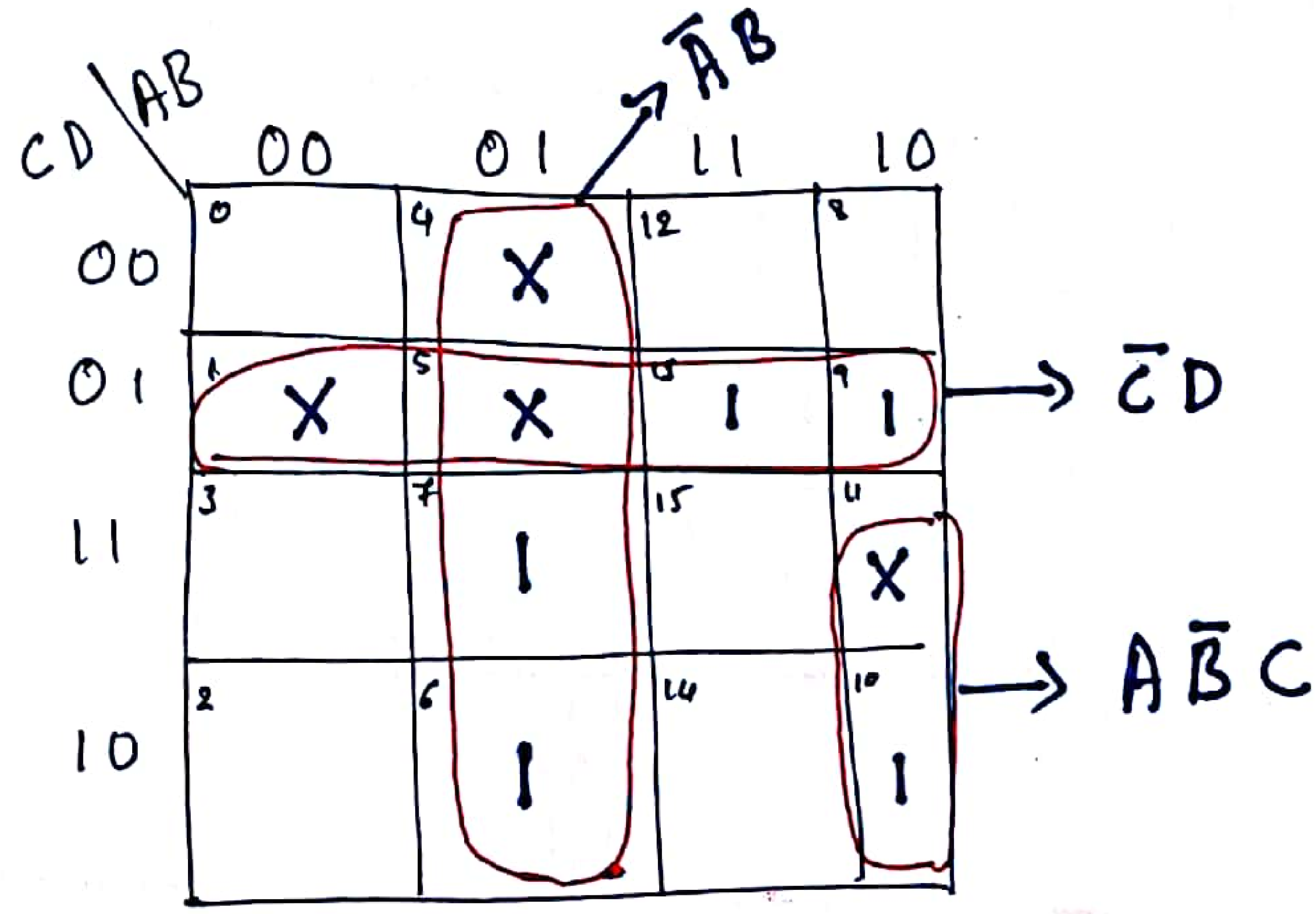
(1)

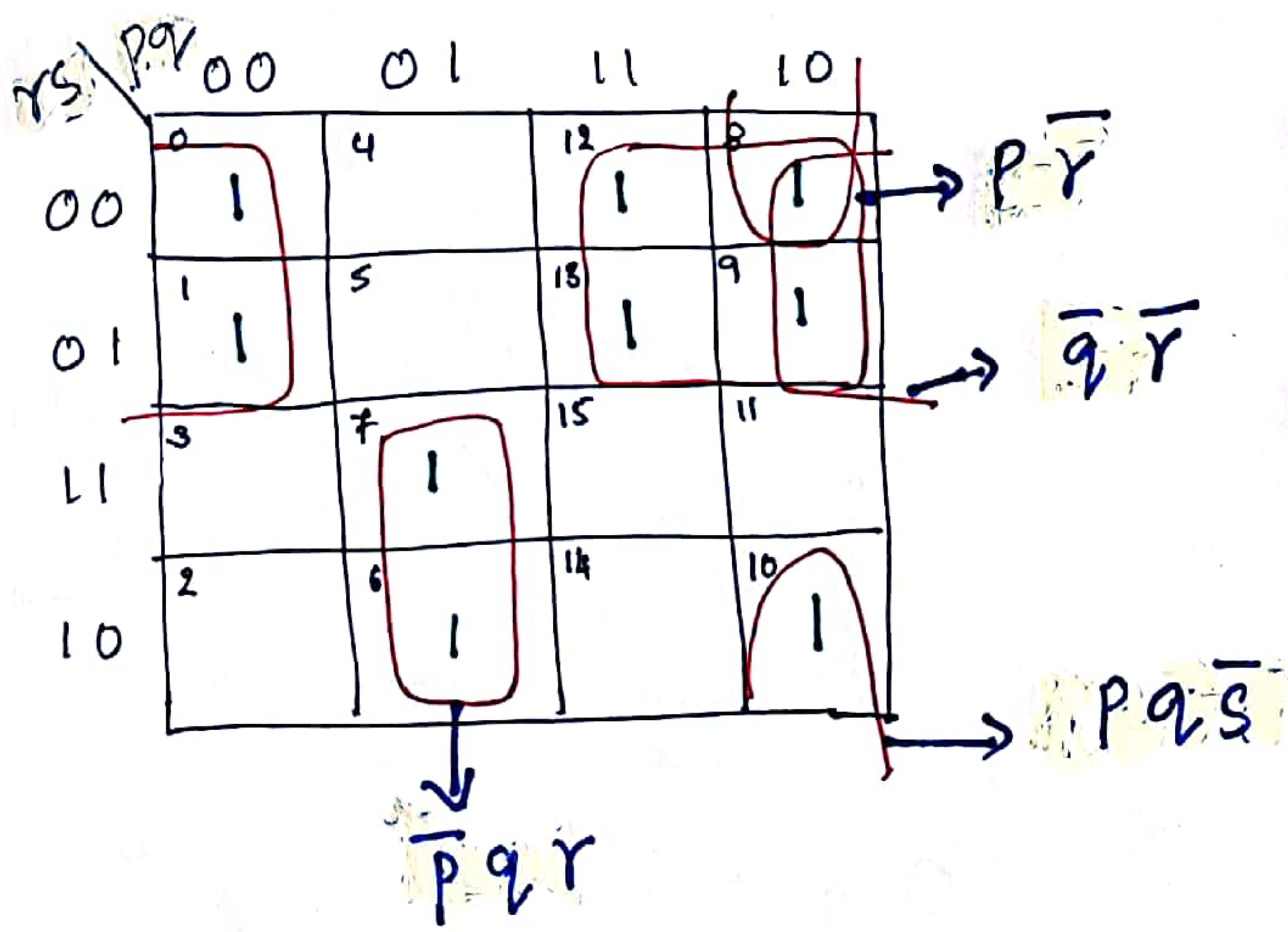
(1) Find the minimal SOP of the following Boolean functions using K-map.

(a) $F(a,b,c,d) = \Sigma m(6,7,9,10,13) + d(1,4,5,11)$



$$\boxed{Y = F(a,b,c,d) = \bar{A}B + \bar{C}D + A\bar{B}C}$$

(b) $F(P,q,r,s) = \Sigma m(6,7,9,10,13) + d(0,1,8,12)$



$$\boxed{F(P,q,r,s) = p\bar{r} + \bar{q}\bar{r} + pq\bar{s} + \bar{p}qr}$$

(c) $F(A, B, C, D) = \sum m(6, 8, 9, 10, 11, 12, 13, 14, 15)$

CD \ AB	00	01	11	10
00	0	4	12: 1	8: 1
01	1	5	13: 1	9: 1
11	3	7	15: 1	11: 1
10	2	6: 1	14: 1	10: 1

→ A

$BC\bar{D}$

$$\boxed{F(A, B, C, D) = A + BC\bar{D}}$$

(d) $F(A, B, C, D) = \sum m(1, 3, 5, 7, 8, 10, 12, 14)$

CD \ AB	00	01	11	10
00	0	4	12: 1	8: 1
01	1: 1	5: 1	13	9
11	3: 1	7: 1	15	11
10	2	6	14: 1	10: 1

$\bar{A}D$ ←

$A\bar{D}$

$$\boxed{F(A, B, C, D) = A\bar{D} + \bar{A}D}$$

(e) $F(a,b,c,d) = \Sigma m(5,6,7,12,13) + d(4,9,14,15)$

cd \ ab	00	01	11	10
00	0	4 X	12 1	8
01	1	5 1	13 1	9 X
11	3	7 1	15 X	11
10	2	6 1	14 X	10

→ $A\bar{C}D$ $(a\bar{c}d)$

$B(b)$

$$\boxed{F(a,b,c,d) = a \quad + b}$$

2. Find the minimum Sum-of-products expression for each function. Underline the essential prime implicants in your answer and tell which minterm makes each one essential.

(a) $f(a,b,c,d) = \Pi M(1,9,11,12,14)$

cd \ ab	00	01	11	10
00	0 1	4 1	12	8 1
01	1	5 1	13 1	9
11	3 1	7 1	15 1	11
10	2 1	6 1	14	10 1

$\bar{a}\bar{c}\bar{d}$ ←

$\bar{a}b$ ←

bd ←

$\bar{a}b$ ←

$a\bar{b}\bar{d}$ ←

→ Converting the given expression to Sum of product expression

$\Sigma m(0,2,3,4,5,6,7,8,10,13,15)$

→ prime implicants are

$\underline{\bar{a}\bar{c}\bar{d}} + \bar{a}b + \underline{bd} + \underline{\bar{a}b} + \underline{a\bar{b}\bar{d}}$

→ Vertical squad grouping is not required because the terms are covered by other squads & pairs.

→ $\bar{a}b$ is not a essential prime implicant because its been covered by $\bar{a}\bar{c}\bar{d}$, bd, $\bar{a}b$.

cd \ ab	00	01	11	10
00	[0] 1	[4] 1	[12]	[8] 1
01	[1]	[5] 1	[13] 1	[9]
11	[3] 1	[7] 1	[15] 1	[11]
10	[2] 1	[6] 1	[14]	[10] 1

$\bar{a}\bar{c}\bar{d}$

→ bd

$\bar{a}c$

→ $a\bar{b}\bar{d}$

Essential prime implicants are

$\bar{a}\bar{c}\bar{d}$, $\bar{a}c$, bd, $a\bar{b}\bar{d}$ → minterms included are 8, 10

minterms included 0, 4

minterms included are 3, 2, 7, 6

minterms included are 5, 7, 13, 15

(b) $f(a,b,c,d) = \Pi M(5,7,13,14,15) \cdot \Pi D(1,2,3,9)$

→ minimum sum of product expression for the given the given problem is

$f(a,b,c,d) = \Sigma m(0,4,6,8,10,11,12) + d(1,2,3,9)$

CD \ AB	00	01	10	11
00	[0] 1	[4] 1	[12] 1	[8] 1
01	[1] X	[5]	[13]	[9] X
10	[3] X	[7]	[15]	[11] 1
11	[2] X	[6] 1	[14]	[10] 1

→ $\bar{C}\bar{D}$

→ 1

$\bar{A}CD$

Minterms are: 0, 4, 12, 18

minterms are 2, 6

$$\boxed{Y = \bar{C}\bar{D} + \bar{A}CD}$$

3. Get a minimized expression for

$$Y = F(A, B, C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C$$

using K-map, EVM and quine Mc-cluskey.

K-Map

$\boxed{\text{Given minterms are: } 0, 1, 3, 5}$

A \ BC	00	01	11	10
0	0: 1	1: 1	3: 1	2:
1	4:	5: 1	7:	6:

$\bar{A}\bar{B}$

$\bar{A}C$

$\bar{B}C$

$$\boxed{Y = F(A, B, C, D) = \bar{A}\bar{B} + \bar{B}C + \bar{A}C}$$

EVM: Method.

A	B	C (MEV)	Y	Map Entred value
0	0	0	1	1
0	0	1	1	
0	1	0	0	C
0	1	1	1	
1	0	0	0	C
1	0	1	1	
1	1	0	0	0
1	1	1	0	

Represent in K-map

B \ A	0	1
0	1	C
1	C	0

$\boxed{= \bar{A}\bar{B}}$

* Replace 1's with X's

B \ A	0	1
0	X	C
1	C	0

4. Get Simplified Expression for

$Y = F(A, B, C, D, E) = \Sigma m(0, 2, 3, 5, 7, 9, 11, 13, 14, 16, 18, 24, 26, 28, 30)$

minterms	Binary Representation
0	0 0 0 0 0
2	0 0 0 1 0
3	0 0 0 1 1
5	0 0 1 0 1
7	0 0 1 1 1
9	0 1 0 0 1
11	0 1 0 1 1
13	0 1 1 0 1
14	0 1 1 1 0
16	1 0 0 0 0
18	1 0 0 1 0
24	1 1 0 0 0
26	1 1 0 1 0
28	1 1 1 0 0
30	1 1 1 1 0

5 variable Expression.

Hence $2^5 = 32$ combination.

← after obtaining the binary representation for the given minterms Group them based on number of 1's present in the B.R

Step 1:

Group	Minterms	Binary Representation
G_0	m_0 ✓	0 0 0 0 0
G_1	m_2 ✓	0 0 0 1 0
	m_{16} ✓	1 0 0 0 0
G_2	m_3 ✓	0 0 0 1 1
	m_5 ✓	0 0 1 0 1
	m_9 ✓	0 1 0 0 1
	m_{18} ✓	1 0 0 1 0
	m_{24}	1 1 0 0 0
G_3	m_7 ✓	0 0 1 1 1
	m_{11} ✓	0 1 0 1 1
	m_{13} ✓	0 1 1 0 1
	m_{14} ✓	0 1 1 1 0
	m_{26} ✓	1 1 0 1 0
	m_{28}	1 1 1 0 0
G_4	m_{30}	1 1 1 1 0

Step 2:

Group	minterms	Binary Representation
G_0 -	$m_0 - m_2$	0 0 0 – 0
	$m_0 - m_{16}$	– 0 0 0 0
G_1	$m_2 - m_3$	0 0 0 1 –
	$m_2 - m_{18}$	– 0 0 1 0
	$m_{16} - m_{18}$	1 0 0 – 0
	$m_{16} - m_{24}$	1 – 0 0 0
G_2	$m_3 - m_7$	0 0 – 1 1
	$m_3 - m_{11}$	0 – 0 1 1
	$m_5 - m_7$	0 0 1 – 1
	$m_5 - m_{13}$	0 – 1 0 1
	$m_9 - m_{11}$	0 1 0 – 1
	$m_9 - m_{13}$	0 1 – 0 1
	$m_{18} - m_{26}$	1 – 0 1 0
	$m_{24} - m_{26}$	1 1 0 – 0
	$m_{24} - 28$	1 1 – 0 0
G_3	$m_{14} - m_{30}$	– 1 1 1 0
	$m_{26} - m_{30}$	1 1 – 1 0
	$m_{28} - m_{30}$	1 1 1 – 0

Step 3

Groups	minterms	Binary Representation A B C D E	
G_0	$m_0 - m_2 - m_{16} - m_{18}$	~~- 0 0 - 0~~	Eliminate Duplicate term
	$m_0 - m_{16} - m_2 - m_{18}$	- 0 0 - 0	
G_1	$m_{16} - m_{18} - m_{24} - m_{26}$	~~1 - 0 - 0~~	eliminate Duplicate
	$m_{16} - m_{24} - m_{18} - m_{26}$	1 - 0 - 0	
G_2	$m_{24} - m_{28} - m_{26} - m_{30}$	~~1 1 - - 0~~	Eliminate Duplicate
	$m_{24} - m_{26} - m_{28} - m_{30}$	1 1 - - 0	

So prime Implicants are:

$$\bar{B}\bar{C}\bar{E} + A\bar{C}\bar{E} + AB\bar{E}$$

Representing them in PI chart

	PI	0	2	3	5	7	9	11	13	14	16	18	24	26	28	30
0, 2, 16, 18	$\bar{B}\bar{C}\bar{E}$	X	X								X	X				
16, 18, 24, 26	~~$A\bar{C}\bar{E}$~~										~~X~~	~~X~~	~~X~~	~~X~~		
24, 26, 28, 30	$AB\bar{E}$												X	X	X	X

Hence $\boxed{Y = \bar{B}.\bar{C}\bar{E} + A\bar{C}\bar{E} + AB\bar{E}}$

5. a. Find all Prime Implicants of the following function and then find all minimum solutions using Petrick's method.

(a) $F(A, B, C, D) = \sum m(9, 12, 13, 15) + d(1, 4, 5, 7, 8, 11, 14)$

minterms		Binary Repre
m_1	1	0 0 0 1
m_4	4	0 1 0 0
m_5	5	0 1 0 1
m_7	7	0 1 1 1
m_8	8	1 0 0 0
m_9	9	1 0 0 1
m_{11}	11	1 0 1 1
m_{12}	12	1 1 0 0
m_{13}	13	1 1 0 1
m_{14}	14	1 1 1 0
m_{15}	15	1 1 1 1

Step 1

G_0	m_1	0 0 0 1 ✓
	m_4	1 0 0 0 ✓
	m_8	1 0 0 0 ✓
G_1	m_5	0 1 0 1 ✓
	m_9	1 0 0 1 ✓
	m_{12}	1 1 0 0
G_2	m_7	0 1 1 1
	m_{11}	1 0 1 1
	m_{13}	1 1 0 1
	m_{14}	1 1 1 0
G_3	m_{15}	1 1 1 1

Step 2

G_0	$m_1 - m_5$	0 – 0 1 ✓
	$m_1 - m_9$	– 0 0 1 ✓
	$m_4 - m_9$	1 0 0 – ✓
	$m_4 - m_{12}$	1 – 0 0 ✓
	$m_8 - m_9$	1 0 0 – ✓
	$m_8 - m_{12}$	1 – 0 0 ✓

G_1	PI	$m_5 - m_7$	0 – 0 1 ($\bar{A}C D$)
		$m_5 - m_{13}$	– 1 0 1 ✓
		$m_9 - m_{11}$	1 0 – 1
		$m_9 - m_{13}$	1 – 0 1 ✓
		$m_{12} - m_{13}$	1 1 0 – ✓
	PI	$m_{12} - m_{14}$	1 1 – 0 ($AB\bar{D}$)

~~G_1 $m_{12}-m_{14}-m_{13}-m_{15}$ 1 1 - -~~

Step3

Groups	minterms	B.R.	
G_0	$m_1-m_5-m_9-m_{13}$	- - 0 1	$\rightarrow \bar{C}D$
	~~$m_1-m_9-m_5-m_{13}$~~	~~- - 0 1~~	
	$m_4-m_9-m_{12}-m_{13}$	1 - 0 -	$\rightarrow A\bar{C}$
	~~$m_4-m_{12}-m_9-m_{13}$~~	~~1 - 0 -~~	
	$m_8-m_9-m_{12}-m_{13}$	1 - 0 -	$\rightarrow A\bar{C}$
	~~$m_8-m_{12}-m_9-m_{13}$~~	~~1 - 0 -~~	
G_1	$m_5-m_{13}-m_7-m_{15}$	- 1 - 1	$\rightarrow BD$
	~~$m_9-m_{11}-m_{13}-m_{15}$~~	~~1 - - 1~~	
	$m_9-m_{13}-m_{11}-m_{15}$	1 - - 1	$\rightarrow AD$
	$m_{12}-m_{13}-m_{14}-m_{15}$	1 1 - -	$\rightarrow AB$

Representing them in PI chart & Naming each row as $P_1, P_2 \ldots$

		9	12	13	15
P_1	$\bar{C}D$	X		X	
P_2	$A\bar{C}$	X	X	X	
P_3	$A\bar{C}$	X	X	X	
P_4	BD			X	X
P_5	AD	X		X	X
P_6	AB		X	X	X

G_1	$m_5 - m_7$ ✓	0 – 0 1
	$m_5 - m_{13}$ ✓	– 1 0 1
	$m_9 - m_{11}$ ✓	1 0 – 1
	$m_9 - m_{13}$ ✓	1 – 0 1
	$m_{12} - m_{13}$ ✓	1 1 0 –
	$m_{12} - m_{14}$ ✓	1 1 – 0
G_2	$m_7 - m_{15}$ ✓	– 1 1 1
	$m_{11} - m_{15}$ ✓	1 – 1 1
	$m_{13} - m_{15}$ ✓	1 1 – 1
	$m_{14} - m_{15}$ ✓	1 1 1 –

Step3

Groups	Minterms	B.R.	
G_0	$m_1 - m_5 - m_9 - m_{13}$	– – 0 1	→ $\bar{C}D$
	~~$m_1 - m_9 - m_5 - m_{13}$~~	~~– – 0 1~~	
	$m_4 - m_9 - m_{12} - m_{13}$	1 – 0 –	→ $A\bar{C}$
	~~$m_4 - m_{12} - m_9 - m_{13}$~~	~~1 – 0 –~~	
	~~$m_8 - m_9 - m_{12} - m_{13}$~~	~~1 – 0 –~~	→ ~~$A\bar{C}$~~
	~~$m_8 - m_{12} - m_9 - m_{13}$~~	~~1 – 0 –~~	
G_1	$m_5 - m_{13} - m_7 - m_{15}$	– 1 – 1	→ BD
	~~$m_9 - m_{11} - m_{13} - m_{15}$~~	~~1 – – 1~~	
	$m_9 - m_{13} - m_{11} - m_{15}$	1 – – 1	→ AD
	~~$m_{12} - m_{13} - m_{14} - m_{15}$~~	~~1 1 – –~~	

Eliminate the Duplicate Entries

To cover minterm $\boxed{9}$ we must choose $\boxed{P_1 + P_2 + P_3 + P_5}$

To cover minterm $\boxed{12}$ —— " —— $\boxed{P_2 + P_3 + P_6}$

To —— " — $\boxed{13}$ —— " —— $\boxed{P_1 + P_2 + P_3 + P_4 + P_5 + P_6}$

To —— " —— $\boxed{15}$ —— " —— $\boxed{P_4 + P_5 + P_6}$

So that

$$(P_1 + P_2 + P_3 + P_5)\,(P_2 + P_3 + P_6)\,(P_1 + P_2 + P_3 + P_4 + P_5 + P_6)$$
$$(P_4 + P_5 + P_6)$$

$$= (P_1 . P_2 + P_1 P_3 + P_1 P_6 + P_2 + \underline{P_2 P_3} + P_2 P_6 + \underline{P_3 . P_2} + P_3 + P_3 P_6$$
$$+ P_5 P_2 + P_5 P_3 + P_5 P_6)\,(P_1 + P_2 + P_3 + P_4 + P_5 + P_6)$$
$$(P_4 + P_5 + P_6)$$

$$= (P_1 . P_2 + P_1 . P_3 + P_1 . P_6 + P_2 + P_2 P_3 + P_2 P_6 + P_3 + P_3 P_6$$
$$+ P_5 P_2 + P_5 P_3 + P_5 P_6)\,(P_1 P_4 + P_1 P_5 + P_1 P_6$$
$$+ P_2 P_4 + P_2 P_5 + P_2 P_6 + P_3 P_4 + P_3 P_5 + P_3 P_6 + P_4 + \boxed{P_4 P_5}$$
$$+ \boxed{P_4 P_6} + \boxed{P_5 P_4} + P_5 + \boxed{P_5 P_6} + \boxed{P_6 P_4} + \boxed{P_5 P_6} + P_6)$$

$$= (P_1 . P_2 + P_1 . P_3 + P_1 . P_6 + P_2 + P_2 P_3 + P_2 P_6 + P_3 + P_3 P_6$$
$$+ P_5 P_2 + P_5 P_3 + P_5 P_6)\,(P_1 . P_4 + P_1 P_5 + P_1 P_6 + P_2 P_4 + P_2 P_5$$
$$+ P_2 P_6 + P_3 P_4 + P_3 P_5 + P_3 P_6 + P_4 + P_4 P_5 + P_4 P_6 + P_5 + P_6 + P_5 P_6)$$

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$$= \boxed{P_1 . P_2 . P_4} + P_1 P_2 P_5 + P_1 P_2 P_6 + \boxed{P_1 P_2 P_4} + P_1 P_2 P_5$$

$$+ P_1 P_2 P_6 + P_1 P_2 P_3 P_4 + P_1 P_2 P_3 P_5 + P_1 P_2 P_3 P_6 + \boxed{P_1 . P_2 . P_4}$$

$$+ P_1 P_2 P_4 P_5 + \boxed{P_1 P_2 P_4} P_6 + P_1 . P_2 P_5 + P_1 P_2 P_4 P_6$$

$$+ P_1 . P_2 P_5 + \underset{P_1 P_2}{P_6} + P_4 \; P_1 P_2 P_5 . P_6$$

Continue like this untill you get the final solution.