{\kappa}(\tau) = 288$.

 $\xi = ((1,1),(1,1),(2,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q^2 1)(q 1)q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^3$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = q 2.$
- $\bullet \ \ [X(\tau)] = q^{19} q^{18} 3 \, q^{17} + 2 \, q^{15} + 4 \, q^{14} + 2 \, q^{13} 2 \, q^{12} 3 \, q^{11} 3 \, q^{10} + q^9 + 2 \, q^8$
- $m_{\kappa}(\tau) = 144$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^2$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{\mathrm{irr}}] = 1.$
- $[X(\tau)] = q^{17} q^{15} q^{14} q^{13} + q^{12} + q^{11} + q^{10} q^{8}$
- $m_{\kappa}(\tau) = 288$.

 $\xi = ((1,1),(1,1),(1,1),(1,1)), \quad \sigma_A = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4), \quad \sigma_B = (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4).$

- $[\mathcal{M}_{\tau}] = (q-1)^3 q^3$.
- $[\mathcal{G}_{\tau}] = (q-1)^4$.
- $[\mathfrak{M}_{\tau}^{irr}] = 1.$
- $[X(\tau)] = q^{18} q^{16} q^{15} q^{14} + q^{13} + q^{12} + q^{11} q^9$
- $m_{\kappa}(\tau) = 576$.

Total count of $\kappa = ((\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4), (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4))$

$$\begin{split} [X_{\kappa}^{\mathrm{red}}] &= 32\,q^{21} + 196\,q^{20} + 164\,q^{19} - 164\,q^{17} - 58\,q^{16} + 386\,q^{15} + 296\,q^{14} + 94\,q^{13} - 264\,q^{12} - 406\,q^{11} - 8\,q^{10} + 182\,q^9 + 270\,q^8 - 144\,q^6, \\ [X_{\kappa}^{\mathrm{irr}}] &= q^{24} + 6\,q^{23} + 19\,q^{22} + 10\,q^{21} - 125\,q^{20} - 68\,q^{19} + 106\,q^{18} + 260\,q^{17} + 129\,q^{16} - 344\,q^{15} - 277\,q^{14} - 88\,q^{13} + 265\,q^{12} + 406\,q^{11} + 8\,q^{10} - 182\,q^9 - 270\,q^8 + 144\,q^6, \\ [X_{\kappa}] &= q^{24} + 6\,q^{23} + 19\,q^{22} + 42\,q^{21} + 71\,q^{20} + 96\,q^{19} + 106\,q^{18} + 96\,q^{17} + 71\,q^{16} + 42\,q^{15} + 19\,q^{14} + 6\,q^{13} + q^{12}, \\ [\mathfrak{M}_{\kappa}] &= q^9 + 6\,q^8 + 20\,q^7 + 17\,q^6 - 98\,q^5 - 26\,q^4 + 38\,q^3 + 126\,q^2 - 144. \end{split}$$

Summary

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\begin{split} [X_{(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3),(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4)}^{\text{ir}}] &= q^{18} - 4\,q^{17} + 5\,q^{16} - q^{15} - 3\,q^{14} + 3\,q^{13} - 5\,q^{12} + 7\,q^{11} - 2\,q^{10} + q^9 - 6\,q^7 + 4\,q^6. \\ [X_{(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2),(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)}^{\text{ir}}] &= 0. \\ [X_{(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2),(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)}^{\text{ir}}] &= q^{18} - 3\,q^{17} + 4\,q^{16} - 2\,q^{15} - 3\,q^{14} + 3\,q^{13} - 3\,q^{12} + 7\,q^{11} - 2\,q^{10} - q^8 - 5\,q^7 + 4\,q^6. \\ [X_{(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_2),(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4)}^{\text{ir}}] &= q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^9 - 19\,q^8 - 18\,q^7 + 18\,q^6. \\ [X_{(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3),(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)}^{\text{ir}}] &= q^{18} - 3\,q^{17} + 4\,q^{16} - 2\,q^{15} - 3\,q^{14} + 3\,q^{13} - 3\,q^{12} + 7\,q^{11} - 2\,q^{10} - q^8 - 5\,q^7 + 4\,q^6. \\ [X_{(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3),(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)}^{\text{ir}}] &= q^{20} + 2\,q^{19} - 11\,q^{18} + 4\,q^{17} + 17\,q^{16} - 13\,q^{15} - 4\,q^{14} - 9\,q^{13} + 4\,q^{12} + 21\,q^{11} + 5\,q^9 - 22\,q^8 - 10\,q^7 + 15\,q^6. \\ [X_{(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3),(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4)}^{\text{ir}}] &= q^{22} + 5\,q^{21} + 6\,q^{20} - 40\,q^{19} - 13\,q^{18} + 57\,q^{17} + 51\,q^{16} - 35\,q^{15} - 74\,q^{14} - 32\,q^{13} + 25\,q^{12} + 93\,q^{11} + 38\,q^{10} - 30\,q^9 - 82\,q^8 - 18\,q^7 + 48\,q^6. \\ [X_{(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),(\epsilon_1,\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)}^{\text{ir}}] &= q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^9 - 19\,q^8 - 18\,q^7 + 18\,q^6. \\ [X_{(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)}^{\text{ir}}] &= q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^9 - 19\,q^8 - 18\,q^7 + 18\,q^6. \\ [X_{(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)}^{\text{ir}}] &= q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^9 - 19\,q^8 - 18\,q^7 + 18\,q^6. \\ [X_{(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4),(\epsilon_1,\epsilon_1,\epsilon_2,\epsilon_3)}^{\text{ir}}] &= q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{15} - 6\,q^{15} + 25\,q^{11} - 3\,q^
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Final result representations.

 $[X_4^{\rm irr}] = (q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^9 - 19\,q^8 - 18\,q^7 + 18\,q^6)C_{(1,1,1,1),(2,2)} + (q^{18} - 4\,q^{17} + 5\,q^{16} - q^{15} - 3\,q^{14} + 3\,q^{13} - 5\,q^{12} + 7\,q^{11} - 2\,q^{10} + q^9 - 6\,q^7 + 4\,q^6)C_{(1,1,1,1),(1,3)} + (q^{22} + 5\,q^{21} + 6\,q^{20} - 40\,q^{19} - 13\,q^{18} + 57\,q^{17} + 51\,q^{16} - 35\,q^{15} - 74\,q^{14} - 32\,q^{13} + 25\,q^{12} + 93\,q^{11} + 38\,q^{10} - 30\,q^9 - 82\,q^8 - 18\,q^7 + 48\,q^6)C_{(1,1,2),(1,1,1,1)} + (q^{22} + 5\,q^{21} + 6\,q^{20} - 40\,q^{19} - 13\,q^{18} + 57\,q^{17} + 51\,q^{16} - 35\,q^{15} - 74\,q^{14} - 32\,q^{13} + 25\,q^{12} + 93\,q^{11} + 38\,q^{10} - 30\,q^9 - 82\,q^8 - 18\,q^7 + 48\,q^6)C_{(1,1,1),(1,1,2)} + (q^{24} + 6\,q^{23} + 19\,q^{22} + 10\,q^{21} - 125\,q^{20} - 68\,q^{19} + 106\,q^{18} + 260\,q^{17} + 129\,q^{16} - 344\,q^{15} - 277\,q^{14} - 88\,q^{13} + 265\,q^{12} + 406\,q^{11} + 8\,q^{10} - 182\,q^9 - 270\,q^8 + 144\,q^6)C_{(1,1,1,1),(1,1,1,1)} + (q^{18} - 3\,q^{17} + 4\,q^{16} - 2\,q^{15} - 3\,q^{14} + 3\,q^{13} - 3\,q^{12} + 7\,q^{11} - 2\,q^{10} - q^8 - 5\,q^7 + 4\,q^6)C_{(2,2),(1,1,2)} + (q^{18} - 3\,q^{17} + 4\,q^{16} - 2\,q^{15} - 3\,q^{14} + 3\,q^{13} - 3\,q^{12} + 7\,q^{11} - 2\,q^{10} - q^8 - 5\,q^7 + 4\,q^6)C_{(1,1,2),(2,2)} + (q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^9 - 19\,q^8 - 18\,q^7 + 18\,q^6)C_{(2,2),(1,1,1,1)} + (q^{18} - 4\,q^{17} + 5\,q^{16} - q^{15} - 3\,q^{14} + 3\,q^{13} - 5\,q^{12} + 7\,q^{11} - 2\,q^{10} - q^8 - 5\,q^7 + 4\,q^6)C_{(1,1,2),(2,2)} + (q^{20} + 4\,q^{19} - 12\,q^{18} - 4\,q^{17} + 24\,q^{16} - 11\,q^{15} - 3\,q^{14} - 7\,q^{13} - 6\,q^{12} + 25\,q^{11} - 3\,q^{10} + 11\,q^9 - 19\,q^8 - 18\,q^7 + 18\,q^6)C_{(2,2),(1,1,1,1)} + (q^{18} - 4\,q^{17} + 5\,q^{16} - q^{15} - 3\,q^{14} + 3\,q^{13} - 5\,q^{12} + 7\,q^{11} - 2\,q^{10} - q^8 - 5\,q^7 + 4\,q^6)C_{(1,1,2),(1,1,2)} + (q^{18} - 4\,q^{17} + 17\,q^{16} - 13\,q^{15} - 4\,q^{14} - 9\,q^{13} + 4\,q^{15} + 21\,q^{11} + 5\,q^9 - 22\,q^8 - 10\,q^7 + 15\,q^6)C_{(1,1,2),(1,1,2)} +$

Final result characters.

 $[\mathfrak{M}_{4}^{\mathrm{irr}}] = (q^{5} + 4\,q^{4} - 11\,q^{3} + q^{2} + 18\,q - 18)C_{(1,1,1,1),(2,2)} + (q^{3} - 4\,q^{2} + 6\,q - 4)C_{(1,1,1,1),(1,3)} + (q^{7} + 5\,q^{6} + 7\,q^{5} - 34\,q^{4} + 34\,q^{2} + 18\,q - 48)C_{(1,1,2),(1,1,1,1)} + (q^{7} + 5\,q^{6} + 7\,q^{5} - 34\,q^{4} + 34\,q^{2} + 18\,q - 48)C_{(1,1,1),(1,1,2)} + (q^{9} + 6\,q^{8} + 20\,q^{7} + 17\,q^{6} - 98\,q^{5} - 26\,q^{4} + 38\,q^{3} + 126\,q^{2} - 144)C_{(1,1,1,1),(1,1,1,1)} + (q^{3} - 3\,q^{2} + 5\,q - 4)C_{(2,2),(1,1,2)} + (q^{3} - 3\,q^{2} + 5\,q - 4)C_{(1,1,2),(2,2)} + (q^{5} + 4\,q^{4} - 11\,q^{3} + q^{2} + 18\,q - 18)C_{(2,2),(1,1,1,1)} + (q^{3} - 4\,q^{2} + 6\,q - 4)C_{(1,3),(1,1,1,1)} + (q^{5} + 2\,q^{4} - 10\,q^{3} + 7\,q^{2} + 10\,q - 15)C_{(1,1,2),(1,1,2)}.$

References

[1] Á. González-Prieto and V. Muñoz, Motive of the $SL_4(\mathbb{C})$ -character variety of torus knots, arXiv.