M-2 Multi, encoder, decoder.

Assignment - 1

1\. Let p, q and r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following:

Ans. (i) $(p \vee q) \vee r$.

$(0 \vee 0) \vee 1 \equiv 1$

(ii) $(p \wedge q) \wedge r$

$(0 \wedge 0) \wedge 1 \equiv 0$

(iii) $(p \wedge q) \rightarrow r$

$(0 \wedge 0) \rightarrow 1 \equiv 1$.

(iv) $p \rightarrow (q \wedge r)$.

$0 \rightarrow (0 \wedge 1) \equiv 0 \rightarrow 0 \equiv 1$.

(v) $p \wedge (r \rightarrow q)$

$0 \wedge (1 \rightarrow 0) \equiv 0 \wedge 0 \equiv 0$.

(vi) $p \rightarrow (q \rightarrow \neg r)$

$0 \rightarrow (0 \rightarrow 0) \equiv 0 \rightarrow 1 \equiv 1$.

2\. Using the laws of logic, Prove that

$[(\neg p \vee \neg q) \wedge (F_0 \vee p) \wedge p] \Leftrightarrow p \wedge \neg q$

Solⁿ. LHS $(\neg p \vee \neg q) \wedge p \wedge p$ (identity law).

$(\neg p \vee \neg q) \wedge p$ (idempotent law).

$(\neg p \wedge p) \vee (\neg q \wedge p)$ (distributive law).

$F_0 \vee (\neg q \wedge p)$ (inverse law)

$\neg q \wedge p$ (identity law)

$p \wedge \neg q \equiv \text{RHS}$ (Commutative law)

3\. Define dual of a logical statement. Verify the principle of duality for

$[\neg(p \wedge q) \rightarrow \neg p \vee (\neg p \vee q)] \Leftrightarrow (\neg p \vee q)$.

Ans. The dual of a compound proposition that contains only the logical operators $\vee$, $\wedge$, and $\neg$ is the

Compound proposition obtained by replacing each $\vee$ by $\wedge$, each $\wedge$ by $\vee$, each T by F, and each F by T. The dual of s is denoted as S.

$$[\neg(p\wedge q) \to \neg p \vee (\neg p \vee q)]^d \Leftrightarrow [(\neg p \vee q)]^d$$

$[\neg(p\wedge q) \to (\neg p \vee \neg p) \vee (\neg p \vee q)]^d$ [distributive]

$[\neg(p\wedge q) \to \neg p \vee (\neg p \vee q)]^d$

$[(p\wedge q) \vee (\neg p \vee (\neg p \vee q))]^d$ (property)

$[(p\wedge q) \vee ((\neg p \vee \neg p) \vee q)]^d$ [Associative law]

$[(p\wedge q) \vee (\neg p \vee q)]^d$ [Idempotent law]

$\cancel{((p\wedge q)\vee\neg p) \vee ((p\wedge q)\vee q)}$ ~~[Distributive]~~

$\cancel{((q\wedge p)\vee\neg p) \vee ((p\wedge(q\vee q))}$ ~~[Associative]~~

$\cancel{(q\wedge p\vee\neg p)) \vee (p)}$ ~~[commutative]~~

$\cancel{(q\wedge T_0) \vee (p)}$ [

$\cancel{((q\wedge p)\vee\neg p)} \vee (p\wedge q)$

$\cancel{((p\wedge q)\vee\neg p) \vee ((p\wedge q)\vee q)}$

$[((p\wedge q)\vee q)\vee\neg p]^d$ [Associative, Commutative]

$[(q\vee\neg p)]^d$ [Absorption law]

$[(\neg p\vee q)]^d$ [Commutative law]

$\equiv$ RHS.

Q4. Define tautology. Show that $[(p \vee q) \wedge [(p \to r) \wedge (q \to r)]] \to r$ is a tautology by constructing truth table.

sol^n The Statement which is always true for any Condition is called tautology.

p	q	r	$p \vee q$ (c)	$p \to r$ (a)	$q \to r$ (b)	$a \wedge b$	$c \wedge (a \wedge b)$ (d)	$d \to r$
1	1	1	1	1	1	1	1	1
1	1	0	1	1	0	0	0	1
1	0	1	1	0	1	0	0	1
1	0	0	1	0	1	0	0	1
0	1	1	1	1	1	1	1	1
0	1	0	1	1	0	0	0	1
0	0	1	0	1	1	1	0	1
0	0	0	0	1	1	1	0	1

Hence the given statement is always true for any arrangement p, q, r it's called tautology.

Q5. For any statements p and q, Prove the following:

(i) $\neg(p \downarrow q) \iff (\neg p \uparrow \neg q)$

~~LHS $(\neg p \uparrow \neg q) \iff$ RHS~~ ~~[Demorgan's]~~

(ii) $\neg(p \uparrow q) \iff (\neg p \downarrow \neg q)$

(i) $\neg(p \downarrow q) \iff (\neg p \uparrow \neg q)$.

$\neg(\neg(p \vee q))$ [def^n].

$\neg(\neg p \wedge \neg q)$ [demorgan's]

$(\neg p \uparrow \neg q)$ [def^n of NAND].

(ii) $\neg(p \uparrow q) \iff (\neg p \downarrow \neg q)$.

$\neg(\neg(p \wedge q))$ [def^n of NAND]

$\neg(\neg p \vee \neg q)$ [demorgan's].

$\neg p \downarrow \neg q$ [def^n of NOR]

Q6. Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow (R \vee S)$.

	Using step	Step	Rule Premises
1	---	$(C \vee D) \rightarrow \neg H$	P 1
2	---	$\neg H \rightarrow (A \wedge \neg B)$	P 2
3	---	$(A \wedge \neg B) \rightarrow (R \vee S)$	P 3
4	1,2,3	$(C \vee D) \rightarrow (R \vee S)$	Rule of syllogism
5.	---	$C \vee D$	P 4
6.	5	$(R \vee S)$	Modus Tollens

Q7. Test the validity of the following argument: if Ravi studies, then he will pass in DMS. if Ravi does not play cricket, then he will study. Ravi failed in DMS. Therefore, Ravi played cricket.

Answer:- $p \rightarrow$ Ravi studies

$q \rightarrow$ He will pass in DMS

$r \rightarrow$ Ravi plays cricket

premises

$P_1 \equiv p \rightarrow q$ $\quad P_2 \equiv \neg r \rightarrow p$ $\quad P_3 \equiv \neg q$

$\therefore r$.

	using step	Step	Rule
1	---	$p \rightarrow q$	P-1
2	---	$\neg q$	P-3
3	1, 2	$\neg p$	Modus tollens
4	---	$\neg r \rightarrow p$	P-3
5	3,4.	r	Modus tollens

$\therefore$ Ravi played Cricket this argument is valid.

Q8. Find whether the following argument is valid or not:
if a triangle has two equal sides then it is isoceles.
if a triangle is isoceles, then it has two equal angles.
The ~~angles~~ triangle ABC does not have two equal angles
Therefore, ABC does not have two equal sides.

Ans. p → A Δ has two equal sides.
q → it is isoceles
r → it has two equal angles.

Premies:-

$P_1 \equiv p \to q$ $\quad P_2 \equiv q \to r$ $\quad P_3 \equiv \neg r$

∴ ABC does not have two equal sides $(\neg p)$.

	using step	Step	Rule
1	--	$p \to q$	P-1
2	---	$q \to r$	P-2
3	1,2	$p \to r$	Rule of syllogism
4	----	$\neg r$	P-3
5	3,4.	$\neg p$	Modus tollens

Hence the given argument is valid.

Q9. Consider the following statements with a set of all real numbers as the universe.

$P(x): x \geq 0$, $q(x): x^2 \geq 0$, $r(x): x^2-3x-4=0$,

$s(x): x^2-3>0$

Determine the truth values of the following.

(a) $\exists x, P(x) \wedge q(x)$

Let $x = 2$.

$P(2) \wedge q(2)$.

$(2 \geq 0) \wedge (4 \geq 0)$

T $\wedge$ T.

$\equiv$ True.

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(b) $\forall x, P(x) \to q(x) \equiv \neg P(x) \vee q(x)$

$q(x): x^2 \geq 0$ is always true.

$\neg P(x) \vee (\text{True})$

$\equiv$ True.

(c) $\forall x, q(x) \to s(x) \equiv \neg q(x) \vee s(x)$.

$q(x)$ is always true.

$F \vee s(x) \equiv s(x)$.

for $x = 0$

$s(0) \equiv 0 - 3 > 0$

False.

(d) $\forall x, r(x) \vee s(x)$.

$(x^2 - 3x - 4 = 0) \vee (x^2 - 3 > 0)$

$((x-4)(x+1) = 0) \vee (x^2 - 3 > 0)$

for $x = 0$

$((-4)(1) = 0) \vee (0 - 3 > 0)$.

$F \vee F$

False.

(e) $\exists x, P(x) \wedge r(x)$.

$(x \geq 0) \wedge ((x-4)(x+1) = 0)$

for $x = 4$

$P(4)$ is true.

$r(4)$ is true.

$T \wedge T \equiv$ True.

Q.10. Give a direct proof, indirect proof and proof by contradiction for:

"For all integers k and l, if k and l are both even then k + l is even".

$\forall K, l \quad p \equiv K \wedge l$ is even.

$q = K + l$ is even

given $p \to q$.

(i) Direct proof of condition ..

Assume p is true.

~~using step~~	step	Rule
1 ~~...~~	~~$p \to q$~~	~~P-1~~
2 ~~..~~	~~p~~	~~P-2~~
3 ~~1, 2~~	q	Modus ponens

$\Rightarrow$ K and l are both even

$\Rightarrow K = 2m, \ l = 2n$, where $m, n \in I$

$k = 2m + 2n$

$K = 2(m+n)$

$k + l$ is even

q is true $\qquad p \to q$ is true.

∴ The given statement is true by dP.

(ii) Indirect proof :-

$p =$ K and l are even

$q = K + l$ is even

Assume $\neg q$ is true.

$K + l$ is not even $m, n \in I$

Assume $K = 2m \quad l = 2n + 1$

$K + l = 2m + 2n + 1$

$= 2(m+n) + 1$

$=$ odd.

$\neg p$ is true.

$\neg q \to \neg p$ is true and

hence $p \to q$ is true

The given statement is true by indirect proof

(iii) Contradiction.

Assume $p \to q$ is false.

i.e. p is true q is false

$K + l$ is ~~even~~ odd

~~Assume $K = 2m + 1 \quad m, n \in I$~~

~~and $l = 2n$~~

~~but K and l never be ...~~

~~if $K + l$ is odd~~

~~so $K + l = 2(m) + 1$~~

~~$m = (K + l) - 1$ is odd~~

~~q is false~~

$p \vee q$
$\neg q$

Q11. Establish the validity of the following argument using the rules of inference:

$p,\ p \to r,\ p \to (q \vee \neg r),\ \neg q \vee \neg s \quad \therefore s.$

1	----	$p \to r$	P-2
2	---	p	P-1
3	1, 2	r	Modus pones.
4	---	$p \to (q \vee \neg r)$	P-3
5	2, 4	$(q \vee \neg r)$	Modus Pones.
6	5, 3	q	Rule of disjunctive syllogism
7	---	$(\neg q \vee \neg s)$	P-3
8	6, 7	$\neg s$	Disjunctive amplification

$\therefore$ given argument is not valid.

Q12. Prove by mathematical induction that for all positive integers $n \geq 1$,

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

proof: Consider $S(n)$: $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2$

Basic step: $S(1) = 1^2 = \frac{1(1)(3)}{3}$ is true Clearly.

So it is verified that $S(n)$ is true for $n=1$.

Induction Step:

Assume $S(k)$: $1^2 + 3^2 + 5^2 + \cdots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$

is true.

$2k^2+2k+3k+3 \quad 2k+2-1$

$2k(2k+1) \quad 3(k+1)$

$$S(k+1) = 1^2+3^2+5^2+ \cdots (2k-1)^2 + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$$

$$= \frac{(2k+1)}{3}\left(k(2k-1) + 3(2k+1)\right)$$

$$= \frac{(2k+1)}{3}(2k^2-k+6k+3)$$

$$= \frac{(2k+1)}{3}(2k^2+5k+3)$$

$$= \frac{(2k+1)(k+1)(2k+3)}{3}$$

$$= \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$$

it is proved that $S(k+1)$ is true.
Therefore, by the principle of mathematical induction, $S(n)$ is true for any $n \geq 1$.

Q.13. prove by Mathematical induction

$$1^3+2^3+3^3+ \cdots + n^3 = \frac{n^2(n+1)^2}{4} \quad \forall n \in N$$

sol^n proof:

Basic step: $S(1) = 1^3 = \frac{1^2(1+1)^2}{4}$ is true clearly

So, it is verified that $S(n)$ is true for $n=1$.

Induction step:

Assume $S(k)$: $1^3+2^3+3^3 \cdots + k^3 = \frac{k^2(k+1)^2}{4}$ is true.

$$S(K+1) = 1^3 + 2^3 + 3^3 - - - K^3 + (K+1)^3$$

$$= \frac{K^2 (K+1)^2}{4} + (K+1)^3$$

$$= \frac{K^2 (K+1)^2 + 4(K+1)^3}{4}$$

$$= \frac{(K+1)^2}{4} (K^2 + 4(K+1))$$

$$= \frac{(K+1)^2}{4} (K^2+4K+4).$$

$$= \frac{(K+1)^2 (K+2)^2}{4}$$

$$= \frac{(K+1)^2 ((K+1)+1)^2}{4}$$

it is proved that $S(K+1)$ is true.

$\therefore$ By principle of mathematical induction $S(n)$ is true for any. $n \geq 1$. or $n \in N$.

Q14. prove by mathematical induction:

$$\frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

sol^n:- Let $S(n) = \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \dots \frac{1}{(3n-1)(3n+2)}$

Basic step: $S(1) = \frac{1}{2\cdot5} = \frac{1}{6+4}$ it is true clearly.

it is verified that $S(n)$ is true for $n=1$.

Induction step: Assume $S(K)$: $\frac{1}{2\cdot5} + \frac{1}{5\cdot8} \dots + \frac{1}{(3K-1)(3K+2)} = \frac{K}{6K+4}$

is true.

$$S(k+1) = \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \dots + \frac{1}{(3k-1)(3k+2)} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{6k+4} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{1}{3k+2}\left(\frac{k}{2} + \frac{1}{(3k+5)}\right)$$

$$= \frac{1}{(3k+2)}\left(\frac{3k^2+5k+2}{6k+10}\right)$$

$$= \frac{1}{(3k+2)}\left(\frac{(3k+2)(k+1)}{6k+10}\right)$$

$$= \frac{k+1}{6(k+1)+4}$$

it is proved that $S(k+1)$ is true.

$\therefore$ by the principle of MI, $S(n)$ is true for any $n \geq 1$.

Q15 | Prove by Mathematical induction that:

$$1\cdot3 + 2\cdot4 + 3\cdot5 + \dots + n(n+2) = \frac{1}{6}n(n+1)(2n+7).$$

Sol^n. Let $S(n) = 1\cdot3 + 2\cdot4 \dots + n(n+2)$.

Basic step. $S(1) = 1\cdot3 = \frac{1}{6} 1 (2)(9)$ it is true clearly.

Induction step $S(k) = 1\cdot3 + 2\cdot4 \dots k(k+2)$ is true (Assume).

$S(k+1) = 1\cdot3 + 2\cdot4 \dots k(k+2) + (k+1)(k+3)$ ~~is true~~

$$= \frac{1}{6}k(k+1)(2k+7) + (k+1)(k+3)$$

$$= \frac{k(k+1)(2k+7) + 6(k+1)(k+3)}{6}$$

$$= \frac{1}{6} \frac{(k+1)(2k^2+13k+18)}{6}$$

$$= \frac{(k+1)(k+2)(2(k+1)+7)}{6}$$

it is proved that $S(k+1)$ is true.

$\therefore$ by principle of MI, $S(n)$ is true for all $n \geq 1$.