5.2 Test of significance - t test

Working rule:

- Write the null hypothesis H_0 and the alternative hypothesis H_1.
- Find the calculated value using $|t| = \left|\frac{\bar{x}-\mu}{S.E\ (\bar{x})}\right|$, Where $S.E(\bar{x}) = \sqrt{\frac{s^2}{n-1}}$

$$(or)$$

$$|t| = \left|\frac{\overline{x_1}-\overline{x_2}}{SE(\overline{x_1}-\overline{x_2})}\right|, \text{ where } S.E(\overline{x_1}-\overline{x_2}) = \sqrt{\frac{s_1^2}{n_1-1}+\frac{s_2^2}{n_2-1}}$$

- Find the critical value using the table at $n-1$ or n_1+n_2-2 degrees of freedom.
- If calculated value < critical value, Accept H_0. H_0 is the conclusion.
- If calculated value > critical value, Reject H_0. H_1 is the conclusion.

t distribution: Critical values of t

Degrees of freedom		*Significance level*					
	Two-tailed test:	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646

1. **A Machinist making engine parts with axle diameter of 0.7 inches. A random sample of 10 parts shows mean diameter 0.742 inches with a SD of 0.04 inches. On the basis of this sample, would you say that the work is inferior at 5% level of significance?**

$$[t_{(.05,\ 9)} = 2.26]$$

Since n = 10, apply t test.

By data, $\bar{x} = 0.742, s = 0.04, \mu = 0.7, \alpha = 0.05$

H_0: $\mu = 0.7$, The work is not inferior.

$$S.E(\bar{x}) = \sqrt{\frac{s^2}{n-1}} = \sqrt{\frac{0.04^2}{9}} = 0.0133$$

$$|t| = \left|\frac{\bar{x}-\mu}{S\,E(\bar{x})}\right| = \left|\frac{0.742-0.7}{0.0133}\right| = 3.1579$$

Calculated value of $t = 3.1579$

$\alpha = 0.05, \gamma = n - 1 = 9.$

Therefore, Critical value of $t = 2.26$

Since calculated value > critical value,

Reject H_0.

Therefore, the work is inferior.

2. The nine items of the sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53 and 51. Does the mean of these differ significantly from the assumed mean of 47.5?

$[t_{(.05,\ 8)} = 2.31]$

Since n = 9, apply t test.

By data, $\mu = 47.5, \alpha = 0.05$

$$\bar{x} = \frac{\sum x}{n} = \frac{442}{9} = 49.11$$

$$s^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{21762}{9} - 49.11^2 = 6.2079$$

$$S.E(\bar{x}) = \sqrt{\frac{s^2}{n-1}} = \sqrt{\frac{6.2079}{8}} = 0.8809$$

$H_0: \mu = 47.5$, There is no significant difference from the assumed mean 47.5

$$|t| = \left|\frac{\bar{x}-\mu}{S.E(\bar{x})}\right| = \left|\frac{49.11-47.5}{0.8809}\right| = 1.8276$$

Therefore, calculated value of t = 1.8276

$\alpha = 0.05,\ \gamma = n - 1 = 8,$

Therefore, Critical value of t = 2.31

Since calculated value < critical value,

Accept H_0.

∴There is no significant difference from the assumed mean 47.5

3. **A random sample of 10 boys had the following IQ: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Does this data support the hypothesis that the population mean of IQ's is 100 at 5% level of significance?** $[t_{(.05,\ 9)} = 2.26$

Since n = 10, apply t - test.

$$\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$$

$$s^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{96312}{10} - 97.2^2 = 183.36$$

$$S.E(\bar{x}) = \sqrt{\frac{s^2}{n-1}} = \sqrt{\frac{183.36}{9}} = 4.5136$$

H_0: $\mu = 100$, The population mean of IQ's is 100.

$$|t| = \left|\frac{\bar{x}-\mu}{SE(\bar{x})}\right| = \left|\frac{97.2-100}{4.5136}\right| = 0.6203$$

Therefore, calculated value of t = 0.6203

$\alpha = 0.05, \qquad \gamma = n - 1 = 9.$

Therefore, Critical value of t = 2.26

Since calculated value < critical value,

Accept H_0.

Therefore, the population mean IQ is100

at 5% level of significance.

4. **A certain stimulus administered to each of 12 patients resulted in the following increases of blood pressure: 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6. Can it be concluded that the stimulus will in general be accompanied by an increase in blood pressure?**

$$[t_{(.05,\ 11)} = 2.2$$

Since n = 12, apply t - test.

$$\bar{d} = \frac{\sum d}{n} = \frac{31}{12} = 2.5833$$

$$s^2 = \frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2 = \frac{185}{12} - \left(\frac{31}{12}\right)^2 = 8.7433$$

$$S.E(\bar{d}) = \sqrt{\frac{s^2}{n-1}} = \sqrt{\frac{8.7433}{11}} = 0.8915$$

H_0: $\mu = 0$

The stimulus will not increase in blood pressure.

$$|t| = \left|\frac{\bar{d}-\mu}{SE(\bar{d})}\right| = \left|\frac{2.5833-0}{0.8915}\right| = 2.8977$$

Therefore, calculated value of t = 2.8977

$\alpha = 0.05, \gamma = n - 1 = 11$

Critical value of t = 2.2

Since calculated value > critical value,

Reject H_0.

∴The stimulus will increase blood pressure.

5. **Eleven students were given a test in statistics. They were given a month's further tuition and a second test of equal difficulty was held at the end of it. Do the marks give evidence that the students have benefitted by extra coaching?**

Boys:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Marks I test:	**23**	**20**	**19**	**21**	**18**	**20**	**18**	**17**	**23**	**16**	**19**
Marks II test:	**24**	**19**	**22**	**18**	**20**	**22**	**20**	**20**	**23**	**20**	**17**

$[t_{(.05,\ 10)} = 2.23]$

Since n = 11, apply t test.

$\Sigma d = \Sigma(x_2 - x_1) = 11$

$\bar{d} = \frac{\Sigma d}{n} = 1$

$s^2 = \frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2$

$= \frac{61}{11} - 1$

$= \frac{50}{11}$

$S.E\ (\bar{x}) = \sqrt{\frac{s^2}{n-1}}$

$= \sqrt{\frac{50/11}{10}}$

$= \sqrt{\frac{50}{110}}$

$= 0.6742$

H_0: $\mu = 0$, The students did not have benefitted by extra coaching.

$|t| = \left|\frac{\bar{d}-\mu}{SE(\bar{d})}\right| = \left|\frac{1-0}{0.6742}\right| = 1.4832$

Therefore, calculated value of t = 1.4832

$\alpha = 0.05, \gamma = n - 1 = 10.$

Therefore, critical value of t = 2.23

Since calculated value < critical value,

Accept H_0. ∴ The students did not have benefit by extra coaching.

6. A group of boys and girls were given an intelligent test. The mean score SD's and numbers in each group are as follows:

	Mean	**S.D**	**n**
Boys	**124**	**12**	**18**
Girls	**121**	**10**	**14**

Is the mean score of boys significantly different from that of girls? $[t_{(.05,30)} = 2.04]$

Since $n_1 = 18$, $n_2 = 14$, apply t test.

By data, $\overline{x_1} = 124, \overline{x_2} = 121,$

$s_1 = 12, s_2 = 10$

$$SE(\overline{x_1} - \overline{x_2}) = \sqrt{\frac{s_1^2}{n_1-1} + \frac{s_2^2}{n_2-1}}$$

$$= \sqrt{\frac{144}{17} + \frac{100}{13}}$$

$$= 4.0203$$

H₀: $\mu_1 = \mu_2$, the mean score of boys does not differ significantly from that of girls.

$$|t| = \left|\frac{\overline{x_1} - \overline{x_2}}{S.E(\overline{x_1} - \overline{x_2})}\right| = \left|\frac{124 - 121}{4.0203}\right| = 0.7462$$

Therefore, calculated value of t = 0.77

$\alpha = 0.05, \gamma = n_1 + n_2 - 2 = 30$

Therefore, critical value of t = 2.04

Since calculated value < critical value, Accept H_0.

∴The mean score of boys does not differ significantly from that of girls.

7. **The means of two random samples of sizes 9 and 7 are 196.42 and 198.82 respectively. The sum of squares of the deviations from the mean are 26.94 and 18.73 respectively. Can the sample be considered to have been drawn from the same normal population?** $[t_{(.05,\ 14)} = 2.14]$

Since $n_1 = 9$, $n_2 = 7$, apply t test.

By data, $\overline{x_1} = 196.42, \overline{x_2} = 198.$

$s_1^2 = \frac{26.94}{9}, s_2^2 = \frac{18.73}{7}$

$$SE(\overline{x_1} - \overline{x_2}) = \sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}$$

$= 0.9056$

H_0: $\mu_1 = \mu_2$, sample is drawn from the same normal population.

$$|t| = \left|\frac{\overline{x_1} - \overline{x_2}}{SE(\overline{x_1} - \overline{x_2})}\right| = \left|\frac{196.42 - 198.82}{0.9056}\right| = 2.6502$$

Therefore, calculated value of t = 2.6502

$\alpha = 0.05, \gamma = n_1 + n_2 - 2 = 9 + 7 - 2 = 14$

Critical value of t = 2.14

Since calculated value < critical value,

Accept H_0.

∴ Sample is drawn from the same normal population.

8. **From a random sample of 10 pigs fed on diet A, the increases in weight in a certain period were 10, 6, 16, 17, 13, 12, 8, 14, 15, 9 lbs. For another random sample of 12 pigs fed on diet B, the increases in the same period were 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 lbs. Test whether diets A and B differ significantly as regards their effect on increases in weight?** $[t_{(.05,\ 14)} = 2.09]$

Since $n_1 = 10$, $n_2 = 12$, apply t test.

By data, $\overline{x_1} = \frac{\Sigma x_1}{n_1} = \frac{120}{10} = 12$

$$\overline{x_2} = \frac{\Sigma x_2}{n_2} = \frac{180}{12} = 15$$

$$s_1^2 = \frac{\Sigma {x_1}^2}{n_1} - \left(\frac{\Sigma x_1}{n_1}\right)^2 \qquad s_2^2 = \frac{\Sigma {x_2}^2}{n_2} - \left(\frac{\Sigma x_2}{n_2}\right)^2$$

$$= \frac{1560}{10} - 144 \qquad = \frac{3014}{12} - 225$$

$$= 12 \qquad = 26.17$$

$$SE(\overline{x_1} - \overline{x_2}) = \sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}$$

$$= \sqrt{\frac{12^2}{9} + \frac{26.17^2}{11}} = 1.93$$

$H_0: \mu_1 = \mu_2$,

diets A and B do not differ significantly.

$$|t| = \left|\frac{\overline{x_1} - \overline{x_2}}{SE(\overline{x_1} - \overline{x_2})}\right| = \left|\frac{12-15}{1.93}\right| = 1.6$$

Therefore, calculated value =1.6

$\alpha = 0.05, \gamma = n_1 + n_2 - 2 = 10 + 12 - 2 = 20$

Critical value = 2.09

Since calculated value < critical value,

Accept H_0.

∴Diets A and B do not differ significantly.

Home work

9. A machine is expected to produce nails of length 3 inches. A random sample of 25 nails gave an average length of 31. Inches with standard deviation 0.3. Can it be said that the machine is producing nails as per specifications? Given $t_{0.05}(24) = 2.064$

10. Two horses A and B were tested according to the time (seconds) to run a particular race with the following results:

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	

Test whether you can discriminate between the two horses.

Given that $t_{0.05} = 2.20$ for 11 degrees of freedom

Home work

11. Two types of batteries are tested for their length of life and the following results were obtained:

	Battery A	Battery B
Mean	500	500
Variance	100	121
Sample size	10	10

Check whether there is a significant difference between two means. $[t_{0.05}(18) = 0.086]$

12. A sample of 12 measurements of the diameter of a metal ball gave the mean 7.38 mm with standard deviation 1.24 mm. Find 99% confidence limits for actual diameter. $[t_{0.01}(11) = 3.11]$

Note: Confidence limits for the mean are $\bar{x} \pm \frac{s}{\sqrt{n-1}} t_{\alpha}(\gamma)$