Knapsack problem:

"Given 'n' objects and a knapsack (bag) with capacity 'm', where each object i has a weight "W_i" and profit associated with it "P_i"

- The objective is to obtain filling of knapsack that maximizes the total profit earned.

i,e

$$\text{Maximize} \sum_{1 \le i \le n} P_i x_i$$

$$\text{Subject to} \sum_{1 \le i \le n} W_i x_i \le m$$

$$\text{where } 0 \le x_i \le 1 \quad \& \quad 1 \le i \le n$$

A feasible solution is any set $\{x_1 \dots x_n\}$ satisfying 2 and 3

- An optimal solution is a feasible solution for which 1 is maximized.

① Consider the following instance of Knapsack problem & find all feasible and hence an optimal solution

$n = 3$, $m = 20$ $(P_1, P_2, P_3) = \{25, 24, 15\}$

$(w_1, w_2, w_3) = \{18, 15, 10\}$

I feasible solution:

- Consider decreasing order of profit P_i

i,e P_1 has highest value = 25,

so place it in Knapsack first

set $x_1 = 1$ & profit $P_i x_i = 25 * 1 = 25$ is earned.

So now, Knapsack capacity = $20 - 18 = 02$

Next largest profit is $P_2 = 24$

but $W_2 = 15$

Knapsack capacity = 02, but $W_2 = 15$

which doesnt fit into knapsack.

Hence $\frac{2}{15}$ can fill Knapsack

$$\sum_{i=1 \text{ to } 3} w_i x_i = 18 \times 1 + 15 \times \frac{2}{15} = 20$$

$$\sum_{i=1 \text{ to } 3} P_i X_i = 25 \times 1 + 24 \times \frac{2}{15} = 28.2$$

Solution vector $(x_1, x_2, x_3) = (1, 2/15, 0)$

Feasible Solution = 28.2

II feasible solution:

- Consider "<u>increasing order of weight w_i</u>"

i.e object 3 has least weight $w_3 = 10$

Hence place object 3 into knapsack &

set $x_3 = 1$ & profit $p_i x_i = P_3 x_3 = 15 \times 1 = 15$

Knapsack capacity reduces to

$20 - 10 = 10$

- Next least weight is $w_2 = 15$, but capacit now is 10, so it does not fit

Hence use fraction $10/15 = 2/3$ to fill knapsack.

Now $\sum_{i=1 \text{ to } 3} w_i x_i = 10 \times 1 + 15 \times \frac{2}{3} = 20$

$\sum_{i=1 \text{ to } 3} P_i x_i = 15 \times 1 + 24 \times \frac{2}{3} = 31$

Solution Vector $(x_1, x_2, x_3) = (0, 2/3, 1)$

and feasible Solution $= 31$

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III feasible Solution:

Consider decreasing order of ratio "P_i/w_i"

$P_1/w_1 = \frac{25}{18} = 1.38$

$P_2/w_2 = \frac{24}{15} = 1.6$

$P_3/w_3 = 15/10 = 1.5$

- According to decreasing order of P_i/w_i first consider object 2 with weight 15 & place it in knapsack & set $x_2 = 1$

& profit $P_i x_i = 24 \times 1 = 24$ is earned

capacity reduces to $20 - 15 = 5$

- Next object 3 is considered i.e $W_3 = 10$. which does not fit into knapsack and hence a fraction $5/10 = 1/2$ is used to fill knapsack $\therefore x_3 = 1/2$

Now $\sum_{i=1 \text{ to } 3} W_i X_i = 15 \times 1 + 10 \times \frac{1}{2} = 20$

$\sum_{i=1 \text{ to } 3} P_i X_i = 24 \times 1 + 15 \times 1/2 = 31.5$

Solution vector $(x_1, x_2, x_3) = (0, 1, 1/2)$

Feasible solution $= 31.5$

- Now comparing all three feasible solution, it can be concluded that the 3rd feasible solution i,e 31.5 is maximum

- Hence the optimal solution is 31.5 with solution vector (x_1, x_2, x_3) $= (0, 1, 1/2)$

* Obtain the optimal solution for the following instance of knapsack problem.

① $m = 40$ $n = 3$

$w_i = \{20, 25, 10\}$ $P_i = \{30, 40, 35\}$

$1 \leq i \leq 3$

82.5

② $n = 3$, $m = 20$, $w_i = \{18, 15, 10\}$ $P_i = \{30, 21, 18\}$

32.8

③ $n = 7$, $m = 15$, $w_i = \{2, 3, 5, 7, 1, 4, 1\}$

$P_i = \{10, 5, 15, 7, 6, 18, 3\}$

55.35

Algorithm Greedy_Knapsack (m, n)

// finds the solution vector

// Input : m, the capacity of knapsack,

n, number of objects &

w_i the weights of 'n' objects

// output : the solution vector x

```
{
    for i=1 to n do x[i]=0.0;
        u = m;
    for i=1 to n do
    {
            if (w[i] > u) then break;
            x[i] = 1.0;
            u = u - w[i];
    }
    if (i ≤ n) then
        x[i] = u/w[i];
}
```