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RNS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS

III Semester – II Test – Nov-Dec 2020 (Common to all branches)

Transform Calculus, Fourier Series and Numerical Techniques (18MAT31)

Max Marks: 50 Time: 8.30-10:10 AM Date: 30/11/2020

NOTE: Answer *Five* full questions.

Qn. No.		Questions	Marks	BCL	CO
1	a)	Obtain the Fourier series of the function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $[0,2\pi]$ and hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$	5	L1, L3	CO2
	b)	Obtain the half range Fourier Sine series for the function $f(x) = x\sin x$ in $[0,\pi]$	5	L1, L3	CO2
		OR			
2	a)	Find the Fourier series expansion of the function $f(x) = \begin{cases} x & 0 < x < \pi \\ 2\pi - x & \pi < x < 2\pi \end{cases}$ hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$	5	L1, L3	CO2
	b)	Obtain the half range Fourier Cosine series for the function $f(x) = (x-1)^2$ in $[0,1]$	5	L1, L3	CO2
3	a)	Expand $f(x) = \sqrt{1 - \cos x}$ in a Fourier series over the interval $[-\pi, \pi]$	5	L1, L3	CO2
	b)	Find the Fourier series expansion of the function $f(x) = x$ in the interval $-l < x < l$	5	L1, L3	CO2
		OR			
4	a)	Obtain the Fourier series for the function $f(x) = x\cos x$ in the interval $[-\pi, \pi]$	5	L1, L3	CO2
	b)	Obtain the Fourier series of the function $f(x) = \begin{cases} 2 - x & 0 < x < 4 \\ x - 6 & 4 < x < 8 \end{cases}$	5	L1, L3	CO2
5	a)	A function $f(x)$ of period 2π is specified by the following table x: 0, $\pi/3$, $2\pi/3$, π, $4\pi/3$, $5\pi/3$, 2π $f(x)$: 7.9, 7.2, 3.6, 0.5, 0.9, 6.8, 7.9 Obtain the Fourier series up to first harmonic	5	L1, L3	CO2
	b)	Find the co efficients a_0, a_1, a_2 in the half range Fourier Cosine series for the function given below x: 0, 1, 2, 3, 4, 5, 6 $f(x)$: 4, 8, 15, 7, 6, 2, 4	5	L1, L3	CO2
		OR			

Table for 5 a):

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Table for 5 b):

x	0	1	2	3	4	5	6
$f(x)$	4	8	15	7	6	2	4

Bloom's Cognitive Levels (BCL): L1: Remember, L2: Understand, L3: Apply, L4: Analyze : Evaluate, L6: Create

<table>
<tr><td colspan="6"></td></tr>
<tr><td>6</td><td>a)</td><td>For the function $f(x)$ specified by the following table, find the Fourier coefficients a_0, a_1, b_1
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$f(x)$</td><td>9</td><td>18</td><td>24</td><td>28</td><td>26</td><td>20</td><td>9</td></tr></table></td><td>5</td><td>L1, L3</td><td>CO2</td></tr>
<tr><td></td><td>b)</td><td>In the following table the values of the turning moment T for the set of values of crank angle θ are given. Obtain the first two terms in the half range sine series that represents T
<table><tr><td>θ(deg)</td><td>0</td><td>30</td><td>60</td><td>90</td><td>120</td><td>150</td><td>180</td></tr><tr><td>T</td><td>0</td><td>5.224</td><td>8.097</td><td>7.85</td><td>5.499</td><td>2.626</td><td>0</td></tr></table></td><td>5</td><td>L1, L3</td><td>CO2</td></tr>
<tr><td colspan="6"></td></tr>
<tr><td>7</td><td>a)</td><td>Given $\frac{dy}{dx} = 3x + \frac{y}{2}, y(0) = 1$. Compute $y(0.2)$ by taking $h = 0.2$ using Runge-Kutta method of fourth order.</td><td>5</td><td>L1, L3</td><td>CO4</td></tr>
<tr><td></td><td>b)</td><td>Using Modified Euler's method find $y(0.2)$ correct to four decimal places solving the equation $\frac{dy}{dx} = x - y^2, y(0) = 1, h = 0.1$</td><td>5</td><td>L1, L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>8</td><td>a)</td><td>Using Runge – Kutta method to solve $\frac{dy}{dx} = 3e^x + 2y, y(0) = 0$ at $x = 0.1$ by taking the step length as 0.1</td><td>5</td><td>L1, L3</td><td>CO4</td></tr>
<tr><td></td><td>b)</td><td>Using the Taylor's series method solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ at the point $x = 0.1, 0.2$</td><td>5</td><td>L1, L3</td><td>CO4</td></tr>
<tr><td colspan="6"></td></tr>
<tr><td>9</td><td>a)</td><td>Using Milne's predictor – corrector method find y when $x = 1.4$ given that $\frac{dy}{dx} = x^2 + \frac{y}{2}, y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649,$ $y(1.3) = 2.7514.$ Apply the corrector formula twice.</td><td>5</td><td>L1, L3</td><td>CO4</td></tr>
<tr><td></td><td>b)</td><td>Find the value of y at $x = 4.4$ by Adams Bashforth method given that $5x\frac{dy}{dx} + y^2 - 2 = 0, y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) =$ 1.0142</td><td>5</td><td>L1, L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>10</td><td>a)</td><td>If $\frac{dy}{dx} = 2e^x - y, y(0) = 2, y(0.1) = 2.01, y(0.2) = 2.04, y(0.3) = 2.09$ find $y(0.4)$ correct to four decimal places using Adams Bashforth method.</td><td>5</td><td>L1, L3</td><td>CO4</td></tr>
<tr><td></td><td>b)</td><td>Solve by Milne's method the differential equation $\frac{dy}{dx} + xy^2 = 0$ to compute $y(0.8)$ using the following table
<table><tr><td>x</td><td>0</td><td>0.2</td><td>0.4</td><td>0.6</td></tr><tr><td>y</td><td>2</td><td>1.9231</td><td>1.7241</td><td>1.4706</td></tr></table></td><td>5</td><td>L1, L3</td><td>CO4</td></tr>
</table>

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QUIZ

1. Fourier expansion of an odd function has only ---------- terms.
   a) Cosine b) Sine c) Both cosine and sine d) None

2. If f(x) an odd function in (-π, π), then the graph of f(x) is symmetric about the -----------
   a) x- axis b) y-axis c) origin d) none

3. The mean value of f(x)=cosnx in (0,2 π) is ------ a) $\frac{a_n}{2}$ b) $\frac{b_n}{2}$ c) $\frac{a_0}{2}$ d) none

4. The period of a constant function is a) 2 π b) 2l c) not defined d) none

5. A function f(x) defined for 0<x<1 can be extended to an odd periodic function in (-1, 1) if
   a) f(-x)=-f(x) b) f(-x)= f(x) c) f(-x) ≠ -f(x) ≠ f(x) d) none

6. The term $a_1 cosx + b_1 sinx$In the Fourier series is called
   a) constant term b) first harmonic c) second harmonic d) none

7. The value of $b_n$in the Fourier series of f(x)=|x| in $-\pi$ <x< π , a) 0 b) π/2 c) π d) none

8. Which of the following method gives a polynomial series expression to find the solution of IVP
   a) Runge Kutta method b) Milne's method c) Taylor's method d) none of these

9. Which of the following is the predictor – corrector method
   a) Euler's method b) Taylor's method c) Piccard's method d) Adams - Bashforth method

10. If $\frac{dy}{dx} = xy, y(0) = 1$. To find $y(0.2)$ in one stage we need to take $h = \cdots$
    a) 0.1 b) 0.2 c) 0.3 d) 0.05

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