Permutations

- A permutation is the number of possible arrangements in a set when the order of the arrangements matters.

- The number of permutations of n distinct objects is $n!$ (Taken all at a time)

- The number of circular permutations of n distinct objects is $(n-1)!$

- The number of permutations of size r of n distinct objects is $\frac{n!}{(n-r)!}$

- The number of permutations of n objects of which n_1 are of the first type and n_2 are of the second type is $\frac{n!}{n_1!n_2!}$

Problems:

1. In how many ways can 6 men and 6 women be seated in a row (i) if any person may sit next to any other? (ii) If men and women must occupy alternative seats?

(i) If there is no restriction, 12 persons in a row can sit in $12!$ Ways.

(ii) 6 men in odd places and 6 women in even places can be seated in $6! \times 6!$ ways.

6 men in even places and 6 women in odd places can be seated in $6! \times 6!$ ways.

Therefore, total number of arrangements $= 2 \times 6! \times 6!$

2. In how many ways can three men and three women be seated at a round table if

(i) No restriction is imposed?

(ii) Two particular women must not sit together?

(iii) Each women is to be between two men?

(i) If no restriction imposed, 6 persons can be seated in a round table in $(6-1)! = 5! = 120$ ways.

(ii) Two women can sit together in 2 ways. Consider this as 1 unit.
One unit and 4 remaining persons can sit in $(5-1)! = 4! = 24$ ways.
Therefore, If two women can sit together, total no. of arrangements is $2 \times 24 = 48$
If two women can't sit together, total no. of arrangements is $120 - 48 = 72$

(iii) Three men can be seated in $(3-1)! = 2!$ ways by leaving one seat between them.
Three women can be seated in the remaining 3 seats in $3!$ ways.
Therefore, total number of arrangements is $2! \times 3! = 12$.

3. A student has three books on C++ and four books on Java. In how many ways can he arrange three books on a shelf (i) If there are no restrictions? (ii) If the languages should alternate? (iii) If all the C++ books must be next to each other? (iv) If all the C++ books must be next to each other and all the Java books must be next to each other?

- If there are no restrictions, three books on C++ and four books on Java, totally 7 books can be

  arranged in $7!$ ways.

- Three C++ books in even places and four Java books in odd places can be arranged in

  $3! \times 4! = 144$ ways.

- Three C++ books together can be arranged in $3!$ Ways. Consider this as one unit. Now one unit and four Java books can be arranged in $5!$ ways.

  Therefore, total number of arrangements is $3! \times 5! = 720$.

- Three C++ books can be arranged in $3!$ Ways. Consider this as one unit.

  Four Java books can be arranged in $4!$ Ways. Consider this as one unit.
  Two units can be arranged in 2 ways.

  Therefore, total number of arrangements is $2 \times 3! \times 4!$.

4. Find the number of permutations of the letters of the word MASSASAUGA. In how many of these, all four A's together? How many of them begin with S?

(i) In 10 alphabets, 'S' repeated three times and 'A' repeated 4 times.

Therefore, total no. of permutations $= \frac{10!}{3!\times 4!} = 25{,}200$

(ii) Consider four A's together as one unit. Consider the remaining 6 letters as 6 units. Now, we have 7 units.Out of 7 units, 'S' repeated three times.

Therefore, total no. of permutations $= \frac{7!}{3!} = 840$

(iii) First alphabet is fixed as S. Now, 9 alphabets remaining. In 9 alphabets, 'S' repeated twice and 'A' repeated 4 times.

Therefore, total no. of permutations $= \frac{9!}{2!\times 4!} = 7560$

5. (i) How many arrangements are there for all letters in the word SOCIOLOGICAL? In how many of these arrangements (ii) A and G are adjacent? (iii) All the vowels are adjacent ?

(i) In 12 letters, 'O' repeated thrice and 'C', 'I', 'L' repeated twice each.

Therefore, total number of arrangements= $\frac{12!}{3! \times 2! \times 2! \times 2!} = 99{,}79{,}200$

(ii) A and G together can be arranged in 2 ways. Consider this as one unit.

Consider the remaining 10 letters as 10 units. Now we have 11 units

In 11 units, 'O' repeated thrice and 'C', 'I', 'L' repeated twice each.

Therefore, total number of arrangements= $2 \times \frac{11!}{3! \times 2! \times 2! \times 2!} = 16{,}63{,}200$

(iii) Two I's and three O's together can be arranged in $\frac{5!}{2! \times 3!} = 10$ ways.

Consider this as a single unit. Consider the remaining 6 letters as 6 units.

In 7 units, 'C' and 'L' repeated twice each.

Therefore, total number of arrangements= $10 \times \frac{7!}{2! \times 2!}$

6. (i) Find the number of permutations of the letters of the word MISSISSIPPI

(ii) How many of these begin with I?

(iii) How many of these begin and end with an S?

(i) In 11 letters, ‘S’ and ‘I’ repeated 4 times each and ‘P’ repeated twice.

Therefore, total no. of permutations $= \frac{11!}{4!\times 4!\times 2!} = 34{,}650$

(ii) First letter is fixed as I. Now,10 letters remaining. In 10 letters, ‘S’ repeated four times, ‘I’ repeated thrice and ‘P’ repeated twice.

Therefore, total no. of permutations $= \frac{10!}{4!\times 3!\times 2!} = 12.600$

(iii) Starting and ending letters are fixed as ‘S’. Now, 9 letters remaining. In 9 letters, ‘S’ and ‘P’ repeated twice each and ‘I’ repeated four times.

Therefore, total no. of permutations $= \frac{9!}{2!\times 2!\times 4!} = 3780$

7. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?

We have 7 digits, out of which there are two 4's and two 5's.

Let $n = x_1x_2x_3x_4x_5x_6x_7$. x_1 must be 5 or 6 or 7.

Suppose $x_1 = 5$, remaining 6 digits can be arranged in $\frac{6!}{2!} = 360$ ways.

Suppose $x_1 = 6$, remaining 6 digits can be arranged in $\frac{6!}{2! \times 2!} = 180$ ways.

Suppose $x_1 = 7$, remaining 6 digits can be arranged in $\frac{6!}{2! \times 2!} = 180$ ways.

Therefore total no. of arrangements$= 360 + 180 + 180 = 720$.

8. How many different three-digit numbers can be formed with 3 four's, 4 two's and 2 three's?

We have 9 digits, out of which there are three 4's, four 2's and two 3's.

let $n = x_1x_2x_3$

Suppose $x_1 = 3, x_2 \neq 3,$ remaining 2 digits can be arranged in $2 \times 3 = 6$ ways.

Suppose $x_1 = 3, x_2 = 3,$ remaining digit can be arranged in 2 ways.

Suppose $x_1 = 4,$ remaining 2 digits can be arranged in $3 \times 3 = 9$ ways.

Suppose $x_1 = 2,$ remaining 2 digits can be arranged in $3 \times 3 = 9$ ways.

Therefore, total no. of arrangements $= 6 + 2 + 9 + 9 = 26$

PRACTICE WORK

1. How many 8 digit numbers have one or more repeated digits? Answer: $10^8 - \binom{10}{8}$

2. How many different strings (sequences) of length four can be formed using the letters of the word FLOWER? Answer: $\binom{6}{4}$

3. How many nine letter words can be formed by using the letters of the word DIFFICULT? Answer: $\frac{9!}{2!\times 2!}$

4. Find the number of permutations of all letters of the word BASEBALL if the words are begin and end with a vowel? Answer: 540

5. How many four digit numbers can be formed with the 10 digits 0,1,2,3,4,5,6,7,8,9

a) If repetitions are allowed?

b) Repetitions are not allowed? c) The last digit must be zero and repetitions are not allowed?

Answer: (a) 9000 (b) 4536 (c) 504

6. In how many ways can 7 books be arranged on a shelf if

(a) Any arrangement is allowed

(b) Three particular books must always be together?

(c) Two particular books must occupy the ends? Answer: (a) 5040 (b) 720 (c) 240