2.4 Combinations

- Selecting r objects from a set of $n \geq r$ objects without regard to order is $\binom{n}{r}$

- $\binom{n}{r} = \binom{n}{n-r}$, $\binom{n}{0} = \binom{n}{n} = 1$, $\binom{n}{1} = \binom{n}{n-1} = n$

1. In how many different ways can a committee of 5 teachers and 4 students be selected from 9 teachers and 15 students?

   - No. of ways of selecting 5 teachers from 9 teachers = $\binom{9}{5}$
   - No. of ways of selecting 4 students from 15 students = $\binom{15}{4}$.
   - By product rule, Total no. of different ways = $\binom{9}{5} \times \binom{15}{4} = 1{,}71{,}990$

2. A bag contains 5 red marbles and 6 white marbles. Find the number of ways that 4 marbles can be drawn from the bag if the 4 marbles of the same color?

- No. of ways of selecting 4 red marbles from 5 red marbles = $\binom{5}{4}$
- No. of ways of selecting 4 white marbles from 6 white marbles= $\binom{6}{4}$
- By sum rule, Total no. of different ways = $\binom{5}{4} + \binom{6}{4} = 75$

3. How many arrangements of the letters can be made in the word MISSISSIPPI? How many have no consecutive S's?

- By ignoring fourS's, there are 7 letters remaining.
- Among 7 letters, 'P' repeated twice and 'I' repeated four times.
- No. of arrangements of 7 letters= $\frac{7!}{2!\times 4!} = 105$
- There are 8 possible locations for four S's.
- No. of ways of selecting locations for four S's= $\binom{8}{4}$
- $\left.\begin{array}{r}\text{Therefore, total no. of arrangements}\\ \text{having no adjacent A's}\end{array}\right\} = 105 \times 70 = 7{,}350$

4. Find the number of arrangements of all the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's?

- By ignoring three A's, there are 8 letters remaining.
- Among 8 letters, 'L','S' and 'E' repeated twice each.
- No. of arrangements of 8 letters$= \frac{8!}{2!\times 2!\times 2!} = 5040$
- There are 9 possible locations for three A's.
- No. of ways of selecting locations for three A's $= \binom{9}{3}$
- $\left.\begin{array}{r}\text{Therefore, total no. of arrangements}\\ \text{having no adjacent A's}\end{array}\right\} = 5040 \times 84 = 4{,}23{,}360$

5. From seven consonants and five vowels, how many words consists of four different consonants and three different vowels can be formed?

- No. of ways of selecting 4 consonants from 7 consonants = $\binom{7}{4}$
- No. of ways of selecting 3 vowels from 5 vowels = $\binom{5}{3}$
- No. of arrangements of 4 consonants and 3 vowels = $7!$
- Total no. of possible words = $\binom{7}{4} \times \binom{5}{3} \times 7! = 17{,}64{,}000$

6. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations:
(i) Two particular persons will not attend separately.
(ii) Two particular persons will not attend together.

(i)
- If two particular persons are invited,

  No. of ways of selecting 3 more relatives from the remaining 9 $= \binom{9}{3}$
- If two particular persons are not invited,

  No. of ways of selecting 5 relatives from the remaining 9 relatives $= \binom{9}{5}$
- Total no. of ways of selection $= \binom{9}{3} + \binom{9}{5}$

(ii) Two particular persons will not attend together.

(ii)

- If two particular persons P_1 and P_2 are not invited,
  No. of ways of selecting 5 relatives from the remaining 9 relatives $= \binom{9}{5}$

- If P_1 is invited and P_2 is not invited,
  No. of ways of selecting 4 more relatives from the remaining 9 $= \binom{9}{4}$

- By sum rule, Total no. of ways of selection $= \binom{9}{5} + \binom{9}{4} + \binom{9}{4} = 378.$

7. Find the number of committees of 5 that can be selected from 7 men and 5 women if the committee is to consist of at least one man and at least one woman.

- No. of ways of selecting 5 persons from 7 men and 5 women = $\binom{12}{5} = 792$
- No. of ways of selecting 5 men from 7 men = $\binom{7}{5} = 21$
- No. of ways of selecting 5 women from 5 women = $\binom{5}{5} = 1$
- No. of ways of selecting a committee consisting at least one man and at least one woman = $792 - 21 - 1 = 770$.

8. A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in part C. It is required to answer 7 questions selecting at least two questions from each part. In how many ways can a student select his seven questions for answering?

- There are 4 questions in part A, 5 questions in part B & 6 questions in part C.
- If 2 questions in part A, 2 questions in part B and 3 questions in part C are selected, no. of ways of selecting 7 questions $= \binom{4}{2} \times \binom{5}{2} \times \binom{6}{3} = 1200.$
- If 2 questions in part A, 3 questions in part B and 2 questions in part C are selected, no. of ways of selecting 7 questions $= \binom{4}{2} \times \binom{5}{3} \times \binom{6}{2} = 900.$
- If 3 questions in part A, 2 questions in part B and 2 questions in part C are selected, no. of ways of selecting 7 questions $= \binom{4}{3} \times \binom{5}{2} \times \binom{6}{2} = 600.$
- By sum rule, total no. of ways of selecting 7 questions $= 1200 + 900 + 600$
  $= 2700$