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Inverse Laplace Transform

If $L[f(t)] = \overline{f}(s)$, then $f(t)$ is called the inverse Laplace transform of $\overline{f}(s)\}$ and is denoted by $L^{-1}\{\overline{f}(s)\} = f(t)$.

Here L^{-1} denotes the inverse Laplace transform.

Inverse Laplace transforms

$L[f(t)] = \bar{f}(s)$ then

$f(t) = L^{-1}[\bar{f}(s)]$ is called inverse laplace transform.

Examples:-

$\bar{f}(s) = \frac{1}{s}$ then $L^{-1}[\bar{f}(s)] = L^{-1}\left[\frac{1}{s}\right] = 1$

$$L^{-1}\left[\frac{1}{s^2+4}\right] = L^{-1}\left[\frac{1}{2}\,\frac{2}{s^2+2^2}\right]$$

$$= \frac{1}{2} L^{-1}\left[\frac{2}{s^2+2^2}\right] = \frac{1}{2}\sin 2t$$

$$L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$$

Examples: $L^{-1}\left[\frac{1}{s^3}\right] = \frac{t^{3-1}}{(3-1)!} = \frac{t^2}{2!}$

$$L^{-1}\left[\frac{1}{s^5}\right] = \frac{t^4}{4!}$$

Formulas

$$L^{-1}\left[\frac{1}{(s-a)^n}\right] = \frac{e^{at}\, t^{n-1}}{(n-1)!}$$

$$L^{-1}\left[\frac{1}{(s+a)^n}\right] = \frac{e^{-at}\, t^{n-1}}{(n-1)!}$$

$$L^{-1}\left[\frac{s-a}{(s-a)^2+b^2}\right] = e^{at}\cos bt$$

$$L^{-1}\left[\frac{b}{(s-a)^2+b^2}\right] = e^{at}\sin bt$$

$$L^{-1}\left[\frac{s+a}{(s+a)^2+b^2}\right] = e^{-at}\cos bt$$

$$L^{-1}\left[\frac{b}{(s+a)^2+b^2}\right] = e^{-at}\sin bt$$

$$L^{-1}\left[\frac{s-a}{(s-a)^2-b^2}\right] = e^{at}\cosh bt$$

$$L^{-1}\left[\frac{s+a}{(s+a)^2-b^2}\right] = e^{-at}\cosh bt$$

$$L^{-1}\left[\frac{b}{(s-a)^2-b^2}\right]=e^{at}\sinh bt$$

$$L^{-1}\left[\frac{b}{(s+a)^2-b^2}\right]=e^{-at}\sinh bt$$

Examples

$$L^{-1}\left[\frac{2}{(s-2)^2+4}\right]=e^{2t}\sin 2t$$

$$L^{-1}\left[\frac{s+2}{(s+2)^2+16}\right]=e^{-2t}\cos 4t$$

$$L^{-1}\left[\frac{1}{(s+3)^4}\right]=\frac{e^{-3t}\,t^{4-1}}{(4-1)!}=\frac{e^{-3t}\,t^{3}}{3!}$$

$$L^{-1}\left[\frac{s-4}{(s-4)^2-25}\right]=e^{4t}\cosh 5t$$

$$L^{-1}\left[\frac{1}{(s-2)^2+9}\right]=\frac{1}{3}L^{-1}\left[\frac{3}{(s-2)^2+3^2}\right]=\frac{1}{3}e^{2t}\sin 3t$$

$$L^{-1}\left[\frac{6}{(s+3)^2+16}\right]=\frac{6}{4}L^{-1}\left[\frac{4}{(s+3)^2+4^2}\right]=\frac{6}{4}e^{-3t}\sin 4t$$