CANARA ENGINEERING COLLEGE

BANTWAL, MANGALURU 574219

COMPUTER SCIENCE & ENGINEERING

CLASS

Computer **L**earning **A**dventure in **S**ocial **S**phere

COURSE TITLE (CODE)	ANALOG & DIGITAL ELECTRONICS(18CS33)
MODULE/ CHAPTER	MODULE 2
TOPICS	SOP, K-MAP, QM MERHOD, EVM
FACULTY	ANUPAMA V

"QUALITY EDUCATION AT AFFORDABLE COST"

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Text Book:

1. Charles H Roth and Larry L Kinney, Analog and Digital Electronics, Cengage Learning, 2019

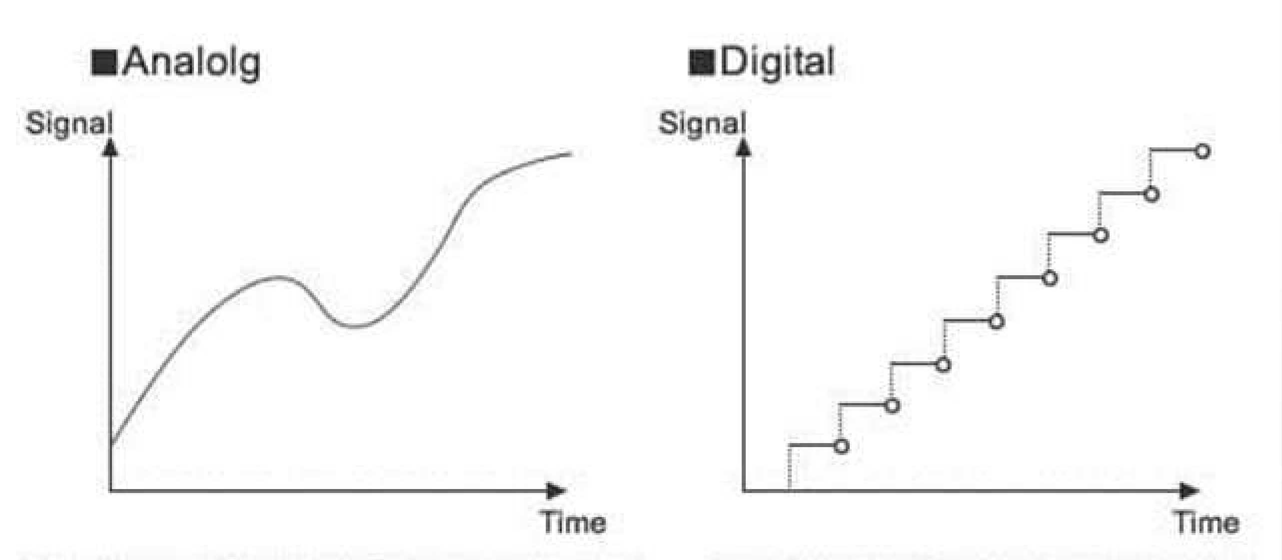
Reference Books:

1. Anil K Maini, Varsha Agarwal, Electronic Devices and Circuits, Wiley, 2012.
2. Donald P Leach, Albert Paul Malvino & Goutam Saha, Digital Principles and Applications, 8th Edition, Tata McGraw Hill, 2015.
3. M. Morris Mani, Digital Design, 4th Edition, Pearson Prentice Hall, 2008.
4. David A. Bell, Electronic Devices and Circuits, 5th Edition, Oxford University Press, 2008

Basics

- **Analog electronics** is a electronics system where signal change continuously.
- **Digital electronics** is a field of electronics involving the study of digital signals and the engineering of devices that use or produce them.
- An **analog circuits** operates on continuous signals.
- A **digital circuits** operates on discrete signals.

Basics

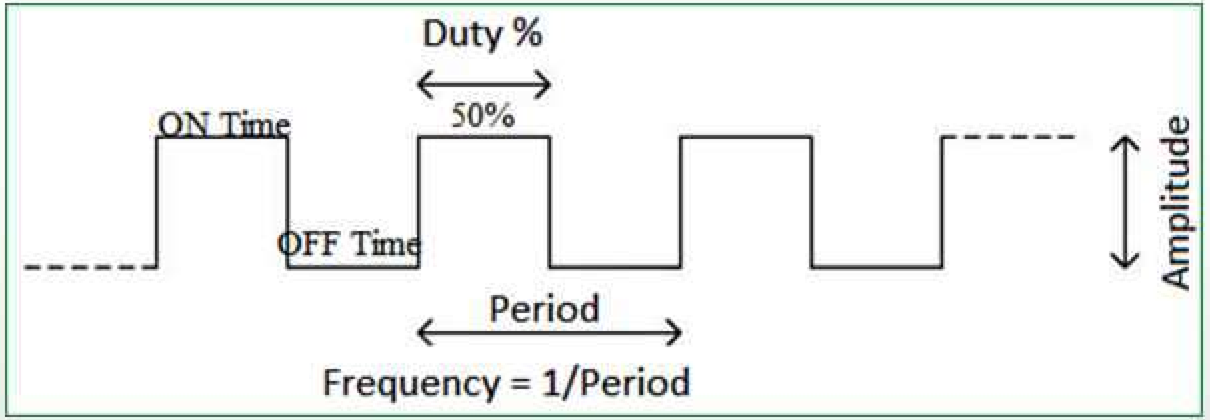


Basics

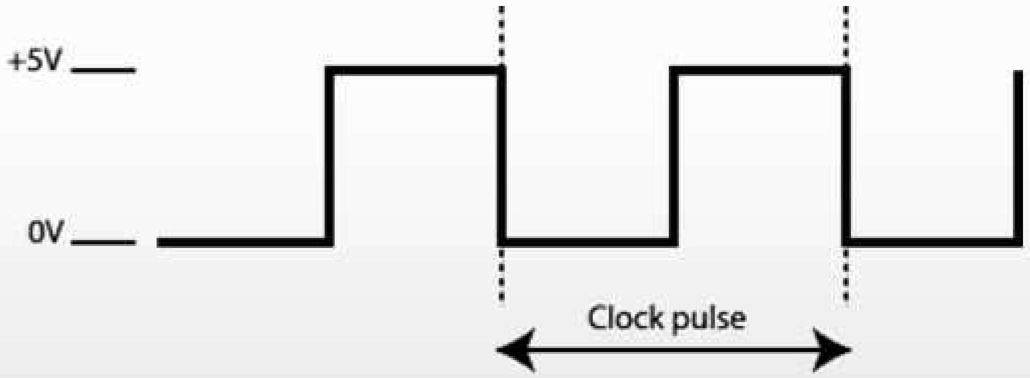
- **Analog signal** is a signal whose amplitude can take any value between given limits. A continuous signal.
- **Digital signal** is a signal whose amplitude can have only given discrete values between defined limits. A signal that changes amplitude in discrete steps.
- **Clock** is a periodic, rectangular waveform used as a basic timing signal.
- **Duty cycle** for a periodic digital signal, the ratio of high level time to the period or the ratio of low level time to the period.

Basics

- Duty Cycle



- Clock



Truth Table

- A table that shows all of the input output possibilities of a logic circuit.

A	Y
0	1
1	0

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Basic Gates

NOT gate: A gate with only one input and a complemented output.

$F = \overline{A}$

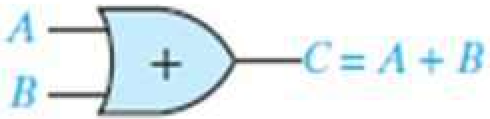
or

$F = A'$

A	F
0	1
1	0

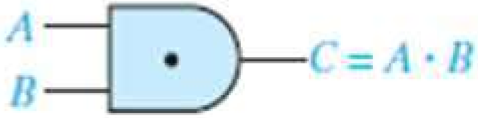
OR gate: A gate with two or more inputs. The output is high when any input is high.

A B	$C = A + B$
0 0	0
0 1	1
1 0	1
1 1	1



AND gate: A gate with 2 or more inputs. The output is high only when all inputs are high.

A B	$C = A \cdot B$
0 0	0
0 1	0
1 0	0
1 1	1



Laws of Boolean algebra

Operations with 0 and 1:

1. $X + 0 = X$ — 1D. $X \cdot 1 = X$
2. $X + 1 = 1$ — 2D. $X \cdot 0 = 0$

Idempotent laws:

3. $X + X = X$ — 3D. $X \cdot X = X$

Involution law:

4. $(X')' = X$

Laws of complementarity:

5. $X + X' = 1$ — 5D. $X \cdot X' = 0$

Commutative laws:

6. $X + Y = Y + X$ — 6D. $XY = YX$

Associative laws:

7. $(X + Y) + Z = X + (Y + Z) = X + Y + Z$ — 7D. $(XY)Z = X(YZ) = XYZ$

Distributive laws:

8. $X(Y + Z) = XY + XZ$ — 8D. $X + YZ = (X + Y)(X + Z)$

DeMorgan's laws:

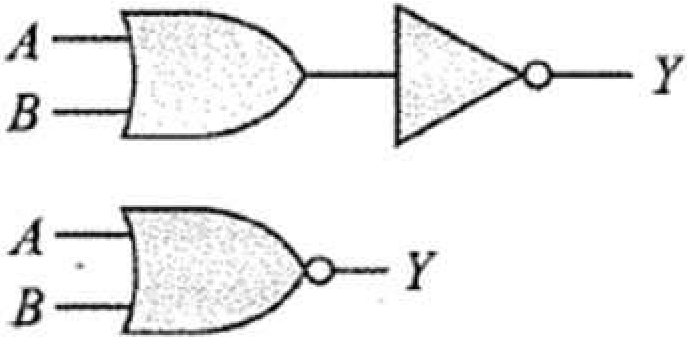
9. $(X + Y)' = X'Y'$ — 9D. $(XY)' = X' + Y'$

Universal gate: NOR

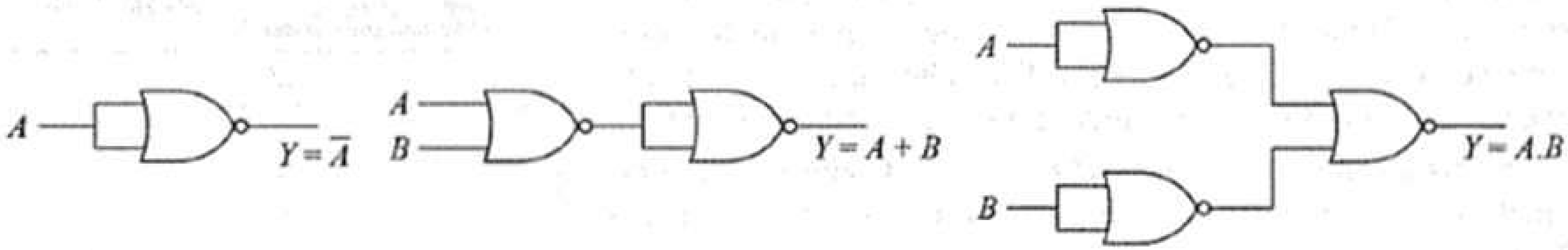
NOR gate: A gate with two or more inputs. The output is low when any input is high.

$Y = \overline{A + B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0



Universality of NOR gate:



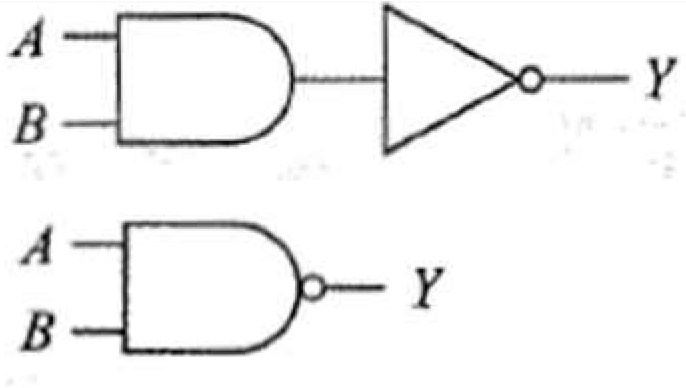
NOT from NOR OR from NOR AND from NOR

Universal Gate: NAND

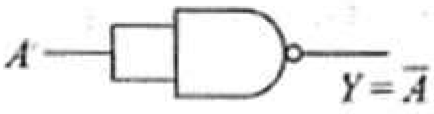
NAND gate: A gate with two or more inputs. The output is low when all input is high.

$Y = \overline{AB}$

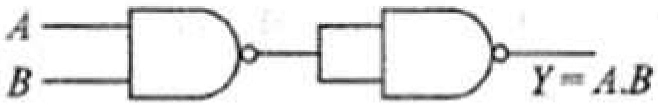
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



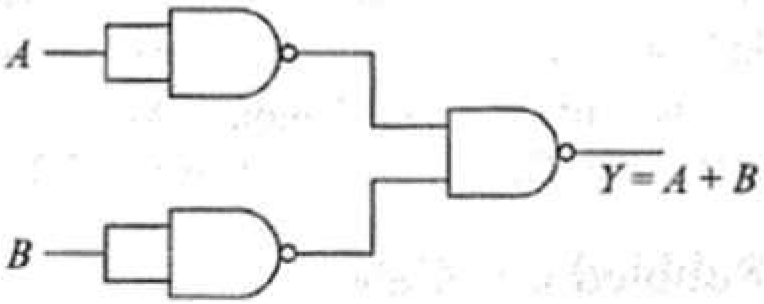
Universality of NAND gate:



NOT from NAND

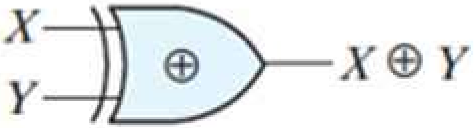
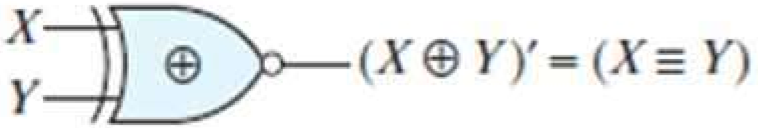


AND from NAND



OR from NAND

XOR and XNOR Gates

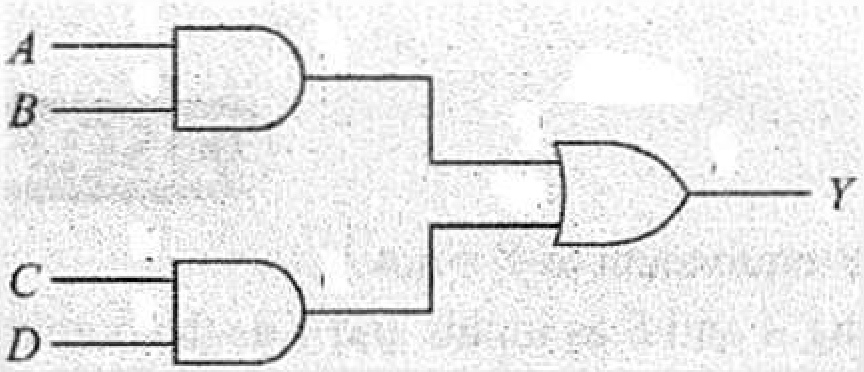
Exclusive-OR gate: A gate with two or more inputs and output is HIGH only when the number of HIGH inputs is odd.	X Y | $X \oplus Y$ 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0	
Equivalence/exclusive-NOR gate: A gate with two or more inputs and output of HIGH only when the number of HIGH inputs is even.	X Y | $X \equiv Y$ 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1	

Assertion-level logic

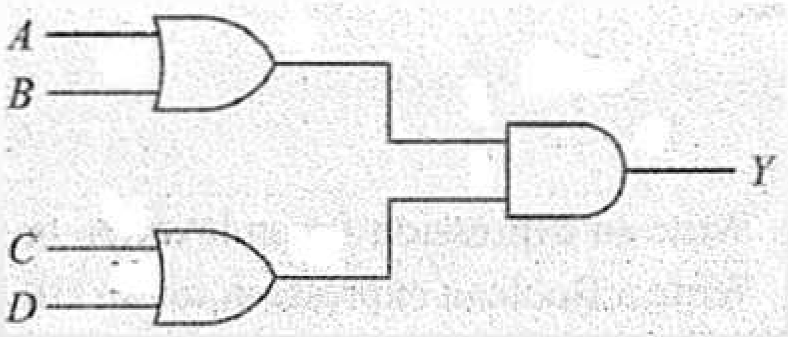
- In a positive logic system, binary 0 stands for low voltage and binary 1 for high voltage.
- In a negative logic system, binary 0 stands for high voltage and binary 1 for low voltage.
- Assert means to activate. If an input line has a bubble on it, you assert the input by making it low. If there is no bubble, you assert the input by making it high.
- Active-low refers to the concept in which a signal must be low to cause something to happen or to indicate that something has happened.

Realize using basic gate.

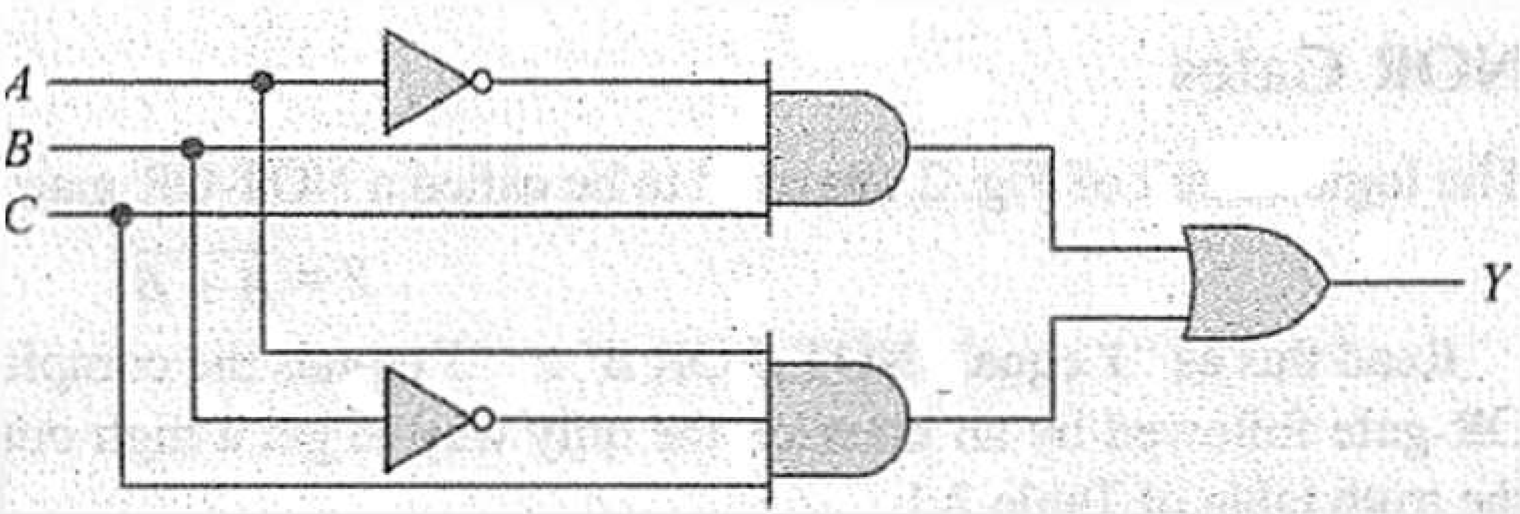
Y=AB+CD



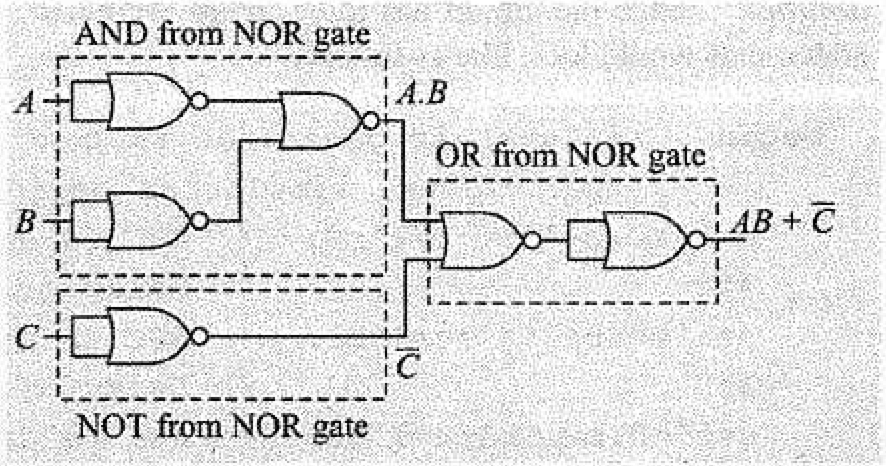
Y=(A+B)(C+D)

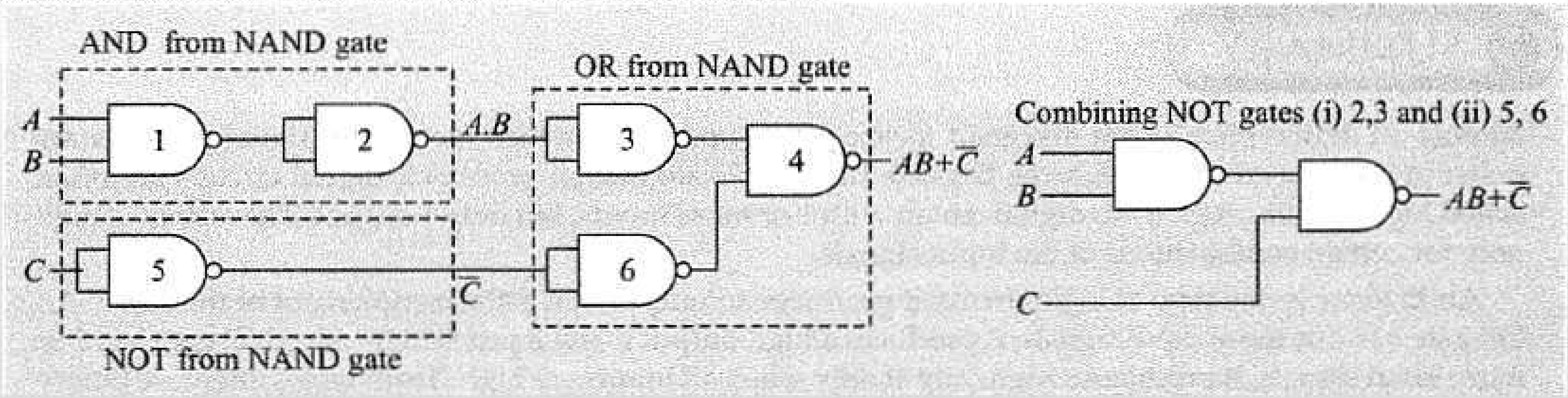


Write Boolean equation and truth table.



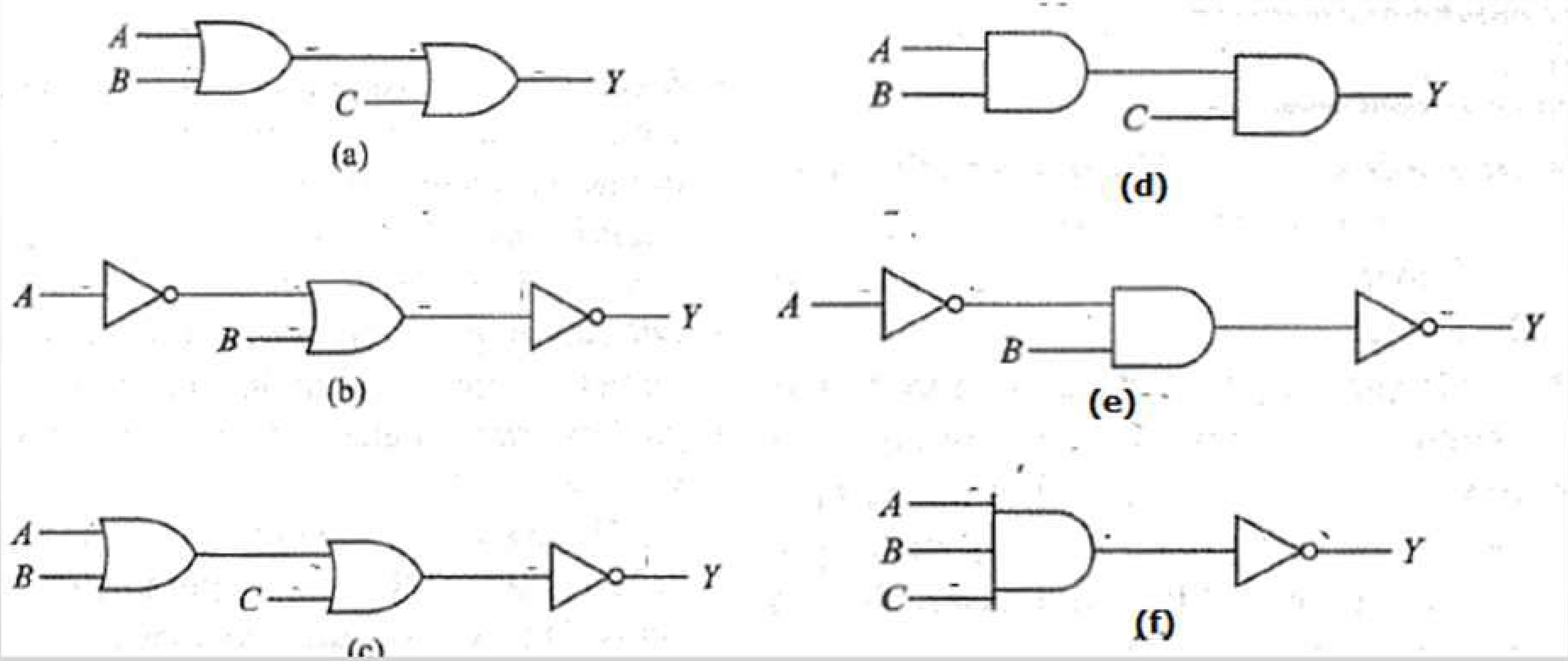
Realize $Y = AB + C$ using only one type of gate.





Solve:

Write Boolean equation and truth table.
Realize using only one type of gate.



Minterm and Maxterm

- **Combinational Circuits** are circuits made up of different types of logic gates. The output of the combinational circuit depends on the values at the input at any given time. The circuits do not make use of any memory or storage device.
- A **literal** is a variable or its complement.
- A **minterm** of n variables is a product of n literals in which each variable appears exactly once in either true or complemented form, but not both.
- A **maxterm** of n variables is a sum of n literals in which each variable appears exactly once in either true or complemented form, but not both.

Minterm and Maxterm (contd…)

- **Example**: Minterms and Maxterms for Three Variables

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

SOP and POS (contd…)

- **minterm expansion or a standard sum of products (SOP)**: A Boolean equation that is the logical sum of logical products. This type of equation applies to an AND-OR circuit.
- **maxterm expansion or standard product of sums (POS)**: A Boolean equation that is the logical product of logical sums. This type of equation applies to an OR-AND circuit.

Steps to get SOP

1. Locate each output 1 in truth table
2. Write the respective minterm
3. Apply OR operation to the minterms

A	B	C	Y	
0	0	0	0	
0	0	1	1	$\overline{A}.\overline{B}.C$
0	1	0	1	$\overline{A}.B.\overline{C}$
0	1	1	1	$\overline{A}.B.C$
1	0	0	0	
1	0	1	0	
1	1	0	1	$A.B.\overline{C}$
1	1	1	0	

$\overline{A}.\overline{B}.C + \overline{A}.B.\overline{C} + \overline{A}.B.C + A.B.\overline{C}$

$\Sigma m(1, 2, 3, 6)$

Steps to get POS

1. Locate each output 0 in truth table
2. Write the respective maxterm
3. Apply AND operation to the maxterms

A	B	C	Y	
0	0	0	0	$A+B+C$
0	0	1	1	
0	1	0	1	
0	1	1	1	
1	0	0	0	$\overline{A}+B+C$
1	0	1	0	$\overline{A}+B+\overline{C}$
1	1	0	1	$\overline{A}+\overline{B}+C$
1	1	1	0	$\overline{A}+\overline{B}+\overline{C}$

$(A+B+C).\ (\overline{A}+B+C).\ (\overline{A}+B+\overline{C}).\ (\overline{A}+\overline{B}+\overline{C})$

$\Pi M(0, 4, 5, 7)$

Conversion between SOP and POS

1. identifying complementary locations,
2. changing mintenn to maxtenn or reverse, and finally
3. changing summation by product or reverse

Ex1: $Y = F(A, B, C) = \Pi M(O, 3, 6)$

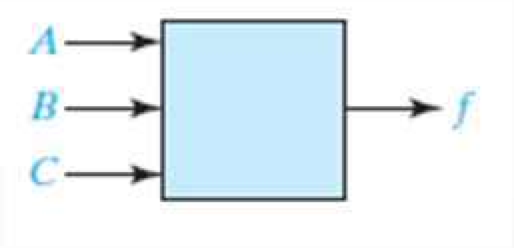
$= \Sigma m(l, 2, 4, 5, 7)$

Ex2: $Y = F(A, B, C) = \Sigma m(3, 5, 6, 7)$

$= \Pi M(O, 1, 2, 4)$

SOP and POS (contd…)

- **Example**: Combinational Circuit with Truth Table is given write SOP and POS expressions.



A	B	C	f	f′
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Minterms: $A'BC$, $AB'C'$, $AB'C$, ABC', ABC

$f = A'BC + AB'C' + AB'C + ABC' + ABC$

$f(A, B, C) = m_3 + m_4 + m_5 + m_6 + m_7$

$f(A, B, C) = \Sigma\, m(3, 4, 5, 6, 7)$ (SOP)

Maxterms: $(A + B + C)$, $(A + B + C')$, $(A + B' + C)$

$f = (A + B + C)(A + B + C')(A + B' + C)$

$f(A, B, C) = M0\ M1\ M2$

$f(A, B, C) = \Pi\, M(0, 1, 2)$ (POS)

SOP and POS (contd…)

- Write minterm and maxterm expansions for the following truth table.

A	B	C	D	X	Y	Z
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	1
1	1	1	1	1	1	0

- Answer:

$$X(A, B, C, D) = \Sigma\, m(7, 10, 11, 13, 14, 15)$$
$$Y(A, B, C, D) = \Sigma\, m(2, 3, 5, 6, 8, 9, 12, 15)$$
$$Z(A, B, C, D) = \Sigma\, m(1, 3, 4, 6, 9, 11, 12, 14)$$

SOP and POS (contd…)

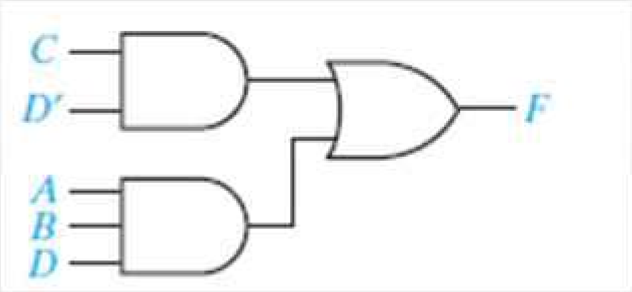
- Write minterm and maxterm expansions for the following truth table.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

- Answer:

$$
\begin{aligned}
F &= \Sigma\, m(2, 6, 10, 13, 14, 15) \\
&= A'B'CD' + A'BCD' + AB'CD' + ABCD' + ABC'D + ABCD \\
&= A'CD' + ACD' + ABD = CD' + ABD
\end{aligned}
$$

- Realization using basic gates



SOP and POS (contd...)

- An **Incompletely specified function** is a Boolean function that only define output values for a subset of its inputs
- Example: Truth Table with Don't-Cares

A	B	C	F
0	0	0	1
0	0	1	X
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	X
1	1	1	1

- $F(A,B,C) = \Sigma\, m(0, 3, 7) + \Sigma\, d(1, 6)$
- $F(A,B,C) = \Pi\, M(2, 4, 5) \cdot \Pi\, D(1, 6)$

SOP and POS (contd…)

- Write minterm and maxterm expansions for the following truth table.

A	B	C	D	Z
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Answer:

$Z = \Sigma\, m(0, 3, 6, 9) + \Sigma\, d(10, 11, 12, 13, 14, 15)$

$Z = \Pi\, m(1, 2, 4, 5, 7, 8) + \Sigma\, \mathrm{D}(10, 11, 12, 13, 14, 15)$

SOP and POS (contd...)

Find the minterm expansion of $f(a, b, c, d) = a'(b' + d) + acd'$.

$$
\begin{aligned}
f &= a'b' + a'd + acd' \\
&= a'b'(c + c')(d + d') + a'd(b + b')(c + c') + acd'(b + b') \\
&= a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + \cancel{a'b'c'd} + \cancel{a'b'cd} \\
&\quad + a'bc'd + a'bcd + abcd' + ab'cd' \qquad (4\text{-}9)
\end{aligned}
$$

Duplicate terms have been crossed out, because $X + X = X$. This expression can then be converted to decimal notation:

$$
\begin{array}{cccccccccccccccc}
f = a'b'c'd' & + & a'b'c'd & + & a'b'cd' & + & a'b'cd & + & a'bc'd & + & a'bcd & + & abcd' & + & ab'cd' \\
0\,0\,0\,0 & & 0\,0\,0\,1 & & 0\,0\,1\,0 & & 0\,0\,1\,1 & & 0\,1\,0\,1 & & 0\,1\,1\,1 & & 1\,1\,1\,0 & & 1\,0\,1\,0
\end{array}
$$

$$f = \Sigma\, m(0, 1, 2, 3, 5, 7, 10, 14) \qquad (4\text{-}10)$$

The maxterm expansion for f can then be obtained by listing the decimal integers (in the range 0 to 15) which do not correspond to minterms of f:

$$f = \Pi\, M(4, 6, 8, 9, 11, 12, 13, 15)$$

SOP and POS (contd...)

Show that $a'c + b'c' + ab = a'b' + bc + ac'$.

We will find the minterm expansion of each side by supplying the missing variables. For the left side,

$$\begin{aligned} a'c(b + b') + b'c'(a + a') + ab(c + c') \\ &= a'bc + a'b'c + ab'c' + a'b'c' + abc + abc' \\ &= m_3 + m_1 + m_4 + m_0 + m_7 + m_6 \end{aligned}$$

For the right side,

$$\begin{aligned} a'b'(c + c') + bc(a + a') + ac'(b + b') \\ &= a'b'c + a'b'c + abc + a'bc + abc' + ab'c' \\ &= m_1 + m_0 + m_7 + m_3 + m_6 + m_4 \end{aligned}$$

Because the two minterm expansions are the same, the equation is valid.

SOP and POS (contd…)

1. Given: $F(a, b, c) = abc' + b'$.
   a) Express F as a minterm expansion. (Use m-notation.)
   b) Express F as a maxterm expansion. (Use M-notation.)
2. Given: $F(a, b, c, d) = (a + b + d)(a' + c)(a' + b' + c')(a + b + c' + d')$
   a) Express F as a minterm expansion. (Use m-notation.)
   b) Express F as a maxterm expansion. (Use M-notation.)
3. Given $f(a, b, c, d) = acd + bd' + a'c'd + ab'cd + a'b'cd'$
   a) Express F as a minterm expansion. (Use m-notation.)
   b) Express F as a maxterm expansion. (Use M-notation.)

Minimum Forms of Switching Functions

- **minimum sum-of-products expression**
  - Minimum product terms
  - Minimum of literals in each term
- AND-OR gates
- minimum two-level gate circuit
  - a minimum number of gates
  - a minimum number of gate inputs.
- **minimum product-of-sums expression**
  - Minimum sum terms
  - Minimum of literals in each term

Minimum sum-of-products expression

Example:

$F(a, b, c) = \Sigma\, m\,(0, 1, 2, 5, 6, 7)$

$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$

$F = a'b'(c'+c) + bc'(a'+a) + ac(b+b')$

$F = a'b' + bc' + ac$

Or

$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$

$F = a'b'c' + a'bc' + a'b'c + ab'c + abc' + abc$

$F = a'c' + b'c + ab$

Minimum product-of-sums expression

Example:

$F(a, b, c, d) = \Pi M(5, 7, 6, 14, 2, 10)$

$F = (a+b'+c+d')(a+b'+c'+d')(a+b'+c'+d)(a'+b'+c'+d)(a+b+c'+d)(a'+b+c'+d)$

$F = (a+b'+d')(b'+c'+d)(b+c'+d)$

$F = (a+b'+d')(c'+d)$

Karnaugh Maps

- Why simplification?
  - Gate count reduces
  - Circuit works faster
  - Less power consumption
- Boolean expression simplification techniques are
  - Algebraic techniques
  - Karnaugh Map/K-Map
  - Entered Variable Map/ MEV/EMV
  - Quine McCluskey
- Disadvantage of Algebraic techniques
  - Difficult to apply in systematic
  - Difficult to identify minimum solution

Karnaugh Maps

- A Karnaugh map provides a pictorial method of grouping together expressions with common factors and simplifying the Boolean expression.
- Karnaugh map of a function specifies the value of the function for every combination of values of the independent variables.

	$\bar{B}$	B
$\bar{A}$	0	1
A	2	3

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	1
$\bar{A}B$	2	3
AB	6	7
$A\bar{B}$	4	5

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

Two Variable K-Map

- No. of Variable = 2
- K-map cells = 2^2=4

	$\overline{B}$	B
$\overline{A}$		
A		

	$\overline{B}$	B
$\overline{A}$	0	1
A	2	3

Two Variable K-Map

- Example: Convert following truth table into K map.
- Y = F(A, B) = Σ m (2, 3)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

	$\overline{B}$	B
$\overline{A}$	0	0
A	1	1

Three Variable K-Map

- No. of Variable = 3
- K-map cells = 2^3=8

	$\overline{B}\,\overline{C}$	$\overline{B}\,C$	$B\,C$	$B\,\overline{C}$
$\overline{A}$	0	1	3	2
A	4	5	7	6

Three Variable K-Map

- Example: Convert following truth table into K map.
- Y = F(A, B, C) = Σ m (2,6,7)

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

	$\overline{B}\,\overline{C}$	$\overline{B}\,C$	$B\,C$	$B\,\overline{C}$
$\overline{A}$	0 $_{0}$	0 $_{1}$	0 $_{3}$	1 $_{2}$
A	0 $_{4}$	0 $_{5}$	1 $_{7}$	1 $_{6}$

Four Variable K-Map

- No. of Variable = 4
- K-map cells = 2^4=16

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	0	1	3	2
$\overline{A}B$	4	5	7	6
AB	12	13	15	14
$A\overline{B}$	8	9	11	10

Four Variable K-Map

- Example: Convert following truth table into K map.
- $Y = F(A, B, C, D) = \Sigma\, m(1,6,7,14)$

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 $_{0}$	1 $_{1}$	0 $_{3}$	0 $_{2}$
$\bar{A}B$	0 $_{4}$	0 $_{5}$	1 $_{7}$	1 $_{6}$
AB	0 $_{12}$	0 $_{13}$	0 $_{15}$	1 $_{14}$
$A\bar{B}$	0 $_{8}$	0 $_{9}$	0 $_{11}$	0 $_{10}$

Pairs

- Two adjacent 1's in the K-map is called a **pair** and it eliminate the **variable** that changes form.

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	0 0	0 1	0 3	0 2
$\overline{A}B$	0 4	0 5	0 7	0 6
AB	0 12	0 13	0 15	0 14
$A\overline{B}$	0 8	1 9	1 11	0 10

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	0 0	0 1	0 3	0 2
$\overline{A}B$	0 4	0 5	0 7	0 6
AB	1 12	0 13	0 15	0 14
$A\overline{B}$	1 8	0 9	0 11	0 10

Pairs

- Two adjacent 1's in the K-map is called a **pair** and it eliminate the **variable** that changes form.

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 0	0 1	0 3	0 2
$\bar{A}B$	0 4	0 5	0 7	0 6
AB	0 12	0 13	0 15	0 14
$A\bar{B}$	1 8	0 9	0 11	1 10

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 0	1 1	0 3	0 2
$\bar{A}B$	0 4	0 5	0 7	0 6
AB	0 12	0 13	0 15	0 14
$A\bar{B}$	0 8	1 9	0 11	0 10

Quads

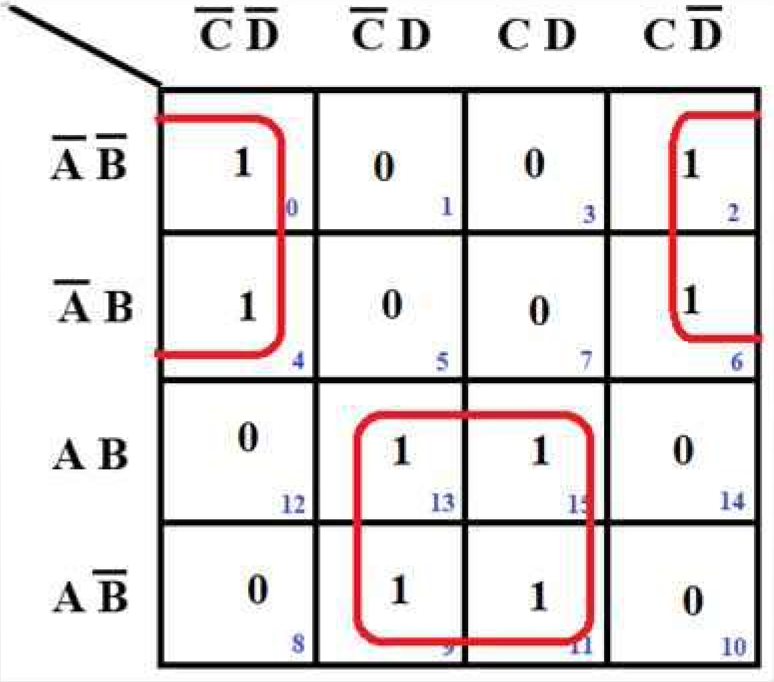
- A **quad** is a group of four l's that are horizontally or vertically adjacent and a quad eliminates two variables and their complements.

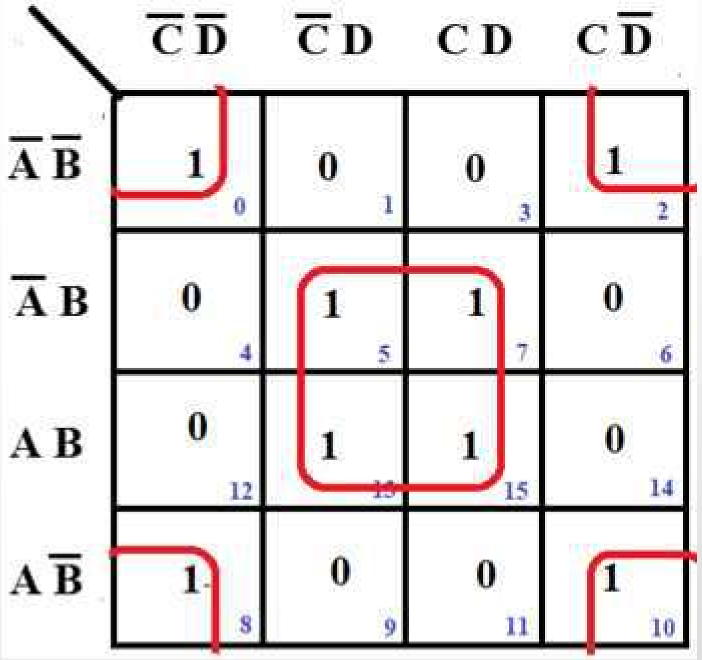
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 $_{0}$	0 $_{1}$	0 $_{3}$	0 $_{2}$
$\bar{A}B$	1 $_{4}$	1 $_{5}$	1 $_{7}$	1 $_{6}$
AB	0 $_{12}$	0 $_{13}$	0 $_{15}$	0 $_{14}$
$A\bar{B}$	0 $_{8}$	0 $_{9}$	0 $_{11}$	0 $_{10}$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 $_{0}$	1 $_{1}$	0 $_{3}$	0 $_{2}$
$\bar{A}B$	0 $_{4}$	1 $_{5}$	0 $_{7}$	0 $_{6}$
AB	0 $_{12}$	1 $_{13}$	0 $_{15}$	0 $_{14}$
$A\bar{B}$	0 $_{8}$	1 $_{9}$	0 $_{11}$	0 $_{10}$

Quads

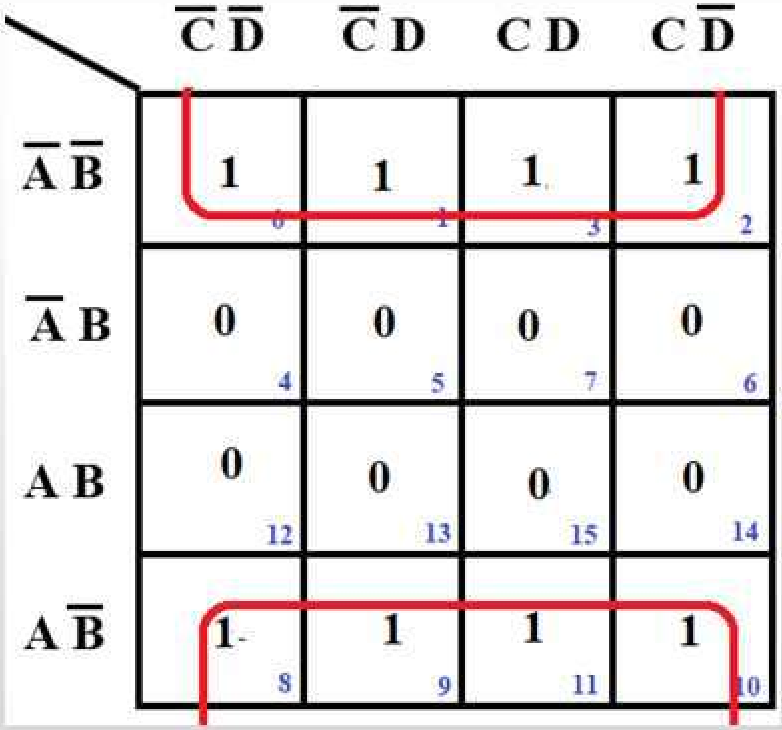
- A **quad** is a group of four l's that are horizontally or vertically adjacent and a quad eliminates two variables and their complements.

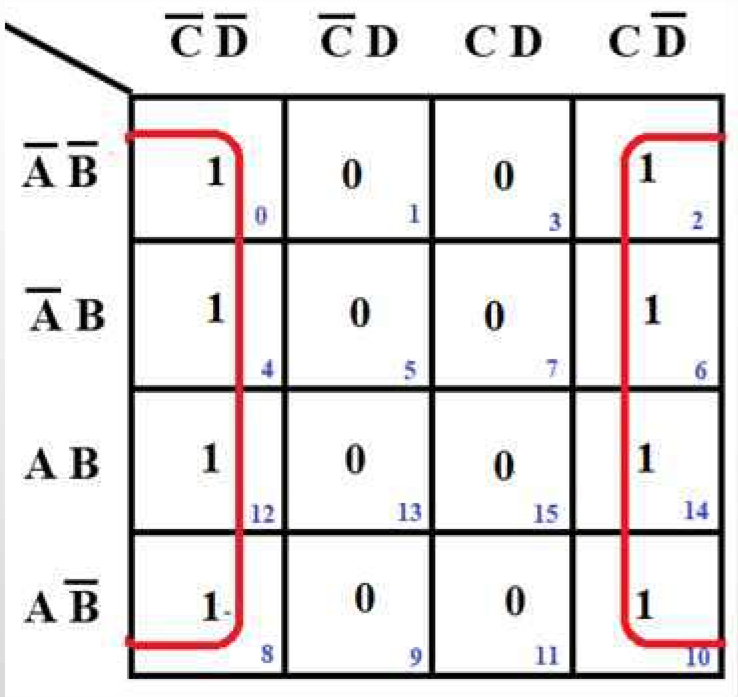




Octets

- An **octet** is a group of 8 l's that are horizontally or vertically adjacent and an octet eliminates three variables and their complements.





Octets

- An **octet** is a group of 8 l's that are horizontally or vertically adjacent and an octet eliminates three variables and their complements.

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0_{0}	1_{1}	1_{3}	0_{2}
$\bar{A}B$	0_{4}	1_{5}	1_{7}	0_{6}
AB	0_{12}	1_{13}	1_{15}	0_{14}
$A\bar{B}$	0_{8}	1_{9}	1_{11}	0_{10}

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0_{0}	0_{1}	0_{3}	0_{2}
$\bar{A}B$	1_{4}	1_{5}	1_{7}	1_{6}
AB	1_{12}	1_{13}	1_{15}	1_{14}
$A\bar{B}$	0_{8}	0_{9}	0_{11}	0_{10}

K- map

- Overlapping of groups

  We are allowed to use the same 1 more than once.

- Rolling of Map

  Roll and overlap to get largest group

- Eliminating redundant group

  Group of 1's which are used in any other group.

  After encircling all possible group, eliminate any redundant group if any

Eliminating redundant group

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 $_{0}$	0 $_{1}$	1 $_{3}$	0 $_{2}$
$\bar{A}B$	1 $_{4}$	1 $_{5}$	1 $_{7}$	0 $_{6}$
AB	0 $_{12}$	1 $_{13}$	1 $_{15}$	1 $_{14}$
$A\bar{B}$	0 $_{8}$	1 $_{9}$	0 $_{11}$	0 $_{10}$

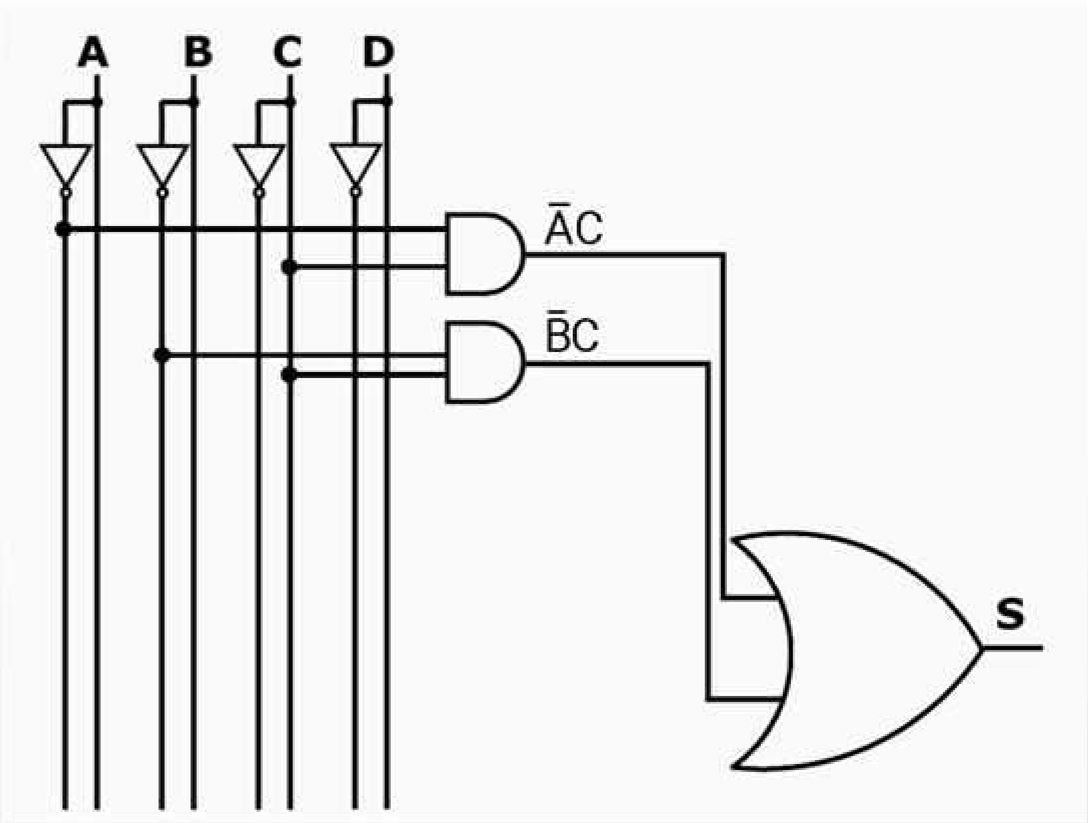
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 $_{0}$	0 $_{1}$	1 $_{3}$	0 $_{2}$
$\bar{A}B$	1 $_{4}$	1 $_{5}$	1 $_{7}$	0 $_{6}$
AB	0 $_{12}$	1 $_{13}$	1 $_{15}$	1 $_{14}$
$A\bar{B}$	0 $_{8}$	1 $_{9}$	0 $_{11}$	0 $_{10}$

Steps in solving SOP using K- map

- Enter 1's on the K map for each minterm given in SOP expression/truth table and enter 0's elsewhere.
- Encircle the octet, quad, pairs. Remember to roll and overlap to get largest possible group.
- If any isolated 1's remain encircle each.
- Eliminate any redundant group.
- Write all product terms for encircled group.
    - All product terms corresponding to encircled group are called implicants.
- OR the all product terms.

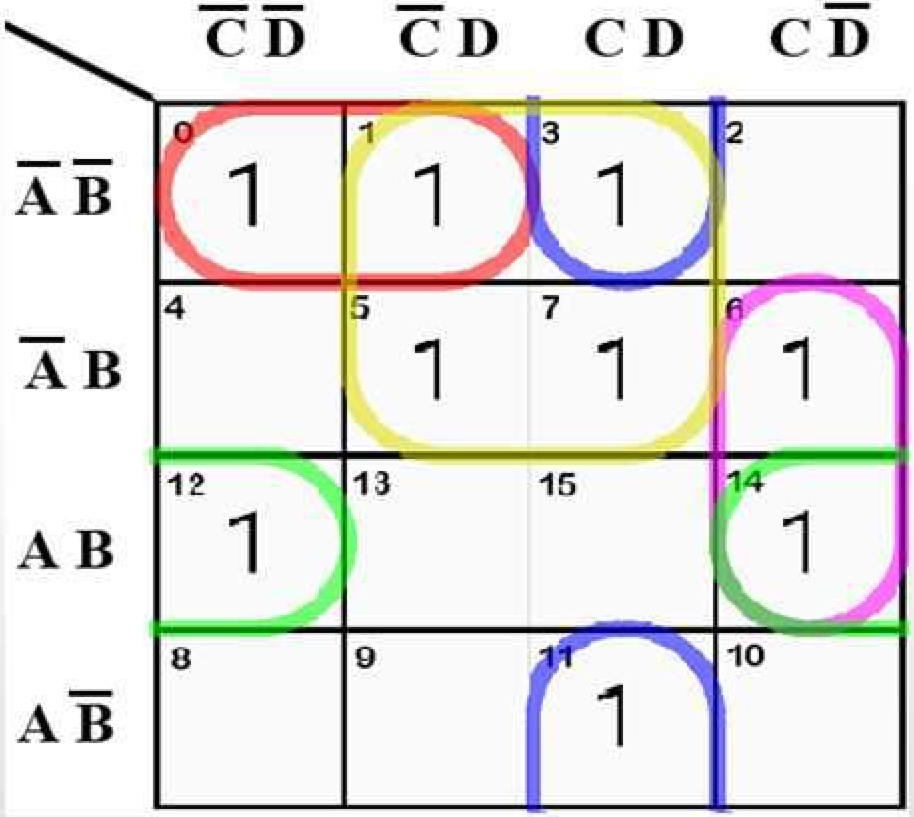
Solve S(A,B,C)=Σm(1,3,5)

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	0	1 1	3 1	2
A	4	5 1	7	6



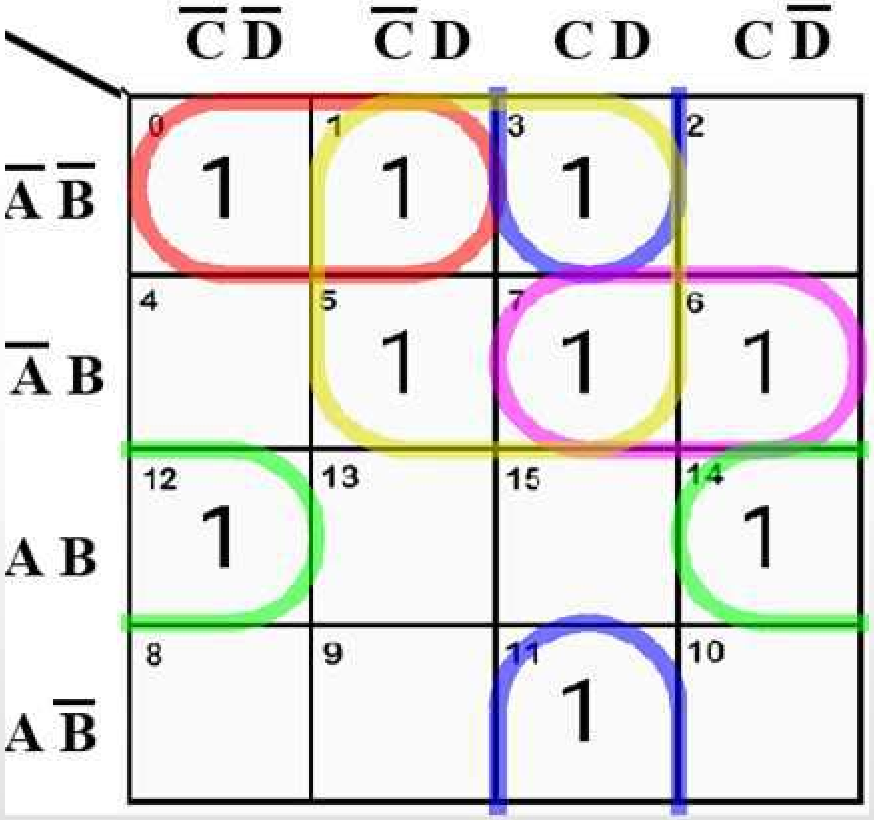
$$S = \bar{A}C + \bar{B}C$$

Solve S(A,B,C)=Σ *m*(0, 1, 3, 5, 6, 7, 11, 12, 14)



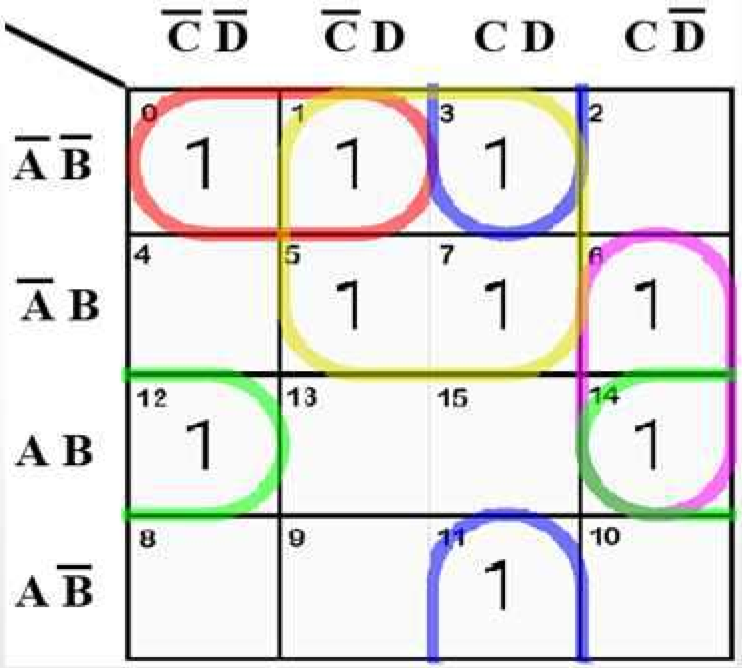
$$S = \bar{A}\bar{B}\bar{C} + \bar{B}CD + BC\bar{D} + AB\bar{D} + \bar{A}D$$

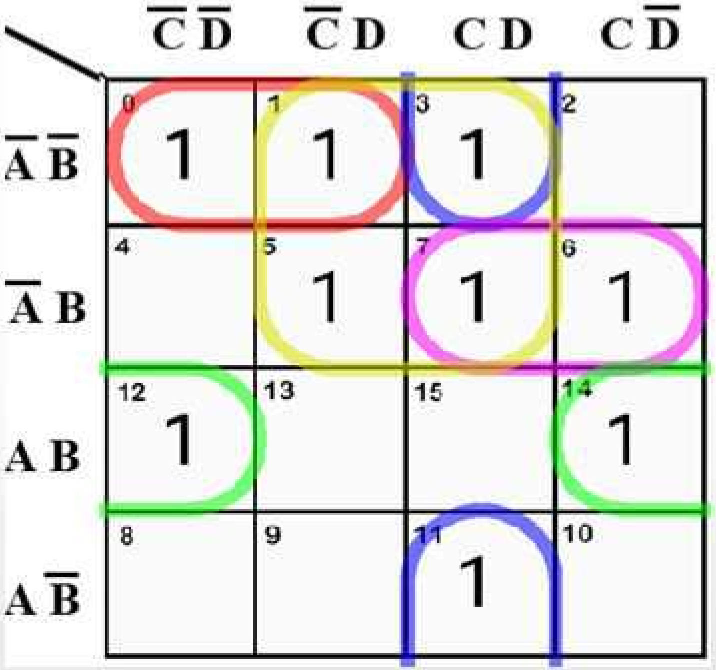
Solve S(A,B,C)=Σ *m*(0, 1, 3, 5, 6, 7, 11, 12, 14)



$S = \bar{A}\bar{B}\bar{C} + \bar{B}CD + \bar{A}BC + AB\bar{D} + \bar{A}D$

Solve S(A,B,C)=Σ *m*(0, 1, 3, 5, 6, 7, 11, 12, 14)





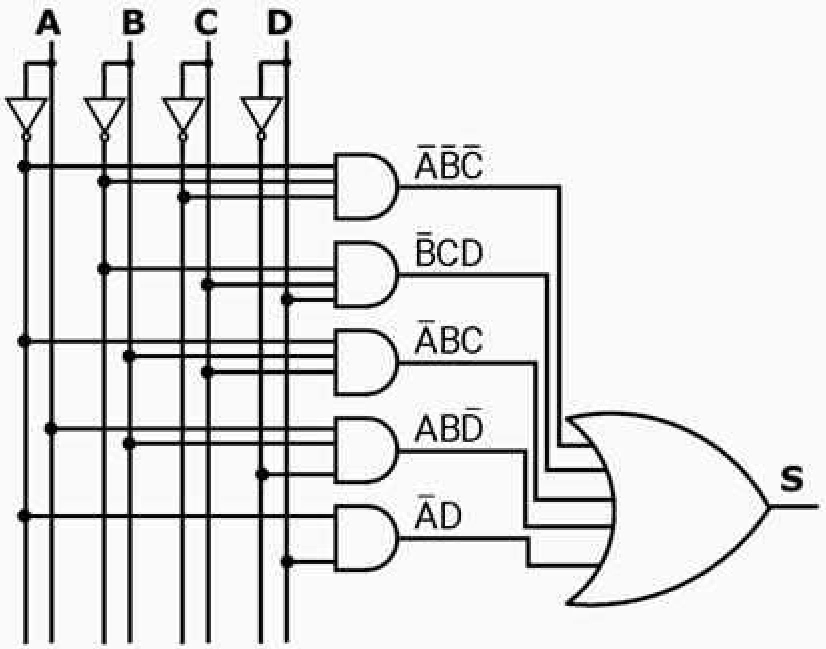
$S = \bar{A}\bar{B}\bar{C} + \bar{B}CD + BC\bar{D} + AB\bar{D} + \bar{A}D$

$S = \bar{A}\bar{B}\bar{C} + \bar{B}CD + \bar{A}BC + AB\bar{D} + \bar{A}D$

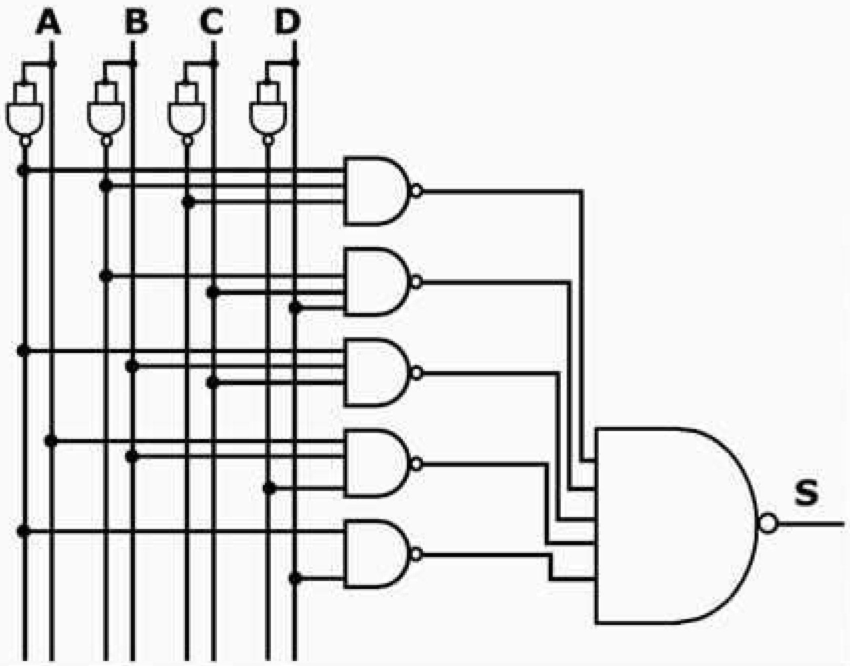
Solve $S(A,B,C)=\Sigma\, m(0, 1, 3, 5, 6, 7, 11, 12, 14)$

$$S = \bar{A}\bar{B}\bar{C} + \bar{B}CD + \bar{A}BC + AB\bar{D} + \bar{A}D$$

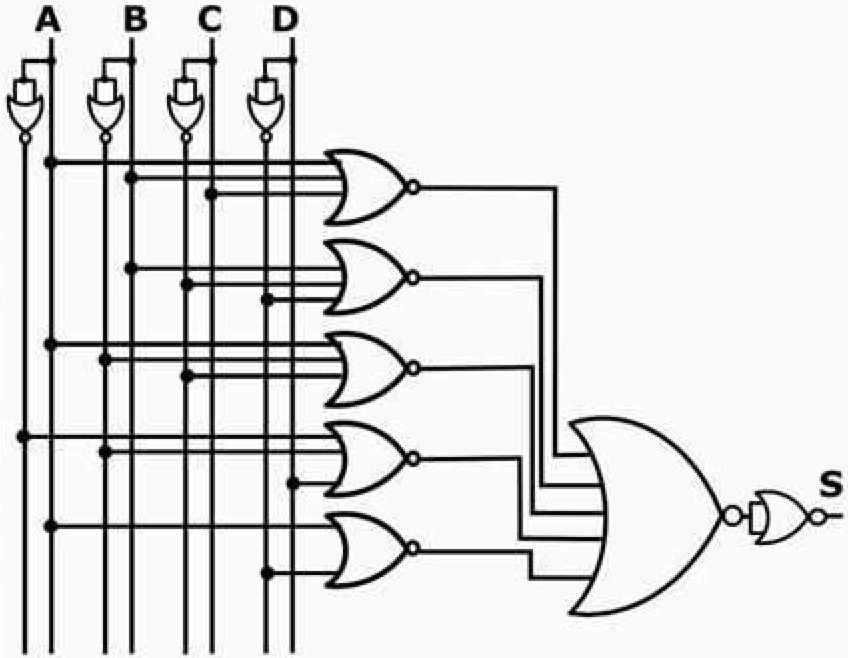
Common Inverted Circuit



Nand Only Circuit



Nor Only Circuit



Solve F(A,B,C,D)=Σ *m*(0,1, 2, 4, 5,6, 8,9,10,12,13)

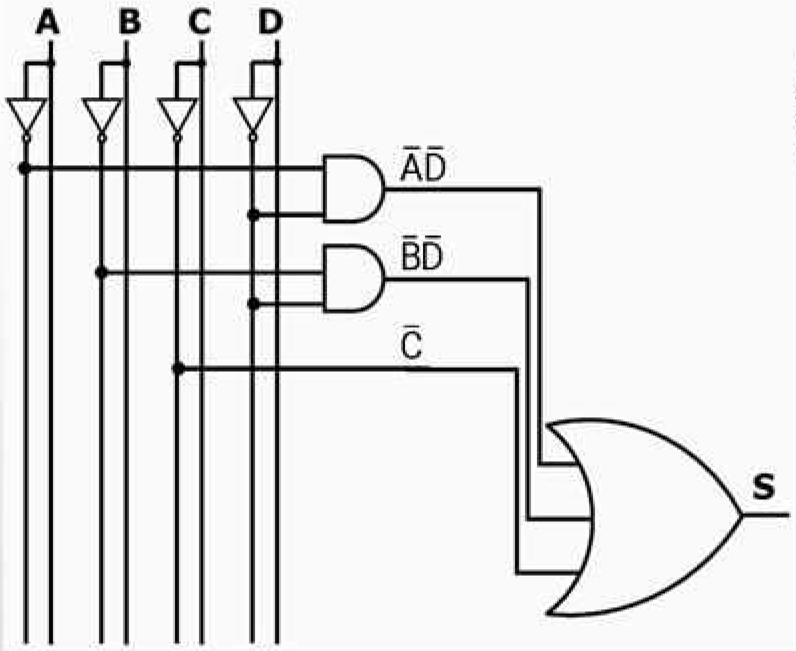
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0: 1	1: 1	3:	2: 1
$\bar{A}B$	4: 1	5: 1	7:	6: 1
AB	12: 1	13: 1	15:	14:
$A\bar{B}$	8: 1	9: 1	11:	10: 1

$S = \bar{A}\bar{D} + \bar{B}\bar{D} + \bar{C}$

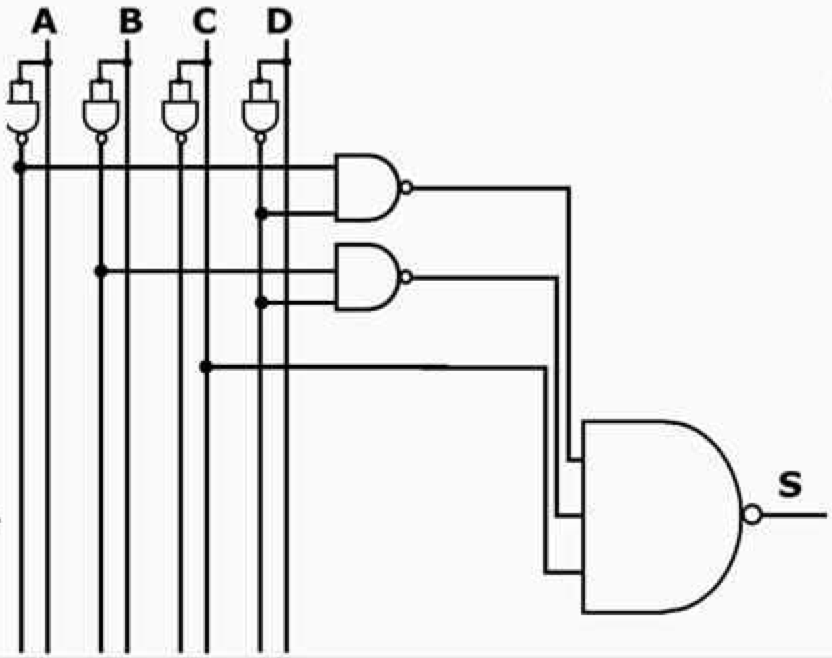
Solve F(A,B,C)=Σ *m*(0,1, 2, 4, 5,6, 8,9,10,12,13)

$S = \bar{A}\bar{D} + \bar{B}\bar{D} + \bar{C}$

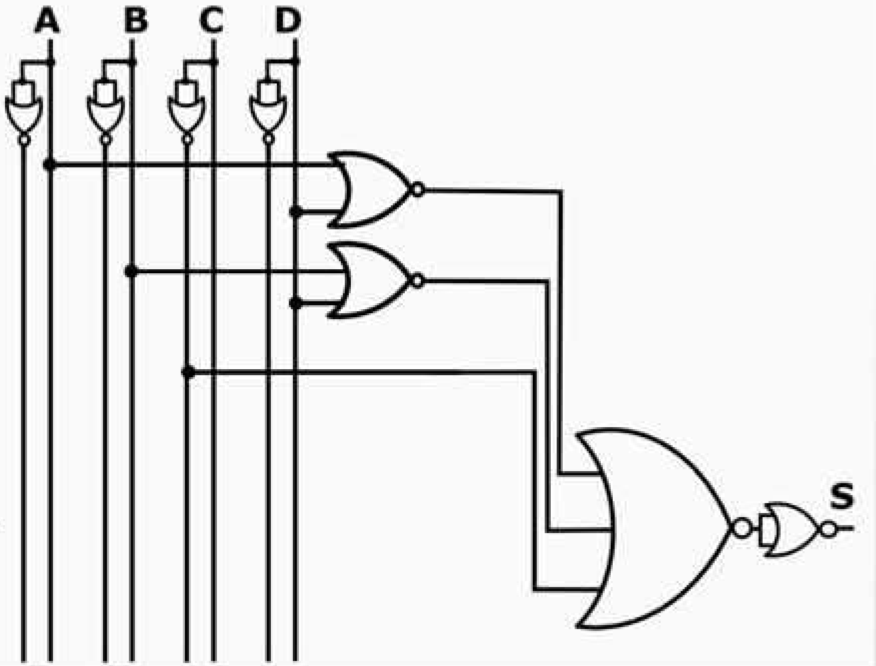
Common Inverted Circuit



Nand Only Circuit



Nor Only Circuit



Solve F(A,B,C,D)=Σm(7,9,10,11,12,13,14,15)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7: 1	6
AB	12: 1	13: 1	15: 1	14: 1
$A\bar{B}$	8	9: 1	11: 1	10: 1

S = BCD + AD + AC + AB

Solve $F(A,B,C,D)=\Sigma m(1,2,3,6,8,9,10,12,13,14)$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1: 1	3: 1	2: 1
$\bar{A}B$	4	5	7	6: 1
AB	12: 1	13: 1	15	14: 1
$A\bar{B}$	8: 1	9: 1	11	10: 1

$$S = \bar{A}\bar{B}D + C\bar{D} + A\bar{C}$$

Steps in solving SOP with don't care condition using K- map

- Draw K map with 0's, 1's and don't care condition.
- Encircle the actual 1's on the K-map in the largest groups by treating the don't cares as 1's.
- After the actual 1's have been included in groups, discard the remaining don't care by considering them as 0's.
- Write all product terms for encircled group.
  - All product terms corresponding to encircled group are called implicants.
- OR the all product term.

Solve F(A,B,C,D)=Σm(7)+d(10,11,12,13,14,15)

	$\overline{C}\,\overline{D}$	$\overline{C}\,D$	$C\,D$	$C\,\overline{D}$
$\overline{A}\,\overline{B}$	0	1	3	2
$\overline{A}\,B$	4	5	7 1	6
$A\,B$	12 X	13 X	15 X	14 X
$A\,\overline{B}$	8	9	11 X	10 X

S = BCD

Solve F(A,B,C,D)=Σm(2,3,5,7,10,12)+d(11,15)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3: 1	2: 1
$\bar{A}B$	4	5: 1	7: 1	6
AB	12: 1	13	15: X	14
$A\bar{B}$	8	9	11: X	10: 1

$$S = AB\bar{C}\bar{D} + \bar{A}BD + \bar{B}C$$

Solve F(A,B,C,D)=Σm(6,7,9,10,13)+d(1,4,5,11)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 X	3	2
$\bar{A}B$	4 X	5 X	7 1	6 1
AB	12	13 1	15	14
$A\bar{B}$	8	9 1	11 X	10 1

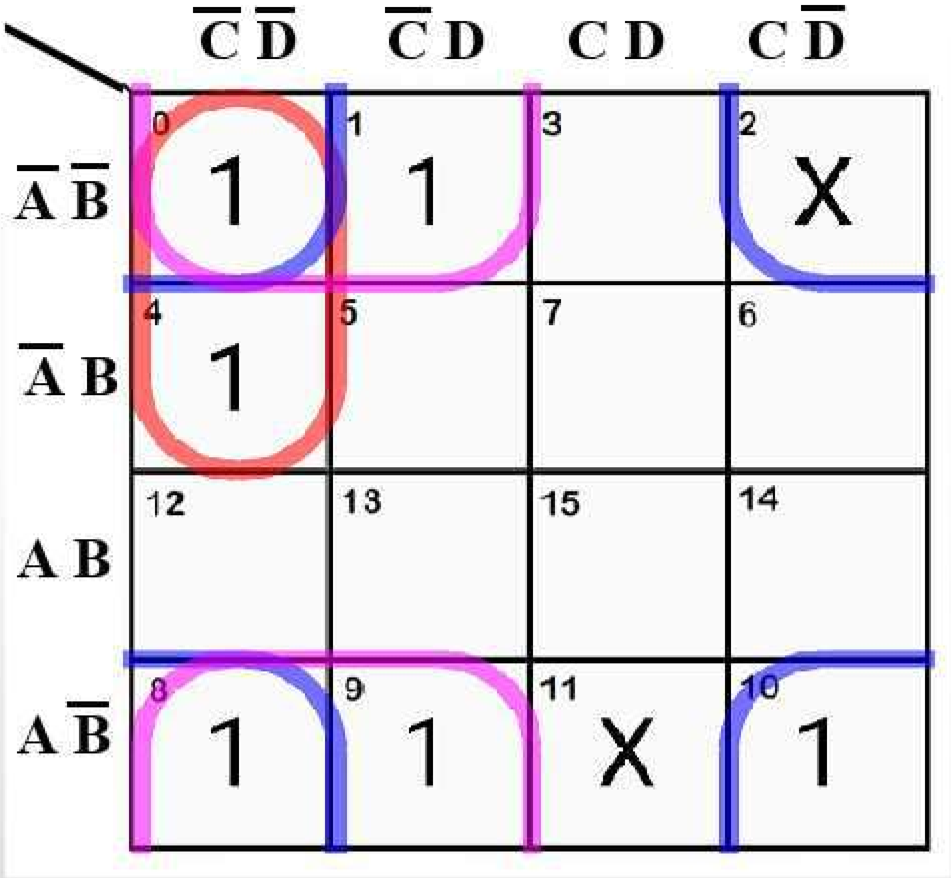
$S = A\bar{B}C + \bar{C}D + \bar{A}B$

Solve F(A,B,C,D)=Σm(0,1,2,4,5,12,14)+d(8,10)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0: 1	1: 1	3:	2: 1
$\bar{A}B$	4: 1	5: 1	7:	6:
AB	12: 1	13:	15:	14: 1
$A\bar{B}$	8: X	9:	11:	10: X

$$S = A\bar{D} + \bar{B}\bar{D} + \bar{A}\bar{C}$$

Solve F(A,B,C,D)=Σm(0,1,4,8,9,10)+d(2,11)



$S = \bar{A}\bar{C}\bar{D} + \bar{B}\bar{D} + \bar{B}\bar{C}$

Solve F(A,B,C,D)=Σm(0,1,4,8,9,10)+d(2,11)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0: 1	1: 1	3:	2: X
$\bar{A}B$	4: 1	5:	7:	6:
AB	12:	13:	15:	14:
$A\bar{B}$	8: 1	9: 1	11: X	10: 1

$$S = \bar{A}\bar{C}\bar{D} + A\bar{B} + \bar{B}\bar{C}$$

Steps in solving POS using K- map

- Enter 0's on the K map for each minterm given in SOP expression/truth table and enter 1's elsewhere.
- Encircle the octet, quad, pairs. Remember to roll and overlap to get largest possible group.
- If any isolated 0's remain encircle each.
- Eliminate any redundant group.
- Write all sum terms for encircled group.
  - All sum terms corresponding to encircled group are called implicants.
- AND the all sum terms.

Solve F(A,B,C,D)=ΠM(0,1,3,4,7)

	$B+C$	$B+\overline{C}$	$\overline{B}+\overline{C}$	$\overline{B}+C$
A	^{0} 0	^{1} 0	^{3} 0	^{2}
$\overline{A}$	^{4} 0	^{5}	^{7} 0	^{6}

$S = (A+\bar{C}) \cdot (B+C) \cdot (\bar{B}+\bar{C})$

	$B+C$	$B+\overline{C}$	$\overline{B}+\overline{C}$	$\overline{B}+C$
A	^{0} 0	^{1} 0	^{3} 0	^{2}
$\overline{A}$	^{4} 0	^{5}	^{7} 0	^{6}

$S = (A+B) \cdot (B+C) \cdot (\bar{B}+\bar{C})$

Solve F(A,B,C,D)=ΠM(0,6,7,8,12,13,14,15)

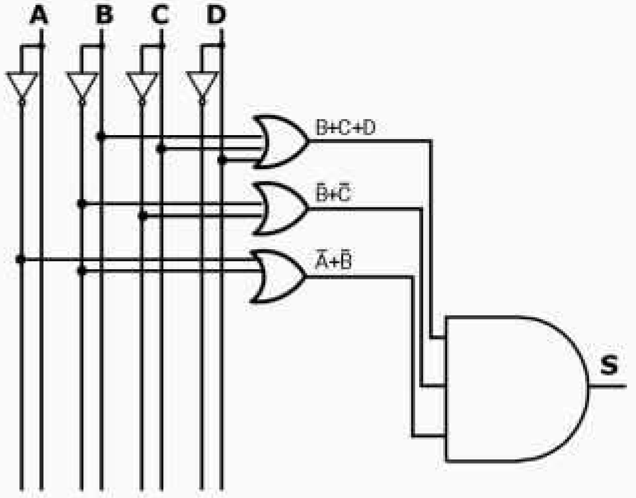
	$C+D$	$C+\overline{D}$	$\overline{C}+\overline{D}$	$\overline{C}+D$
$A+B$	0: 0	1	3	2
$A+\overline{B}$	4	5	7: 0	6: 0
$\overline{A}+\overline{B}$	12: 0	13: 0	15: 0	14: 0
$\overline{A}+B$	8: 0	9	11	10

$S = (B+C+D) \cdot (\bar{B}+\bar{C}) \cdot (\bar{A}+\bar{B})$

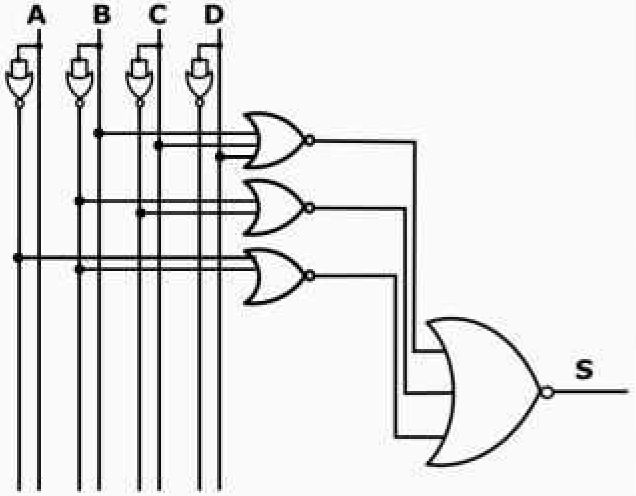
Solve F(A,B,C,D)=ΠM(0,6,7,8,12,13,14,15)

$S = (B+C+D) \cdot (\bar{B}+\bar{C}) \cdot (\bar{A}+\bar{B})$

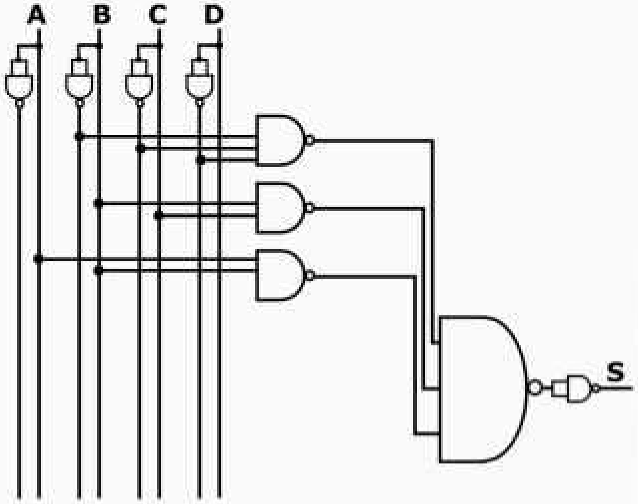
Common Inverted Circuit



Nor Only Circuit



Nand Only Circuit



Steps in solving SOP with don't care condition using K- map

- Draw K map with 0's, 1's and don't care condition.
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- Write all sum terms for encircled group.
  - All sum terms corresponding to encircled group are called implicants.
- AND the all sum terms.

Solve F(A,B,C,D)=ΠM(0,3,4,7,810,12,14).ΠD(2,6)

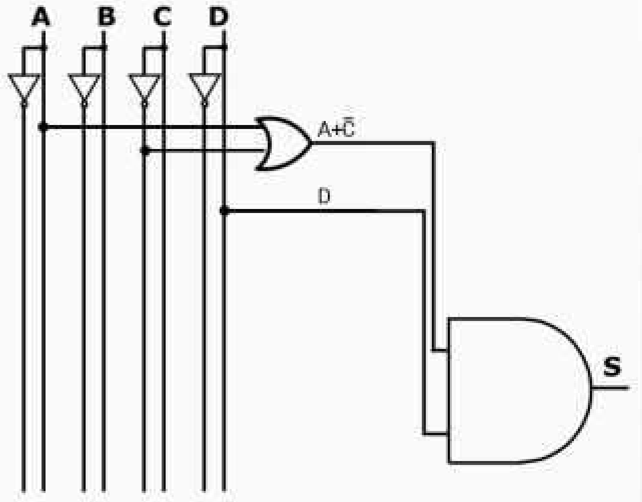
	$C+D$	$C+\overline{D}$	$\overline{C}+\overline{D}$	$\overline{C}+D$
$A+B$	0: 0	1:	3: 0	2: X
$A+\overline{B}$	4: 0	5:	7: 0	6: X
$\overline{A}+\overline{B}$	12: 0	13:	15:	14: 0
$\overline{A}+B$	8: 0	9:	11:	10: 0

$S = (A+\bar{C}) \cdot (D)$

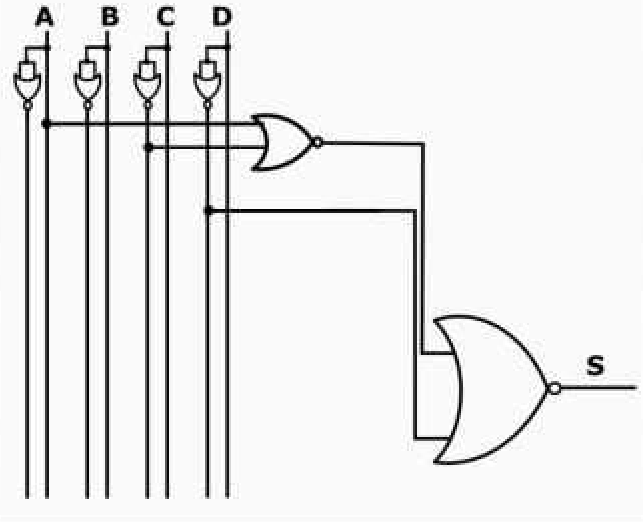
Solve F(A,B,C,D)=ΠM(0,3,4,7,810,12,14).ΠD(2,6)

$S = (A+\bar{C}) \cdot (D)$

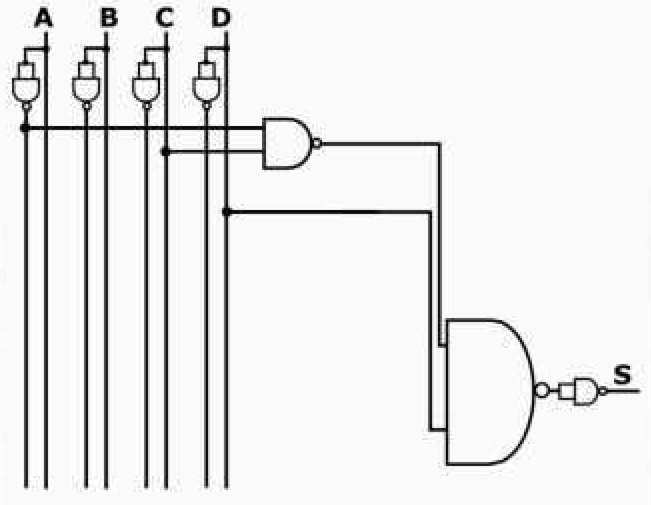
Common Inverted Circuit



Nor Only Circuit



Nand Only Circuit



Solve $F(A,B,C,D)=\Sigma m(6,7,9,10,13)+\Sigma d(1,4,5,11)$

$F(A,B,C,D)=\Pi M(0,2,3,8,12,14,15).\Pi D(1,4,5,11)$

	$C+D$	$C+\overline{D}$	$\overline{C}+\overline{D}$	$\overline{C}+D$
$A+B$	^{0} 0	^{1} X	^{3} 0	^{2} 0
$A+\overline{B}$	^{4} X	^{5} X	^{7}	^{6}
$\overline{A}+\overline{B}$	^{12} 0	^{13}	^{15} 0	^{14} 0
$\overline{A}+B$	^{8} 0	^{9}	^{11} X	^{10}

$S = (\bar{A}+\bar{B}+\bar{C}) \cdot (A+B) \cdot (C+D)$

Find the minimum SOP and POS expression for each function.

1) $f(a, b, c, d) = \Sigma\, m(0, 2, 3, 4, 7, 8, 14)$
2) $f(a, b, c, d) = \Sigma\, m(1, 2, 4, 15) + \Sigma\, d(0, 3, 14)$
3) $f(a, b, c, d) = \Pi\, M(1, 2, 3, 4, 9, 15)$
4) $f(a, b, c, d) = \Pi\, M(0, 2, 4, 6, 8) \cdot \Pi\, D(1, 12, 9, 15)$
5) $f(a, b, c, d) = \Sigma\, m(0, 1, 3, 5, 6, 7, 11, 12, 14)$
6) $f(a, b, c, d) = \Pi\, M(1, 9, 11, 12, 14)$
7) $f(a, b, c, d) = \Pi\, M(5, 7, 13, 14, 15) \cdot \Pi\, D(1, 2, 3, 9)$
8) $f(a, b, c, d) = \Pi\, M(0, 1, 6, 8, 11, 12) \cdot \Pi\, D(3, 7, 14, 15)$
9) $f(a, b, c, d) = \Sigma\, m(1, 3, 4, 11) + \Sigma\, d(2, 7, 8, 12, 14, 15)$

QUINE McCLUSKEY (QM) METHOD

Advantages of QM Method

The Karnaugh map method is an effective way to simplify switching functions which have a small number of variables. When the number of variables is large or if several functions must be simplified, the use of a digital computer is desirable. The *Quine-McCluskey* method provides a systematic simplification procedure which can be readily programmed for a digital computer. The Quine-McCluskey method reduces the min-term expansion (standard sum-of-products form) of a function to obtain a minimum sum of products.

Find the minimum SOP for the function f (a, b, c, d) =∑m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

	Column I		Column II		Column III	
group 0	0	0000 ✓	0, 1	000– ✓	0, 1, 8, 9	–00–
group 1	1	0001 ✓	0, 2	00–0 ✓	0, 2, 8, 10	–0–0
	2	0010 ✓	0, 8	–000 ✓	~~0, 8, 1, 9~~	~~–00–~~
	8	1000 ✓	1, 5	0–01	~~0, 8, 2,10~~	~~–0–0~~
group 2	5	0101 ✓	1, 9	–001 ✓	2, 6, 10, 14	- -10
	6	0110 ✓	2, 6	0–10 ✓	~~2, 10, 6, 14~~	~~- -10~~
	9	1001 ✓	2, 10	–010 ✓		
	10	1010 ✓	8, 9	100– ✓		
group 3	7	0111 ✓	8, 10	10–0 ✓		
	14	1110 ✓	5, 7	01–1		
			6, 7	011–		
			6, 14	–110 ✓		
			10, 14	1–10 ✓		

Prime Implicants

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	X	X					X	⊗		
(0, 2, 8, 10)	$b'd'$	X		X				X		X	
(2, 6, 10, 14)	cd'			X		X				X	⊗
(1, 5)	$a'c'd$		X		X						
(5, 7)	$a'bd$				X		X				
(6, 7)	$a'bc$					X	X				

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	X	X					X	X		
(0, 2, 8, 10)	$b'd'$	X		X				X		X	
(2, 6, 10, 14)	cd'			X		X				X	X
(1, 5)	$a'c'd$		X		X						
(5, 7)	$a'bd$				X		X				
(6, 7)	$a'bc$					X	X				

$$f = b'c' + cd' + a'bd$$

Determine the essential prime implicants using QM method for the function f(a,b,c,d)=∑M(0,1,2,3,10,12,11,13,14,15)

Stage 1		Stage 2		Stage 3	
ABCD		ABCD		ABCD	
0000	(0)√	000-	(0,1)√	00--	(0,1,2,3)
0001	(1)√	00-0	(0,2)√	00--	(0,2,1,3)
0010	(2)√	00-1	(1,3)√	-01-	(2,10,3,11)
0011	(3)√	001-	(2,3)√	1-1-	(10,11,14,15)
1010	(10)√	-010	(2,10)√	1-1-	(10,14,11,15)
1100	(12)√	-011	(3,11)√	11--	(12,13,14,15)
1011	(11)√	101-	(10,11)√	11--	(12,14,13,15)
1101	(13)√	1-10	(10,14)√		
1110	(14)√	110-	(12,13)√		
1111	(15)√	11-0	(12,14)√		
		1-11	(11,15)√		
		11-1	(13,15)√		
		111-	(14,15)√		

	0	1	2	3	10	11	12	13	14	15
A′B′ (0,1,2,3)	√	√	√	√						
B′C (2,3,10,11)			√	√	√	√				
AC (10,11,14,15)					√	√			√	√
AB (12,13,14,15)							√	√	√	√

f=A′B′+b′c+ab or a′b′+ac+ab

PetTrick's Method

Find the minimum SOP form for the function $f(a,b,c)=\sum m(0,1,2,5,6,7)$ using pettricks method.

0	000	✓	0, 1	00–
1	001	✓	0, 2	0–0
2	010	✓	1, 5	–01
5	101	✓	2, 6	–10
6	110	✓	5, 7	1–1
7	111	✓	6, 7	11–

			0	1	2	5	6	7
① →	(0, 1)	$a'b'$	×	×				
	(0, 2)	$a'c'$	×		×			
	(1, 5)	$b'c$		×		×		
② →	(2, 6)	bc'			×		×	
③ →	(5, 7)	ac				×		×
	(6, 7)	ab					×	×

			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2, 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2, 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

$$P = (P1+P2)(P1+P3)(P2+P4)(P3+P5)(P4+P6)(P5+P6) = 1$$

$$P = (P1+P2P3)(P4+P2\ P6)\ (P5+P3P6)$$

$$= (P1P4 + P1P2P6 + P2P3P4 + P2P3P6)\ (\ P5+P3P6)$$

$$= P1P4P5 + P1P2P5P6 + P2P3P4P5 + P2P3P5P6 + P1P3P4P6 + P1P2P3P6 + P2\ P3P4P6 + P2P3P6$$

$$= P1P4P5 + P1P2P5P6 + P2P3P4P5 + P1P3P4P6 + P2P3P6$$

$$=P1P4P5+P1P2P5P6+P2P3P4P5+P1P3P4P6+P2P3P6$$

Because P must be true (P=1) in order to cover all of the minterms, we cantranslate the equation back into words as follows. In order to cover all of the minterms, we must choose rows $P1$ and $P4$ and $P5$, or rows $P1$ and $P2$ and $P5$ and$P6$, or . . . or rows $P2$ and $P3$ and $P6$. Although there are five possible solutions, only two of these have the minimum number of rows. Thus, the two solutions with the minimum number of prime implicants are obtained by choosing rows $P1$, $P4$, and $P5$ or rows $P2$, $P3$, and $P6$.

F=a'b'+bc'+ac or F= a'c'+b'c+ab

Simplification of Incompletely Specified Functions

Find the minimum SOP for the function

$F(A, B, C, D) = \sum m(2, 3, 7, 9, 11, 13) + d(1, 10, 15)$

1	0001	✓
2	0010	✓
3	0011	✓
9	1001	✓
10	1010	✓
7	0111	✓
11	1011	✓
13	1101	✓
15	1111	✓

(1, 3)	00-1	✓
(1, 9)	-001	✓
(2, 3)	001-	✓
(2, 10)	-010	✓
(3, 7)	0-11	✓
(3, 11)	-011	✓
(9, 11)	10-1	✓
(9, 13)	1-01	✓
(10, 11)	101-	✓
(7, 15)	-111	✓
(11, 15)	1-11	✓
(13, 15)	11-1	✓

(1, 3, 9, 11)	-0-1
(2, 3, 10, 11)	-01-
(3, 7, 11, 15)	--11
(9, 11, 13, 15)	1--1

		1	2	3	7	9	10	11	13	15
(1,3,9,11)	b'd	x		x		x		x		
(2,3,10,11)	b'c		x	x			x	x		
(3,7,11,15)	cd			x	x			x		x
(9,11,13,15)	ad					x		x	x	x

$F = B'C + B'D + CD + AD$

Simplification using Entered Variable mapping

Although the Quine-McCluskey method can be used with functions with a fairly large number of variables, it is not very efficient for functions that have many variables and relatively few terms. Some of these functions can be simplified by using a modification of the Karnaugh map method. By using map-entered variables, Karnaugh map techniques can be extended to simplify functions with more than four or five variables. The following Figure shows a four-variable map with two additional variables entered in the squares in the map

$Y=f(a,b,c,d)=\sum m(2,3,4,5,13,15)+d(8,9,10,11)$

a	b	c	d	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	X
1	0	0	1	X
1	0	1	0	X
1	0	1	1	X
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

Conversion of truth table into MEV truth table

a	b	c	d(MEV)	Y	Value to be entered in map
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	1	
0	0	1	1	1	
0	1	0	0	1	
0	1	0	1	1	
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	X	
1	0	0	1	X	
1	0	1	0	X	
1	0	1	1	X	
1	1	0	0	0	
1	1	0	1	1	
1	1	1	0	0	
1	1	1	1	1	

a	b	c	d(MEV)	Y	Value to be entered in map
0	0	0	0	0	D
0	0	0	1	0	
0	0	1	0	1	1
0	0	1	1	1	
0	1	0	0	1	1
0	1	0	1	1	
0	1	1	0	0	0
0	1	1	1	0	
1	0	0	0	X	X
1	0	0	1	X	
1	0	1	0	X	X
1	0	1	1	X	
1	1	0	0	0	d
1	1	0	1	1	
1	1	1	0	0	d
1	1	1	1	1	

a \ bc	00	01	11	10
0	0 (0)	1 (1)	0 (3)	1 (2)
1	X (4)	X (5)	d (7)	d (6)

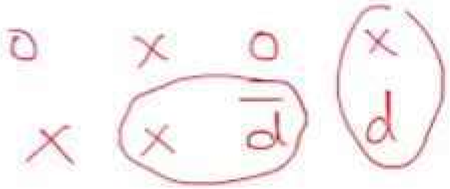
Group 1's with other 1's/X's

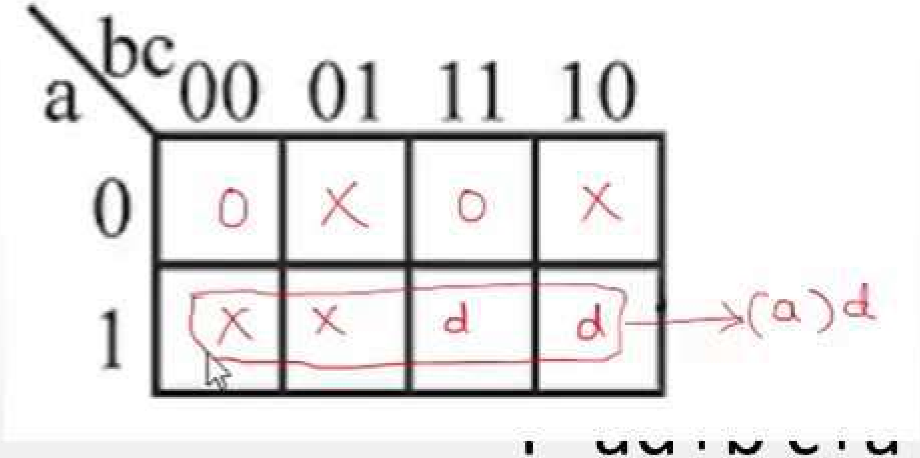
$\bar{a}b\bar{c}$

a \ bc	00	01	11	10
0	0	1	0	1
1	X	X	d	d

$\bar{b}c$

Replace all 1’s by X’s
Group only alike MEV terms





bc’

Thank YOU