RNS INSTITUTE OF TECHNOLOGY
Department of Mathematics

II INTERNAL TEST (CS & IS)

SEM: III   Date: 02-12-2020
SUB: Discrete Mathematical Structures   Time: 1.00 to 2.30 PM
SUB CODE: 18CS36   Max. Marks: 50
Instructions: Answer the following Questions:

Q.NO		Questions	Marks	Bloom's Cognitive level
1	a	Prove the following by mathematical induction: (i) $\sum_{i=1}^{n} i(2^i) = 2 + (n-1)2^{n+1}$, (ii) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$	6	L1, L2, CO2
	b	How many positive integers can be formed using the digits 3, 4, 4, 5, 5, 6, 7 to exceed 5,000,000?	4	L1, L2, CO2
OR				
2	a	Prove by mathematical induction $n! > 2^n$ for n 4.	6	L3, CO2
	b	How many license plates can be made using (i) Either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits? (ii) Either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?	4	L3, CO2
3	a	How many arrangements are there of all the letters in SOCIOLOGICAL? In each of these arrangements how many of the arrangements having (i) A and G are adjacent? (ii) all the vowels are together?	6	L3, CO2
	b	Find the number of non-negative integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 8$	4	L5, CO2
OR				
4	a	A woman has 11 close relatives and she wishes to invite five of them to dinner. In how many ways can she invite them in the following situations: (i) there is no restriction on the choice, (ii) two particular persons will not attend separately, (iii) two particular persons will not attend together.	6	L5, CO2
	b	Find the coefficient of $w^3 x^2 y z^2$ in the expansion of $(2w - x + 3y - 2z)^8$	4	L4, CO2
5	a	Find the number of arrangements of all the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's?	6	L3, CO2
	b	In how many ways can we distribute seven apples and six oranges among 4 children so that each child receives at least one apple?	4	L4, CO2
OR				

6	a	Let A = {2, 3, 4, 6, 8 12, 24} and R be the partial order on divisibility, i.e., xRy $\Rightarrow$ x divides y. Find the (i) least element, (ii) greatest element, (iii) minimal elements, (iv) maximal elements, (v) LUB and GLB of B = {4, 6, 12}.	6	L3, CO3
	b	Write down the Hasse diagram for the positive divisors of 45.	4	L2, CO3
7	a	Define Cartesian product of two sets. For any three non-empty sets A, B and C prove that A × (B − C) = (A × B) − (A × C).	6	L4, CO3
	b	Let $A = \{1,2,3,4\}$ and let R be the relation on A defined by xRy iff "x divides y" (i) Write down R as a set of ordered pairs, (ii) Write the relation matrix M_R, (iii) Draw the digraph of R, (iv) Determine the in degree and out degree of each vertex in the digraph.	4	L4, CO3
		OR		
8	a	Define the following relations with an example: (i) Reflexive, (ii) Irreflexive, (iii) Symmetric, (iv) Antisymmetric, (v) Transitive, and (vi) Equivalence	6	L3, CO3
	b	Define partition of a set. If R is a relation defined on A = {1, 2, 3, 4} by R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3), (3, 3), (4, 4)}, determine the partition induced by the equivalence relation R.	4	L3, CO3
9	a	Let R be an equivalence relation on A and a, b ∈ A, then prove the following: (i) $a \in [a]$, (ii) a R b iff $[a] = [b]$ and (iii) If $[a] \cap [b] \neq \phi$ then $[a] = [b]$.	6	L5, CO3
	b	Let $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$. The relations R and S from A to B are represented by the matrices $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. Determine the relations $\overline{R}$, $\overline{S}$, R ∪ S, R ∩ S, R^c and S^c and their matrix representation.	4	L5, CO3
		OR		
10	a	Let A = {a, b, c}, B = P(A) where P(A) is the power set of A. Let R be a subset relation on B. Show that (B, R) is a POSET and draw its Hasse diagram. Also find its least and greatest elements.	6	L4, CO3
	b	Let $A = \{1,2,3,4\}$ and $R = \{(1,1),(1,2),(2,3)(3,4)\}$, $S = \{(3,1),(4,4),(2,4)(1,4)\}$ be the relations on A. Determine the relations R ∘ S, S ∘ R, R^2 and S^2.	4	L4, CO3

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Multiple choice questions

1. **In how many ways can three different coins be placed in two different purses?**

    (a) 6   (b) 3   (c) 8   (d) 4

2. **How many 6 digit numbers can one make using the digits 1, 3, 3, 7, 7, 8 ?**

    (a) 6!   (b) 4!   (c) 360   (d) 720

3. **How many bytes contain exactly two 1's?**

    (a) 8   (b) 56   (c) 28   (d) 14

4. **In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets at least 1 apple?**

    (a) 104   (b) $7C_4 \times 6C_4$ (c) $7C_3 \times 6C_3$ (d) $6C_3 \times 9C_6$

5. **Find the value of** $\left(\begin{array}{c} 1 \\ 5, 3, 2, 2 \end{array}\right)$.

    (a) 210   (b) $\dfrac{1\ !}{5!\ 3!\ 2!\ 2!}$ (c) 420   (d) Impossible

6. **It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?**

    (a) $5! \times 4!$ (b) $5! \times 4$ (c) 9!   (d) None.

7. **If A = $\{x \in N : x \le 3\}$ and B = $\{x \in W, x < 2\}$, then A $\times$ B =**

    a) {(0, 1), (1, 1), (0, 2), (1, 2), (0, 3), (1, 3)}   b) {(1, 0), (1, 1), (2, 0), (2, 1), (3, 0), (3, 1)}

    c) {(1, 1), (2, 0), (2, 1), (3, 0), (3, 1)}   d) {(1, 0), (1, 1), (0, 2), (1, 2), (3, 0), (3, 1)}

8. **If A = $\{x: x^2 - 5x + 6 = 0\}$, B = $\{2, 4\}$, C = $\{4, 5\}$ then A $\times$ (B $\cap$ C) is**

    a) {(2, 4), (3, 4)}   b) {(4, 2), (4, 3)}

    c) {{2, 4), (3, 4), (4, 4)}   d) {(2, 2), (3, 3), (4, 4), (5, 5)}

9. **Let A = $\{1, 2, 3\}$. Then the relation R = $\{(2, 3)\}$ in A is**

    a) symmetric only   b) transitive only

    c) symmetric and transitive only   d) none of these

10. **Let A = $\{1, 2, 3, 4\}$ and R = $\{(2, 2), (3, 3), (4, 4), (1, 2)\}$ be a relation on A. Then A is**

    a) reflexive   b) symmetric   c) transitive   d) none of these

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