ESTD : 2001
An Institute with a Difference

RNS INSTITUTE OF TECHNOLOGY, BENGALURU - 98

DEPARTMENT OF MATHEMATICS

18MAT31: Transform Calculus, Fourier Series and Numerical Techniques

Assignment - II

<table>
<tr><th>Q. No</th><th>Questions</th><th>Blooms Level</th><th>CO'S</th></tr>
<tr><td>1.</td><td>Obtain the Fourier series of the function $f(x) = x - x^2$ in $-\pi \le x \le \pi$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>2.</td><td>Obtain the Fourier series of the function $f(x) = |x|$ in $(-\pi, \pi)$ and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>3.</td><td>Obtain the Fourier series of the function $f(x) = \begin{matrix} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{matrix}$
Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>4.</td><td>Obtain the Fourier series of the function $f(x) = \frac{\pi - x}{2}$ in $[0, 2\pi]$ and hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>5.</td><td>Find the Fourier series expansion of the function $f(x) = \begin{matrix} \pi x & 0 < x < 1 \\ \pi(2 - x) & 1 < x < 2 \end{matrix}$</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>6.</td><td>Obtain the Fourier series of the function $f(x) = \begin{matrix} 2 - x & 0 < x < 4 \\ x - 6 & 4 < x < 8 \end{matrix}$
Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>7.</td><td>Obtain the half range Fourier Cosine series for the function $f(x) = \sin x$ in $[0, \pi]$</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>8.</td><td>Find the Fourier half range sine series of the function $f(x) = 2x - x^2$ in $[0,3]$</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>9.</td><td>Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data
<table>
<tr><td>$x(deg)$</td><td>0</td><td>45</td><td>90</td><td>135</td><td>180</td><td>225</td><td>270</td><td>315</td></tr>
<tr><td>y</td><td>2</td><td>1.5</td><td>1</td><td>0.5</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td></tr>
</table></td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>10.</td><td>Express y as a Fourier series up to first harmonic given that
<table>
<tr><td>$x(deg)$</td><td>0</td><td>30</td><td>60</td><td>90</td><td>120</td><td>150</td><td>180</td><td>210</td><td>240</td><td>270</td><td>300</td><td>330</td></tr>
<tr><td>y</td><td>1.8</td><td>1.1</td><td>0.3</td><td>0.16</td><td>1.5</td><td>1.3</td><td>2.16</td><td>1.25</td><td>1.3</td><td>1.52</td><td>1.76</td><td>2.0</td></tr>
</table></td><td>L1, L2, L3</td><td>CO2</td></tr>
</table>

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>11.</td><td>The following table gives the variations of periodic current over a period
<table>
<tr><td>$t(sec)$</td><td>0</td><td>$\frac{T}{6}$</td><td>$\frac{T}{3}$</td><td>$\frac{T}{2}$</td><td>$\frac{2T}{3}$</td><td>$\frac{5T}{6}$</td><td>T</td></tr>
<tr><td>$A(amp)$</td><td>1.98</td><td>1.3</td><td>1.05</td><td>1.3</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr>
</table>
Show that there is a constant part of 0.75A in the current and also obtain the amplitude of the first harmonic.</td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>12.</td><td>Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y from the following data
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y</td><td>9</td><td>18</td><td>24</td><td>28</td><td>26</td><td>20</td></tr>
</table></td><td>L1, L2, L3</td><td>CO2</td></tr>
<tr><td>13.</td><td>Using the Taylor's series method solve $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ at the point $x = 0.2, 0.3$. Consider up to 4th degree term</td><td>L1, L2, L3</td><td>CO4</td></tr>
<tr><td>14.</td><td>Using Runge – Kutta method solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2$ by taking the step length as 0.2</td><td>L1, L2, L3</td><td>CO4</td></tr>
<tr><td>15.</td><td>Using Runge – Kutta method solve $\frac{dy}{dx} = y(x + y)$ with $y(0) = 1$ at $x = 0.2$ by taking the step length as 0.2</td><td>L1, L2, L3</td><td>CO4</td></tr>
<tr><td>16.</td><td>Using the modified Euler's method solve the IVP $\frac{dy}{dx} = x + y^2$, $y(0) = 1$ at $x = 0.2$ by taking $h = 0.1$. Carry out two iterations at each step</td><td>L1, L2, L3</td><td>CO4</td></tr>
<tr><td>17</td><td>Using the modified Euler's method solve the IVP $\frac{dy}{dx} = x + y^2$, $y(0) = 1$ at $x = 1$ in steps of 0.5</td><td>L1, L2, L3</td><td>CO4</td></tr>
<tr><td>18</td><td>Using Milne's predictor – corrector method find y when $x = 0.4$ given that $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090$. Apply the corrector formula twice.</td><td>L1, L2, L3</td><td>CO4</td></tr>
<tr><td>19</td><td>Using Milne's predictor – corrector method find y when $x = 1.4$ given that $\frac{dy}{dx} = x^2 + \frac{y}{2}$, $y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514$. Apply the corrector formula twice.</td><td>L1, L2, L3</td><td>CO4</td></tr>
<tr><td>20</td><td>Using Milne's predictor – corrector method find y when $x = 1.4$ given that $\frac{dy}{dx} = x^2(1 + y)$, $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$. Apply the corrector formula twice.</td><td>L1, L2, L3</td><td>CO4</td></tr>
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Multiple choice questions

1.Fourier expansion of an odd function has only ---------- terms.

a) Cosine b) Sine c) Both cosine and sine d) None

2.If $f(x) = x^4 \; in \; (-1,1)$, then the Fourier coefficient b_n =------------

a) 0 b) $\frac{4(-1)^n}{n^2}$ c) $\frac{1-(-1)^n}{n^2}$ d) None

3.Fourier expansion of an even function f(x) in (-π, π) has only ----------- terms.

a) Cosine b) Sine c) Both cosine and sine d) None

4. If f(x)= $\begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ then f(0) =--------------- a) x b) – π c) -π/2 d) none

5. If $(x) = x^2 \; in \; (-2,2)$, f(x+4)=f(x), then a_n =---------

a) $\int_0^2 x^2 cos\frac{n\pi x}{2}$ dx b) $\int_0^4 x^2 cos\frac{n\pi x}{4}$ dx c) $\int_0^2 x^2 cos\frac{n\pi x}{4}$ dx d) none

6. If f(x) an odd function in (-π, π), then the graph of f(x) is symmetric about the -----------

a) x- axis b) y-axis c) origin d) none

7. The mean value of f(x)cosnx in (0,2 π) ---------- a) $\frac{a_n}{2}$ b) $\frac{b_n}{2}$ c) $\frac{a_0}{2}$ d) none

8. The period of a constant function is a) 2 π b) 2l c) not defined d) none

9. A function f(x) defined for 0<x<1 can be extended to an odd periodic function in (-1, 1) if

a) f(-x)=-f(x) b) f(-x)= f(x) c) f(-x) ≠ -f(x) ≠ f(x) d) none

10. If f(x) is defined in (0, l) then the period of f(x) to expand it as a half-range sine series is

a) 2 π b) 2l c) l d) none

11. If x=c is a point of discontinuity then the Fourier series of f(x) at x=c gives f(x)

a) $\frac{1}{2}(f(c-0)+f(c+0))$ b) f(c) c) $\frac{f(c)}{2}$ d) none

12. Period of |sinx| is a)2 π b) 3 π c) π d) none

13. Using sine series for f(x)=1, in 0<x< π , show that $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots \ldots \ldots . =$

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{8}$ d) none

14. The term $a_1 cosx + b_1 sinx$ In the Fourier series is called

a) constant term b) first harmonic c) second harmonic d) none

15. The value of b_n in the Fourier series of f(x)=|x| in $-\pi$ <x< π , a) 0 b) π/2 c) π d) none

16.If Fourier transform of f(x) is F(s) then the inverse formula is a) $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s)e^{-isx}\,dx$

b) $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{-isx}\,dx$ c) $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-isx}\,dx$ d) none

17. Fourier sine transform of 1/x is a) $\frac{s^2}{2}$ b) $\frac{s}{2}$ c) s^2 d) none

18. Fourier cosine transform of e^{-x} is a) $\frac{s}{s^2+1}$ b) $\frac{1}{s^2+1}$ c) $\frac{1}{s^2-1}$ d)none

19. The value of $\int_0^{\infty} \frac{sinx}{x} dx$ is a) $\frac{\pi}{4}$ b) π c) $\frac{\pi}{2}$ d) none

20. $e^{-\frac{x^2}{2}}$ is self-reciprocal in respect of

a) Laplace transform b) Fourier transform c) Z-transform d) none