Module-3
RELATIONS AND FUNCTIONS

PART - 3

RELATION MATRIX, DIGRAPH AND OPERATIONS

Relation Matrix:

Let R be the relation from Set A to set B then the relation matrix is denoted by M_R and is defined by $M_R = [m_{ij}]$, where $m_{ij} = \begin{cases} 1, & \text{if } (a,b) \in R \\ 0, & \text{if } (a,b) \notin R \end{cases}$.

Example: Let R = {(1, 1), (1, 3), (2, 4), (3, 1), (3, 2), (3, 3)} and S = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 4), (3, 2), (3, 4)} be two relations from A = {1, 2, 3} to B = {1, 2, 3, 4} then

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

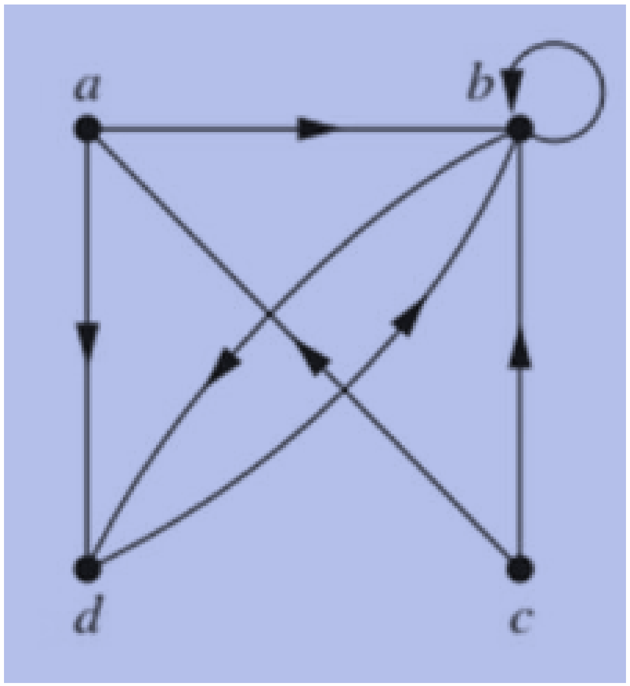
and $M_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

Digraph of a Relation:

- The pictorial representation of a relation R is called **a directed graph or digraph** of R.
- Let R be a relation on set A then the elements of set A are called the **vertices**.
- If $(x, y) \in R$ then draw a directed line from x to y which is called an **edge**.
- An edge of the form (x, x) is represented using an arc from the vertex x back to itself. Such an edge is called a **loop**.
- The number of edges terminating at a vertex is called the **in-degree** of that vertex and the number edges leaving a vertex are called the **out-degree** of that vertex.

Example:

The following figure is the directed graph or digraph for the relation R = {(*a*, *b*), (*a*, *d*), (*b*, *b*), (*b*, *d*), (*c*, *a*), (*c*, *b*), (*d*, *b*)} on set A = { *a*, *b*, *c*, *d* }



Vertices	In-degree	Out-degree
a	1	2
b	4	2
c	0	2
d	2	1

PROBLEMS

Suppose that A = {1, 2, 3} and B = {1, 2}. Let R be the relation from A to B containing (a, b) if a ∈ A, b ∈ B, and a > b. Write the matrix representing R.

Solution:

By definition of R, we have

R = {(2, 1), (3, 1), (3, 2)}.

The matrix for R is $M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

Let A = {a, b, c} and B = {1, 2, 3, 4, 5} and R be a relation from A to B represented by the matrix $M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$. Which ordered pairs are in the relation R?

Solution:

By examining the entries in M_R, we find that

R = {(a, 2), (b, 1), (b, 3), (b, 4), (c, 1), (c, 3), (c, 5)}.

Let A = {a, b, c, d} and R be a relation on A has the matrix $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

Write the elements of R.

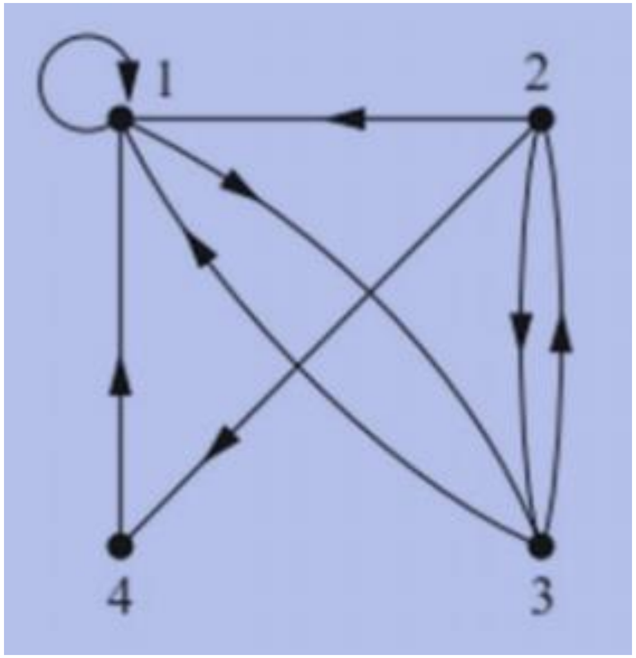
Solution:

By examining the entries in M_R, we find that

R = {(a, a), (b, b), (b, c), (c, a), (c, b), (c, c), (d, b), (d, d)}

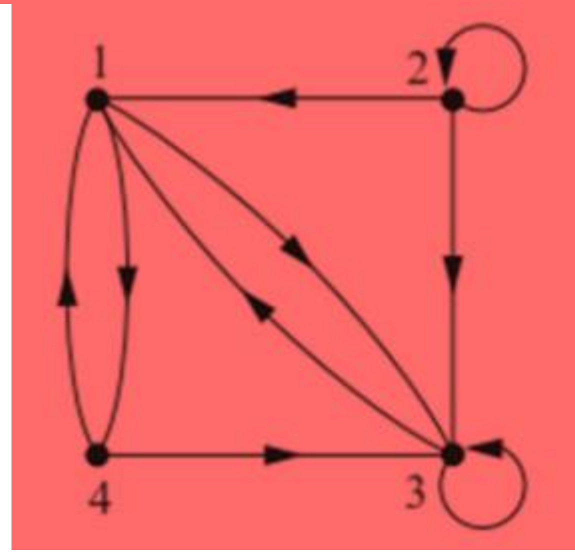
Draw the directed graph of the relation R = {(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)} on the set {1, 2, 3, 4}. Also find in-degree and out-degree of each vertex.

Solution:



Vertices	In-degree	Out-degree
1	4	2
2	1	3
3	2	2
4	1	1

What are the ordered pairs in the relation R represented by the following directed graph. Write its matrix.



The ordered pairs (x, y) in the relation are

R ={(1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 3), (4, 1), (4, 3)}.

The relation matrix is $M_R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Suppose that the relation R on a set is represented by the matrix $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$**.**

Is R reflexive, symmetric, antisymmetric, and/or transitive?

Solution:

Let A = {1, 2, 3}. By examining the entries in M_R, we find that

R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

1) $\forall\, a \in A \Rightarrow (a, a) \in R.\ \therefore$ R is reflexive

2) $\forall\, (a, b) \in R \Rightarrow (b, a) \in R.\ \therefore$ R is symmetric

3) $(1, 2), (2, 1) \in R \Rightarrow 1 \neq 2.\ \therefore$ R is not antisymmetric

4) $(1, 2), (2, 3) \in R \Rightarrow (1, 3) \notin R.\ \therefore$ R is not transitive.

Determine whether the relations for the following directed graphs for the relations R and S are reflexive, symmetric, antisymmetric, and/or transitive.

Solution:

From the first digraph, we find that

$R = \{(a, a), (a, b), (b, b), (b, c), (c, a), (c, b), (c, c)\}$ on set $A = \{a, b, c\}$

$\forall\ x \in A \Rightarrow (x, x) \in R.\ \therefore\ R$ is reflexive

$(a, b) \in R \Rightarrow (b, a) \notin R.\ \therefore\ R$ is not symmetric

$(b, c), (c, b) \in R \Rightarrow b \neq c.\ \therefore\ R$ is not antisymmetric

$(a, b), (b, c) \in R \Rightarrow (a, c) \notin R.\ \therefore\ R$ is not transitive.

From the second digraph, we find that

$S = \{(a, b), (a, c), (a, d), (b, a), (b, b), (c, a), (d, a), (d, d)\}$ on set $A = \{a, b, c, d\}$

1) $\forall\, x \in A \Rightarrow (x, x) \notin S$. $\therefore$ S is not reflexive

2) $\forall\, (x, y) \in S \Rightarrow (y, x) \in S$. $\therefore$ S is symmetric

3) $(a, b), (b, a) \in S \Rightarrow a \neq b$. $\therefore$ S is not antisymmetric

4) $(a, b), (b, a) \in S \Rightarrow (a, a) \notin S$. $\therefore$ S is not transitive.

Operations on Relations:

Union: The union of two relations R_1 and R_2 from set A to set B is denoted by $R_1 \cup R_2$ and is defined as a relation from A to B with the property that $(a, b) \in R_1 \cup R_2$ iff $(a, b) \in R_1$ or $(a, b) \in R_2$.

Intersection: The intersection of two relations R_1 and R_2 from set A to set B is denoted by $R_1 \cap R_2$ and is defined as a relation from A to B with the property that $(a, b) \in R_1 \cap R_2$ iff $(a, b) \in R_1$ and $(a, b) \in R_2$.

Compliment: The complement of a relation R is denoted by R' and is defined as a relation from set A to set B with the property that $(a, b) \in R'$ iff $(a, b) \notin R$.

Difference: The difference of two relations R_1 and R_2 is denoted by $R_1 - R_2$ and is defined as $(a, b) \in R_1 - R_2$ iff $(a, b) \in R_1$ and $(a, b) \notin R_2$.

Converse: The converse of a relation R is denoted by R^c and is defined as a relation from B to A with the property that $(b, a) \in R^c$ iff $(a, b) \in R$.

Example: Let R = {(a, 1), (b, 1), (c, 2), (c, 3)} and S = {(a, 1), (a, 2), (b, 1), (b, 2)} be two relations from A = {a, b, c} and B = {1, 2, 3} then

$R \cup S = \{ (a, 1), (a, 2), (b, 1), (b, 2), (c, 2), (c, 3)\}$

$R \cap S = \{ (a, 1), (b, 1)\}$

$R' = \{(a, 2), (a, 3), (b, 2), (b, 3), (c, 1)\}$

$S' = \{ a, 3), (b, 3), (c, 1), (c, 2), (c, 3)\}$

$R - S = \{(c, 2), (c, 3)\}$

$S - R = \{(a, 2), (b, 2)\}$

$R^c = \{(1, a), (1, b), (2, c), (3, c)\}$

$S^c = \{(1, a), (2, a), (1, b), (2, b)\}$

Let A = {a, b, c} and B = {1, 2, 3} and the relations R = {(a, 1), (b, 1), (c, 2), (c, 3)} and S = {(a, 1), (a, 2), (b, 1), (b, 2)} from A to B. Determine $R \cup S$, $R \cap S$, R', S', $R - S$, $S - R$, R^c, S^c and their matrix representation.

$R \cup S = \{(a, 1),), (a, 2), (b, 1), (b, 2), (c, 2), (c, 3)\}$

$R \cap S = \{(a, 1), (b, 1)\}$

$R' = \{(a, 2), (a, 3), (b, 2), (b, 3), (c, 1)\}$

$S' = \{(a, 3), (b, 3), (c, 1)\ (c, 2), (c, 3)\}$

$R - S = \{(c, 2), (c, 3)\}$

$S - R = \{(a, 2), (b, 2)\}$

$R^c = \{(1, a), (1, b), (2, c), (3, c)\}$

$S^c = \{(1, a), (2, a), (1, b), (2, b)\}$

Their matrix representations are

$$M(R\cup S)=\begin{bmatrix}1&1&0\\1&1&0\\0&1&1\end{bmatrix},\ M(R\cap S)=\begin{bmatrix}1&0&0\\1&0&0\\0&0&0\end{bmatrix},\ M_{R'}=\begin{bmatrix}0&1&1\\0&1&1\\1&0&0\end{bmatrix}$$

$$M_{S'}=\begin{bmatrix}0&0&1\\0&0&1\\1&1&1\end{bmatrix},\quad M(R-S)=\begin{bmatrix}0&0&0\\0&0&0\\0&1&1\end{bmatrix},\quad M(S-R)=\begin{bmatrix}0&1&0\\0&1&0\\0&0&0\end{bmatrix},$$

$$M(R^c)=\begin{bmatrix}1&1&0\\0&0&1\\0&0&1\end{bmatrix},\quad M(S^c)=\begin{bmatrix}1&1&0\\1&1&0\\0&0&0\end{bmatrix}.$$

Let A = {1, 2, 3} and B = {1, 2, 3, 4} and the relations R and S from A to B are represented by the matrices $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ **and** $M_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$**. Determine** $R \cup S$, $R \cap S$, R', S', $R - S$, $S - R$, R^c, S^c **and their matrix representation.**

By examining the entries in M_R and M_S, we find that

$R = \{(1, 1), (1, 3), (2, 4), (3, 1), (3, 2), (3, 3)\}$

$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 1), (3, 2), (3, 3)\}$

$R \cup S = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 1), (3, 2), (3, 3)\}$

$R \cap S = \{(1, 1), (1, 3), (2, 4), (3, 1), (3, 2), (3, 3)\}$

$R' = \{(1, 2), (1, 4), (2, 1), (2, 2), (2, 3), (3, 4)\}$

$S' = \{ (2, 1), (2, 3), (3, 4)\}$

$R - S = \{ \}$

$S - R = \{(1, 2), (1, 4), (2, 2)\}$

$R^c = \{(1, 1), (3, 1), (4, 2), (1, 3), (2, 3), (3, 3)\}$

$S^c = \{(1, 1), (2, 1), (3, 1), (4, 1), (2, 2), (4, 2), (1, 3), (2, 3), (3, 3)\}$

Their matrix representations are

$$M\left(R\cup S\right)=\begin{bmatrix}1&1&1&1\\0&1&0&1\\1&1&1&0\end{bmatrix},\; M\left(R\cap S\right)=\begin{bmatrix}1&0&1&0\\0&0&0&1\\1&1&1&0\end{bmatrix}, M_{R'}=\begin{bmatrix}0&1&0&1\\0&0&0&1\\1&1&1&0\end{bmatrix}$$

$$M_{S'}=\begin{bmatrix}0&0&0&0\\0&1&0&1\\1&1&1&0\end{bmatrix},\;\; M\left(R-S\right)=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\;\; M\left(S-R\right)=\begin{bmatrix}0&1&0&1\\0&1&0&0\\0&0&0&0\end{bmatrix},$$

$$M\left(R^{c}\right)=\begin{bmatrix}1&0&1\\0&0&1\\1&0&1\\0&1&0\end{bmatrix},\;\; M\left(S^{c}\right)=\begin{bmatrix}1&0&1\\1&1&1\\1&0&1\\1&1&0\end{bmatrix}.$$

If R and S are reflexive relations then show that $R \cup S$ and $R \cap S$ are also reflexive.

Let R and S be reflexive relations on set A.

i.e., $\forall\ a \in A \Rightarrow (a, a) \in R$ and $(a, a) \in S$

$\Rightarrow (a, a) \in R \cup S$ and $(a, a) \in R \cap S$

$\therefore\ R \cup S$ and $R \cap S$ are also reflexive.

If R and S are symmetric relations then show that $R \cup S$ and $R \cap S$ are also symmetric.

Let R and S be symmetric relations on set A.

i.e., $\forall\ (a,b) \in R \Rightarrow (b,a) \in R$ and $\forall\ (a,b) \in S \Rightarrow (b,a) \in S$ ---- (1)

(i) Let $(a,b) \in R \cup S$

$\Rightarrow \quad (a,b) \in R$ or $(a,b) \in S$

$\Rightarrow \quad (b,a) \in R$ or $(b,a) \in S$ [by using (1)]

$\Rightarrow \quad (b,a) \in R \cup S$

$\therefore$ $R \cup$ is symmetric.

(ii) Let $(a,b) \in R \cap S$

$\Rightarrow \quad (a,b) \in R$ and $(a,b) \in S$

$\Rightarrow \quad (b,a) \in R$ and $(b,a) \in S$ [by using (1)]

$\Rightarrow \quad (b,a) \in R \cap S$

$\therefore$ $R \cap S$ is symmetric.

If R and S are antisymmetric relations then show that $R \cap S$ is also antisymmetric and $R \cup S$ need not be antisymmetric.

Let R and S be antisymmetric relations on set A.

i.e., $\forall\ (a, b) \in R, (b, a) \in R \Rightarrow a = b$ and

$\forall\ (a, b) \in S, (b, a) \in S \Rightarrow a = b$ ---- (1)

(i) Let $(a, b) \in R \cap S$ and $(b, a) \in R \cap S$

$\Rightarrow\quad (a, b) \in R$ and $(a, b) \in S$ & $(b, a) \in R$ and $(b, a) \in S$

$\Rightarrow\quad (a, b) \in R, (b, a) \in R$ and $(a, b) \in S, (b, a) \in S$

$\Rightarrow\quad a = b$ [by using (1)]

$\therefore$ $R \cap S$ is antisymmetric.

(ii) Let $R = \{(1, 1), (1, 2), (1, 3), (2, 3)\}$ and

$S = \{(2, 1), (2, 2), (2, 3), (3, 1)\}$

be two antisymmetric relations on set $A = \{1, 2, 3\}$ then

$R \cup S = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1)\}$

is not antisymmetric.

$\therefore$ $R \cup S$ need not be antisymmetric.

If R and S are transitive relations then show that $R \cap S$ is also transitive and $R \cup S$ need not be transitive.

Let R and S be transitive relations on set A.

i.e., $\forall \ (a, b), (b, c) \in R \Rightarrow (a, c) \in R$ and

$\forall \ (a, b), (b, c) \in S \Rightarrow (a, c) \in S$ ---- (1)

(i) Let $(a, b), (b, c) \in R \cap S$

$\Rightarrow \quad (a, b), (b, c) \in R$ and $(a, b), (b, c) \in S$

$\Rightarrow \quad (a, c) \in R$ and $(a, c) \in S$ [by using (1)]

$\Rightarrow \quad (a, c) \in R \cap S$

$\therefore R \cap S$ is transitive

(ii) Let $R = \{(1, 1), (1, 2), (1, 3), (2, 3)\}$ and $S = \{(2, 1), (2, 2), (2, 3), (3, 1)\}$ be two transitive relations on set $A = \{1, 2, 3\}$ then

$R \cup S = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1)\}$ is not transitive.

$\therefore R \cup S$ need not be transitive.

If R and S are equivalence relations then show that R $\cap$ S is also equivalence and R $\cup$ S need not be equivalence.

(i) Let R and S be equivalence relations on set A.

i.e., R and S are reflexive, symmetric and transitive.

$\Rightarrow$ R $\cap$ S is reflexive, symmetric and transitive.

$\therefore$ R $\cap$ S is equivalence relation.

(ii) Since R and S are equivalence relations, they are transitive. But R $\cup$ S need not be transitive. Hence R $\cup$ S need not be equivalence.

Let R be a non-empty relation on a set A then prove that if R satisfies any two of the properties: irreflexive, symmetric and transitive then it cannot satisfy the third.

Case 1: Suppose, R is irreflexive and symmetric.

To prove: R is not transitive.

Assume that R is transitive.

Let $(a, b) \in R$

$\Rightarrow$ $(b, a) \in R$ (by symmetry)

$\Rightarrow$ $(a, a) \in R$ (by transitive)

$\Rightarrow$ R is not irreflexive.

This is not true because R is irreflexive.

$\therefore$ R cannot be transitive when it is irreflexive and symmetric.

Case 2: Suppose, R is irreflexive and transitive.

To prove: R is not symmetric.

Assume that R is symmetric.

Let $(a, b) \in R$

$\Rightarrow$ $(b, a) \in R$ (by symmetry)

$\Rightarrow$ $(a, a) \in R$ (by transitive)

$\Rightarrow$ R is not irreflexive.

This is not true because R is irreflexive.

$\therefore$ R cannot be symmetric when it is irreflexive and transitive.

Case 3: Suppose, R is symmetric and transitive.

To prove: R is not irreflexive.

Let $(a, b) \in R$

$\Rightarrow$ $(b, a) \in R$ (by symmetry)

$\Rightarrow$ $(a, a) \in R$ (by transitive)

$\Rightarrow$ R is not irreflexive.

Composition of Relations:

Let R be a relation from set A to set B and S be a relation from set B to set C then the composition of two relations R and S is denoted by S o R and is defined as a relation from A to C with the property that $(x, z) \in S \circ R$ iff $(x, y) \in R$ and $(y, z) \in S$.

$$\text{i.e., } S \circ R = \{(x, z) \mid (x, y) \in R \text{ and } (y, z) \in S\}$$

Similarly, the composition of the relations S and R is defined as

$$R \circ S = \{(x, z) \mid (x, y) \in S \text{ and } (y, z) \in R\}$$

Note:

- In general, $R \circ S \neq S \circ R$. i.e., the composition of the relations is not commutative.
- The composition of the relations is associative.

  i.e., $R_1 \circ (R_2 \circ R_3) = (R_1 \circ R_2) \circ R_3$
- The composition of R with itself is $R \circ R$ and is also denoted by R^2.
- In general, R^n is a relation on A defined recursively by $R^n = R^{n-1} \circ R$ for $n \geq 2$.

Example: Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 3), (3, 4)} and S = {((1, 4), (2, 4), (3, 1), (4, 4)} be relations from A to B then

$S \circ R = \{(1, 4), (2, 1), (3, 4)\}$

$R \circ S = \{(3, 1), (3, 2)\}$

$R^2 = R \circ R = \{(1, 1), (1, 2), (1, 3), (2, 4)\}$

$S^2 = S \circ S = \{(1, 4), (2, 4), (3, 4), (4, 4)\}$

What is the composite of the relations R and S, where R is the relation from {1, 2, 3} to {1, 2, 3, 4} with R = {(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)} and S is the relation from {1, 2, 3, 4} to {0, 1, 2} with S = {(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)}?

$S \circ R = \{(1, 0), (1, 1), (2, 1), (2, 2), (3, 0), (3, 1)\}$

If $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (1, 3), (2, 4), (4, 4)\}$ and $S = \{ (1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)\}$ be relations on A then find R o S, S o R, R^2 and S^2. Also write their matrices.

$S \circ R = \{(1, 3), (1, 4)\}$

$R \circ S = \{(1, 2), (1, 3), (1, 4), (2, 4)\}$

$R^2 = R \circ R = \{(1, 4), (2, 4), (4, 4)\}$

$S^2 = S \circ S = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$

Their Matrices are

$$M(S \circ R) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad M(R \circ S) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M(R^2) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad M(S^2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Let A = {a, b, c} and R and S be relations on A whose matrices are given below:

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the composite relations R o S, S o R, R^2 and S^2. Also write their matrices.

By examining the entries in M_R and M_S, we find that

R = {(a, a), (a, c), (b, a), (b, b), (b, c), (c, b)}

S = {(a, a), (b, b), (b, c), (c, a), (c, c)}

S o R = {(a, a), (a, c), (b, a), (b, b), (b, c), (c, b), (c, c)}

R o S = {(a, a), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}}

R^2 = R o R = {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}

S^2 = S o S = {(a, a), (b, a), (b, b), (b, c), (c, a), (c, c)}

Their Matrices are

$$M\left(\mathrm{S}\circ\mathrm{R}\right)=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 1\\ 0 & 1 & 1\end{bmatrix},\qquad M\left(\mathrm{R}\circ\mathrm{S}\right)=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix},$$

$$M\left(R^{2}\right)=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix},\qquad M\left(\mathrm{S}^{2}\right)=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$$

If $A = \{1, 2, 3, 4\}$ and R be a relation on A defined by $R = \{(1, 2), (1, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$, find R^2 and R^3. Also write their matrices.

$R^2 = R \circ R = \{(1, 2), (1, 3), (1, 4), (3, 2), (3, 3), (3, 4)\}$

$R^3 = R^2 \circ R = \{(1, 2), (1, 3), (1, 4), (3, 2), (3, 3), (3, 4)\}$

Their Matrices are

$$M\left(R^2\right) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad M\left(R^3\right) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$