Module- I
IV Part

METHODS OF PROOFS

1. Direct Proofs
2. Proof by Contraposition

   (Indirect Proofs):
3. Proof by Contradiction

Direct Proofs: A direct proof shows that a conditional statement $p \rightarrow q$ is true by showing that if p is true, then q must also be true.

Working Procedure:

Given a conditional statement $p \rightarrow q$,

First step: Assume that p is true;

Second step: By using rules of inference/ laws of logic and other known facts, infer that q is true.

Third step: It follows that the given conditional $p \rightarrow q$ is true.

Give a direct proof of the theorem "If n is an odd integer, then n^2 is odd."

Solution:

Given, $P \to Q$, where P is "n is an odd integer" and Q "n^2 is odd."

Assume that P is true.

i.e., n is an odd integer

$\Rightarrow n = 2k + 1$ where $k \in Z$

Now, $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2m + 1$

where $m = 2k^2 + 2k$

$\Rightarrow n^2 = 2m + 1$ which is odd

$\therefore$ Q is true.

It follows that $P \to Q$ is true.

Proof by Contraposition (Indirect Proofs):

Conditional statement $p \rightarrow q$ is equivalent to its contrapositive, $\neg q \rightarrow \neg p$.

An indirect proof or Proofs by contraposition shows that a conditional statement $p \rightarrow q$ is true if its contrapositive, $\neg q \rightarrow \neg p$, is true.

Working Procedure:

Given a conditional statement $p \rightarrow q$,

First step: Assume that $\neg q$ is true;

Second step: By using rules of inference/ laws of logic and other known facts, infer that $\neg p$ is true.

Third step: It follows that the contrapositive, $\neg q \rightarrow \neg p$, is true, hence the given conditional $p \rightarrow q$ is true.

Prove by Contraposition "If n is an integer then $3n + 2$ is odd ."

Solution:

Given, $P \rightarrow Q$, where P is "n is an integer" and Q "$3n + 2$ is odd."

To prove $P \rightarrow Q$ is true by Contraposition, we need to prove $\neg Q \rightarrow \neg P$ is true.

Assume that $\neg Q$ is true.

i.e., Q is false

$\Rightarrow$ $3n + 2$ is not odd

$\Rightarrow$ $3n + 2$ is even

$\Rightarrow$ $3n + 2 = 2k$ where $k \in Z$

$\Rightarrow$ $3n = 2k - 2$

$\Rightarrow$ $n = (2k - 2) / 3$ which is not integer.

$\therefore$ P is false.

$\Rightarrow$ $\neg P$ is true.

It follows that $\neg Q \rightarrow \neg P$ is true which is equivalent to $P \rightarrow Q$ is true.

Proof by Contradiction:

In this method, we prove a conditional statement $p \to q$ is true by the following steps:

Working Procedure:

Given a conditional statement $p \to q$,

First step: Assume that $p \to q$ is false;

i.e., p is true and q is false

Second step: Start with the hypothesis q is false, by using rules of inference/ laws of logic and other known facts, infer that p is false.

Third step: This contradicts our assumption that p is true.

Hence our assumption is wrong.

It follows that the given conditional $p \to q$ is true.

Give a proof by contradiction of the theorem "If n is an integer then $3n + 2$ is odd ."

Solution:

Given, $P \rightarrow Q$, where P is "n is an integer" and Q "$3n + 2$ is odd."

To prove $P \rightarrow Q$ is true by Contradiction.

Assume that $P \rightarrow Q$ is false;

i.e., P is true and Q is false

Start with Q is false

$\Rightarrow$ $3n + 2$ is not odd

$\Rightarrow$ $3n + 2$ is even

$\Rightarrow$ $3n + 2 = 2k$ where $k \in Z$

$\Rightarrow$ $3n = 2k - 2$

$\Rightarrow$ $n = (2k - 2) / 3$ which is not integer.

$\therefore$ P is false.

This contradicts our assumption that P is true.

Hence our assumption is wrong.

$\therefore$ $P \rightarrow Q$ is true by Contradiction.

Give (i) a direct proof, (ii) an indirect proof (iii) proof by contradiction for the following statements:

1. **"If n is an odd integer, then $n + 9$ is an even integer."**
2. **If m is an even integer then $m + 7$ is an odd integer.**