Module-4
Part - 3

Fitting of a curve of the form $y = ax^b$

$$y = ax^b \qquad \text{----- (1)}$$

Taking *log* on both sides

$$\log y = \log\left(ax^b\right)$$

$$\log y = \log a + b\log x$$

$$Y = A + BX \qquad \text{----- (2)}$$

where $Y = \log y$, $A = \log a$, $B = b$ and $X = \log x$

The normal equations of (2) are

$$\Sigma Y = nA + B\Sigma X \quad \text{and}$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

Solving these equations we obtain A and B from which $a = e^A$ and $b = B$ can be found.

Substituting these values of a and b in (1), we obtain the equation of the best fitting curve.

WORKED EXAMPLES

Fit a curve of the form $y = ax^b$ for the data

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.1	6.8	7.5

$$y = ax^b \qquad \text{----- (1)}$$

Taking *log* on both sides

$$\log y = \log\left(ax^b\right)$$

$$\log y = \log a + b\log x$$

$$Y = A + BX \qquad \text{----- (2)}$$

where $Y = \log y,\ A = \log a,\ B = b$ and $X = \log x$

The normal equations of (2) are

$$\Sigma Y = nA + B\Sigma X \quad \text{and}$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

where $Y = \log y$, $A = \log a$, $B = b$ and $X = \log x$

We prepare a relevant table as follows

	x	y	$X = \log x$	$Y = \log y$	XY	X^2
	1	2.98	0	1.0919	0	0
	2	4.26	0.6931	1.4492	1.0044	0.4804
	3	5.21	1.0986	1.6506	1.8133	1.2069
	4	6.1	1.3863	1.8083	2.5068	1.9218
	5	6.8	1.6094	1.9169	3.0851	2.5909
	6	7.5	1.7918	2.0149	3.6103	3.2105
Σ	21	32.85	6.5792	9.9318	12.0199	9.4098

Here, $n = 6$, $\Sigma X = 6.5792$, $\Sigma Y = 9.9318$, $\Sigma XY = 12.0199$ and $\Sigma X^2 = 9.4098$

Substituting these values in the above normal equations, we get

$$9.9318 = 6A + 6.5792B$$
$$12.0199 = 6.5792A + 9.4098B$$

Solving these equations, we get

$$A = 1.0912, \quad B = 0.5144$$

$$\Rightarrow \quad a = e^A = e^{1.0912} = 2.9778 \quad \text{and } b = B = 0.5144$$

$\therefore$ The equation of the best fitting curve is $y = (2.9778)x^{0.5144}$

Fit a curve of the form $y = ax^b$ for the data

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

$$y = ax^b \qquad \text{----- (1)}$$

Taking *log* on both sides

$$\log y = \log\left(ax^b\right)$$

$$\log y = \log a + b \log x$$

$$Y = A + BX \qquad \text{----- (2)}$$

where $Y = \log y,\ A = \log a,\ B = b$ and $X = \log x$

The normal equations of (2) are

$$\Sigma Y = nA + B\Sigma X \quad \text{and}$$
$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

where $Y = \log y$, $A = \log a$, $B = b$ and $X = \log x$

We prepare a relevant table as follows

	x	y	$X = \log x$	$Y = \log y$	XY	X^2
	1	0.5	0	-0.6931	0	0
	2	2	0.6931	0.6931	0.4804	0.4804
	3	4.5	1.0986	1.5041	1.6524	1.2069
	4	8	1.3863	2.0794	2.8827	1.9218
	5	12.5	1.6094	2.5257	4.0649	2.5903
Σ	15	27.5	4.7874	6.1092	9.0804	6.1993

Here, $n = 5$, $\Sigma X = 4.7874$, $\Sigma Y = 6.1092$, $\Sigma XY = 9.0804$ and $\Sigma X^2 = 6.1993$

Substituting these values in the above normal equations, we get

$$6.1092 = 5A + 4.7874B$$
$$9.0804 = 4.7874A + 6.1993B$$

Solving these equations, we get

$$A = -0.6929, \quad B = 1.9998$$

$$\Rightarrow \quad a = e^{A} = e^{-0.6929} = 0.5 \quad \text{and} \quad b = B = 1.9998$$

$\therefore$ The equation of the best fitting curve is $y = (0.5)x^{1.9998}$

An experiment gave the following values:

v (ft/min)	350	400	500	600
t (min)	61	26	7	2.6

It is known that v and t are connected by the relation $v = at^b$. Find the best possible values of a and b.

$$v = at^b \qquad \text{----- (1)}$$

Taking *log* on both sides

$$\log v = \log\left(a\,t^b\right)$$

$$\log v = \log a + b \log t$$

$$Y = A + BX \qquad \text{----- (2)}$$

where $Y = \log v$, $A = \log a$, $B = b$ and $X = \log t$

The normal equations of (2) are

$$\Sigma Y = nA + B\Sigma X \quad \text{and}$$
$$\Sigma XY = A\Sigma X + B\Sigma X^2$$

where $Y = \log v,\ A = \log a,\ B = b$ and $X = \log t$

We prepare a relevant table as follows

	v	t	$X = \log t$	$Y = \log v$	XY	X^2
	350	61	4.1109	5.8579	24.0812	16.8995
	400	26	3.2581	5.9915	19.5209	10.6152
	500	7	1.9459	6.2146	12.0930	3.7865
	600	2.6	0.9555	6.3969	6.1122	0.9130
Σ	1850	96.6	10.2704	24.4609	61.8073	29.2142

Here, $n = 4$, $\Sigma X = 10.2704$, $\Sigma Y = 24.4609$, $\Sigma XY = 61.8073$ and $\Sigma X^2 = 29.2142$

Substituting these values in the above normal equations, we get

$$24.4609 = 4A + 10.2704B$$
$$61.8073 = 10.2704A + 29.2142B$$

Solving these equations, we get

$$A = 7.0167, \quad B = -0.3511$$

$$\Rightarrow \quad a = e^A = e^{7.0167} = 1115.10 \quad \text{and} \quad b = B = -0.3511$$

EXERCISE

1. Fit a curve of the form $y = ax^b$ for the data

x	20	16	10	11	14
y	22	41	120	89	56

(VTU 2015)

2. Fit a curve of the form $y = ax^b$ for the data

x	1	2	4	6
y	6	4	2	2

3. Predict y at $x = 3.75$, by fitting a curve $y = ax^b$ for the data

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.10	6.80	7.50

4. Fit a curve of the form $y = ax^b$ for the data

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5