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Probability distribution of a random variable

A description giving the values of the random variable X along with the corresponding probabilities is called the probability distribution of the random variable X.

X	x_1	x_2	x_3	...	x_n
$P(X)$	p_1	p_2	p_3	...	p_n

Discrete Probability Distribution

Let a random variable X assume values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ such that

(i) $P(X = x_i) = p_i \geq 0$, for each x_i and

(ii) $\sum_{i=1}^{n} p_i = p_1 + p_2 + \ldots + p_n = 1$.

Then the probability distribution is called discrete probability distribution.

Mean and variance of discrete probability distribution

Let X be a random variable whose possible values $x_1, x_2, ..., x_n$ occur with probabilities $p_1, p_2, ..., p_n$, respectively. The mean of X, denoted by μ, is the number $\sum_{i=1}^{n} x_i p_i$, i.e., the mean of X is the weighted average of the possible values of X, each value being weighted by its probability with which it occurs. The mean of a random variable X is also called the expectation of X, denoted by $E(X)$.

Thus,

$$E(X) = \mu = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + ... + x_n p_n$$

In other words, the mean or expectation of a random variable X is the sum of the products of all possible values of X by their respective probabilities.

The variance of X is denoted by $V(X)$ and is defined by

$$V(X) = \sum_{i=1}^{n} x_i^2 p_i - \mu^2 = E(X^2) - [E(X)]^2$$

The non-negative number

$$\sigma_X = \sqrt{V(X)}$$

is called the standard deviation of the random variable X.

Example : A random variable X has the following probability distribution:

X	-3	-2	-1	0	1	2	3
$P(X)$	k	$2k$	$3k$	$4k$	$3k$	$2k$	k

(i) **Find k**

(ii) **Evaluate $P(X \leq 1)$, $P(X > 1)$, $P(-1 < X \leq 2)$**

(iii) **Find its mean and standard deviation.**

(i) We have, $\sum_{i=1}^{n} p_i = 1$

$\Rightarrow \quad p_1 + p_2 + \ldots + p_8 = 1$

$\Rightarrow \quad k + 2k + 3k + 4k + 3k + 2k + k = 1$

$\Rightarrow \quad 16k = 1$

$\Rightarrow \quad k = \frac{1}{16}$

(ii) $$P(X \le 1) = 1 - P(X > 1)$$

$$= 1 - [P(X = 2) + P(X = 3)]$$

$$= 1 - (2k + k)$$

$$= 1 - 3k$$

$$= 1 - \frac{3}{16} = \frac{13}{16}$$

$$P(X > 1) = P(X = 2) + P(X = 3)$$

$$= 2k + k = 3k = \frac{3}{16}$$

$$P(-1 < X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= 4k + 3k + 2k = 9k = \frac{9}{16}$$

(iii) Mean $\mu = \sum x_i p_i$

$$= (-3)(k) + (-2)(2k) + (-1)(3k) + 0 + (1)(3k) + (2)(2k) + (3)k$$

$$= -3k - 4k - 3k + 3k + 4k + 3k = 0$$

Variance $V = \sum x_i^2 p_i - \mu^2$

$$= (-3)^2(k) + (-2)^2(2k) + (-1)^2(3k) + 0 + (1)^2(3k) + (2)^2(2k) + (3)^2 k$$

$$= 9k + 8k + 3k + 3k + 8k + 9k = 40k = \frac{40}{16} = \frac{5}{2}$$

Standard deviation $\sigma = \sqrt{V} = \sqrt{\frac{5}{2}} = 1.581$

Example A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6
$P(X)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

(i) Find k

(ii) Evaluate $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$

(iii) Find the minimum value of k so that $P(X \leq 2) > 0.3$

(i) We have, $\sum_{i=1}^{n} p_i = 1$

$\Rightarrow \quad p_1 + p_2 + \ldots + p_8 = 1$

$\Rightarrow \quad k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$\Rightarrow \quad 49k = 1$

$\Rightarrow \quad k = \dfrac{1}{49}$

(ii) $P(X<4)=1-P(X\geq 4)$

$$=1-\left[P(X=4)+P(X=5)+P(X=6)\right]$$

$$=1-(9k+11k+13k)$$

$$=1-33k$$

$$=1-\frac{33}{49}=\frac{16}{49}$$

$$P(X\geq 5)=P(X=5)+P(X=6)$$

$$=11k+13k=24k=\frac{24}{49}$$

$$P(3<X\leq 6)=P(X=4)+P(X=5)+P(X=6)$$

$$=9k+11k+13k=33k=\frac{33}{49}$$

(iii) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$$= k + 3k + 5k = 9k > 0.3$$

$$= k > \frac{0.3}{9} = \frac{1}{30}$$

$\therefore$ The minimum value of k is $\frac{1}{30}$.

Example **A random variable *X* has the following probability distribution:**

X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
***P* (*X*)**	**0**	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) **Find k**

(ii) **Evaluate $P(X < 6)$, $P(X \ge 6)$, $P(3 < X \le 6)$**

(iii) **Find the minimum value of x so that $P(X \le x) > \frac{1}{2}$.**

(i) We have, $\sum_{i=1}^{n} p_i = 1$

$\Rightarrow \quad p_1 + p_2 + ... + p_8 = 1$

$\Rightarrow \quad 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow \quad 10k^2 + 9k - 1 = 0$

$\Rightarrow \quad (10k - 1)(k + 1) = 0$

$\Rightarrow \quad k = \frac{1}{10} \qquad [\because p_i \geq 0]$

(ii) $P(X < 6) = 1 - P(X \geq 6)$

$= 1 - [P(X = 6) + P(X = 7)]$

$= 1 - (2k^2 + 7k^2 + k) = 1 - 9k^2 - k = 1 - \frac{9}{100} - \frac{1}{10} = \frac{81}{100}$

$$P(X \geq 6) = P(X = 6) + P(X = 7)$$

$$= 2k^2 + 7k^2 + k \quad = 9k^2 - k \quad = \frac{9}{100} + \frac{1}{10} = \frac{19}{100}$$

$$P(3 < X \leq 6) = P(X = 4) + P(X = 5) + P(X = 6)$$

$$= 3k + k^2 + 2k^2 \quad = 3k + 3k^2 \quad = \frac{3}{10} + \frac{3}{100} = \frac{33}{100}$$

(iii) $$P(X \leq 1) = P(X = 0) + P(X = 1)$$

$$= 0 + k = k = \frac{1}{10} < \frac{1}{2}$$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= 0 + k + 2k = 3k = \frac{3}{10} < \frac{1}{2}$$

$$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= 0 + k + 2k + 2k = 5k = \frac{5}{10} = \frac{1}{2}$$

$$P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= 0 + k + 2k + 2k + 3k = 8k = \frac{8}{10} > \frac{1}{2}$$

$\therefore$ The minimum value of x so that $P(X \le x) > \frac{1}{2}$ is 4.

Example A random variable X has the following probability distribution:

X	−2	−1	0	1	2	3
$P(X)$	0.1	k	0.2	$2k$	0.3	k

(i) Find k

(ii) Evaluate $P(X < 1)$, $P(X > -1)$

(iii) Find its mean and standard deviation.

(i) We have, $\sum_{i=1}^{n} p_i = 1$

$\Rightarrow \quad p_1 + p_2 + \ldots + p_8 = 1$

$\Rightarrow \quad 0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$\Rightarrow \quad 4k + 0.6 = 1$

$\Rightarrow \quad 4k = 0.4$

$\Rightarrow \quad k = 0.1$

(ii) $$P(X<1)=1-P(X\geq 1)$$

$$=1-\left[P(X=1)+P(X=2)+P(X=3)\right]$$

$$=1-(2k+0.3+k)$$

$$=0.7-3k$$

$$=0.7-3(0.1)=0.4$$

$$P(X>-1)=1-P(X\leq -1)$$

$$=1-\{P(X=-2)+P(X=-1)\}$$

$$=1-\{0.1+k\}$$

$$=1-\{0.1+0.1\}=1-0.2=0.8$$

(iii) Mean $\mu = \sum x_i p_i$

$$= (-2)(0.1) + (-1)(k) + 0 + (1)(2k) + (2)(0.3) + (3)k$$

$$= -0.2 - k + 2k + 0.6 + 3k$$

$$= 4k + 0.4 = 4(0.1) + 0.4 = 0.8$$

Variance $V = \sum x_i^2 p_i - \mu^2$

$$= 0.4 + k + 2k + 1.2 + 9k - 0.64$$

$$= 12k + 0.96 = 12(0.1) + 0.96 = 2.16$$

Standard deviation $\sigma = \sqrt{V} = \sqrt{2.16} = 1.47$

EXERCISE

1. A random variable X has the following probability distribution:

X	0	1	2	3	4	5
$P(X)$	k	$5k$	$10k$	$10k$	$5k$	k

(i) Find k

(ii) Find mean and standard and deviation

2. Find the standard deviation for the following probability distribution:

X	8	12	16	20	24
$P(X)$	1/8	1/6	3/8	1/4	1/12

Module-3
Part - 2

Continuous Probability Distribution

For every continuous random variable X, the real number $f(x)$ is said to be continuous probability function or probability density function (p.d.f) if the following conditions are satisfied:

(i) $f(x) \geq 0$ and

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Note: The probability of x lies in the interval (a, b) is defined as

$$P(a \leq x \leq b) = \int_{a}^{b} f(x)\,dx$$

Note: If X is a continuous random variable with p.d.f $f(x)$ then the function $f(x)$ is defined by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

Mean and variance of continuous probability distribution

If X is a continuous random variable with probability density function $f(x)$ where $-\infty < x < \infty$ then the mean, variance and standard deviation of X are given by

Mean $\mu = \int_{-\infty}^{\infty} x f(x) dx$

Variance $V = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$ or $V = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

Standard deviation $\sigma = \sqrt{V}$

Example **Find the constant C such that**

$$f(x)=\begin{cases} Cx^2, & 0 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

is a probability density function. Also compute $P(1 < X < 2)$.

$f(x)$ is (p.d.f) if

(i) $f(x) \ge 0$ and (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From (ii), $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \quad \int_{0}^{3} Cx^2\,dx = 1$$

$$\Rightarrow \qquad \int_0^3 Cx^2\,dx = 1$$

$$\Rightarrow \qquad C\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$\Rightarrow \qquad C\left[\frac{(3)^3 - 0}{3}\right] = 1$$

$$\Rightarrow \qquad C[9] = 1 \qquad \Rightarrow \quad C = \frac{1}{9}$$

We have, $$P(a \le x \le b) = \int_a^b f(x)\,dx$$

$$\Rightarrow \qquad P(1 < x < 2) = \int_1^2 f(x)\,dx$$

$\Rightarrow$ $$P(1 < x < 2) = \int_{1}^{2} f(x)\,dx$$

$$= \int_{1}^{2} \frac{1}{9} x^2\,dx$$

$$= \frac{1}{9}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{9}\left[\frac{(2)^3 - (1)^3}{3}\right] = \frac{7}{27}$$

Example **A random variable X has the density function:**

$$f(x)=\begin{cases} kx^2, & -3 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

Find k and also find $P(X \le 2)$ and $P(X > 1)$.

$f(x)$ is (p.d.f) if

(i) $f(x) \ge 0$ and (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From (ii), $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$\Rightarrow$ $$\int_{-3}^{3} kx^2\,dx = 1$$

$$\Rightarrow \qquad \int_{-3}^{3} kx^2\,dx = 1$$

$$\Rightarrow \qquad k\left[\frac{x^3}{3}\right]_{-3}^{3} = 1$$

$$\Rightarrow \qquad k\left[\frac{(3)^3-(-3)^3}{3}\right] = 1$$

$$\Rightarrow \qquad k[9+9] = 1$$

$$\Rightarrow \qquad k = \frac{1}{18}$$

We have, $$P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

$$\Rightarrow \quad P(X \le 2) = \int_{-\infty}^{2} f(x)\,dx$$

$$= \int_{-3}^{2} \frac{1}{18} x^2\,dx$$

$$= \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2}$$

$$= \frac{1}{18}\left[\frac{(2)^3 - (-3)^3}{3}\right]$$

$$= \frac{35}{54}$$

We have, $$P(X > x) = \int_x^\infty f(x)\,dx$$

$$\Rightarrow \quad P(X > 1) = \int_1^\infty f(x)\,dx$$

$$= \int_1^3 \frac{1}{18} x^2\, dx$$

$$= \frac{1}{18}\left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{18}\left[\frac{(3)^3 - (1)^3}{3}\right] = \frac{13}{27}$$

Example 3.2.13: A random variable *X* has the density function:

$$f(x)=\begin{cases} k(x+1), & -1<x<1 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) *k*, (ii) mean and variance of *X*.

$f(x)$ is (p.d.f) if

(i) $f(x)\geq 0$ and (ii) $\int_{-\infty}^{\infty} f(x)dx=1$

From (ii), $\int_{-\infty}^{\infty} f(x)dx=1$

$\Rightarrow$ $\int_{-1}^{1} k(x+1)dx=1$

$$\Rightarrow \qquad \int_{-1}^{1} k(x+1)\,dx = 1$$

$$\Rightarrow \qquad k\left[\frac{x^2}{2}+x\right]_{-1}^{1} = 1$$

$$\Rightarrow \qquad k\left[\left(\frac{(1)^2}{2}+1\right)-\left(\frac{(-1)^2}{2}-1\right)\right]=1$$

$$\Rightarrow \qquad 2k = 1$$

$$\Rightarrow \qquad k = \frac{1}{2}$$

We have, Mean $\mu = \int_{-1}^{1} x f(x)\,dx$

$$= \int_{-1}^{1} x\left[\frac{1}{2}(x+1)\right]dx$$

$$= \frac{1}{2}\int_{-1}^{1} \left(x^2 + x\right) dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\left(\frac{1}{3} + \frac{1}{2}\right) - \left(\frac{-1}{3} + \frac{1}{2}\right)\right] = \frac{1}{3}$$

We have, Variance $V = \int_{-1}^{1} (x-\mu)^2 f(x) dx$

$$= \int_{-1}^{1} \left(x - \frac{1}{3}\right)^2 \left[\frac{1}{2}(x+1)\right] dx$$

$$= \frac{1}{2} \int_{-1}^{1} \left(x^2 - \frac{2x}{3} + \frac{1}{9}\right)(x+1) dx$$

$$= \frac{1}{2} \int_{-1}^{1} \left(x^3 + \frac{x^2}{3} - \frac{5x}{9} + \frac{1}{9}\right) dx$$

$$= \frac{1}{2} \left[\frac{x^4}{4} + \frac{x^3}{9} - \frac{5x^2}{18} + \frac{1}{9} x\right]_{-1}^{1}$$

$$=\frac{1}{2}\left[\left(\frac{1}{4}+\frac{1}{9}-\frac{5}{18}+\frac{1}{9}\right)-\left(\frac{1}{4}-\frac{1}{9}-\frac{5}{18}-\frac{1}{9}\right)\right] \quad =\frac{2}{9}$$

Example : The function $f(x)$ is defined as

$$f(x)=\begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

Is $f(x)$ a probability density function? If so, determine the probability that the variate having this density will fall in the interval (1, 2). Also find the cumulative probability function *F* (2).

(i) Clearly, $f(x) \geq 0$ and

(ii) $$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} (0)dx + \int_{0}^{\infty} e^{-x}\, dx$$

$$= 0 + \int_0^{\infty} e^{-x}\, dx$$

$$= \left[\frac{e^{-x}}{-1} \right]_0^{\infty} = -\left[e^{-\infty} - e^{0} \right] = -(0-1) = 1$$

Hence the function $f(x)$ satisfies the requirements for a density function.

$\therefore$ $f(x)$ is a probability density function

We have, $$P(a \le x \le b) = \int_a^b f(x)\,dx$$

$\Rightarrow$ $$P(1 < x < 2) = \int_1^2 f(x)\,dx$$

$\Rightarrow$ $$P(1<x<2)=\int_1^2 f(x)dx$$

$$=\int_1^2 e^{-x}\,dx$$

$$=\left[\frac{e^{-x}}{-1}\right]_1^2$$

$$=-\left(e^{-2}-e^{-1}\right)=0.233$$

Also we have, $$F(x)=P(X\le x)=\int_{-\infty}^{x} f(x)dx$$

$\Rightarrow$ $$F(2)=P(X\le 2)=\int_{-\infty}^{2} f(x)dx$$

$\Rightarrow$

$$F(2) = P(X \le 2) = \int_{-\infty}^{2} f(x)\,dx$$

$$= \int_{-\infty}^{0} (0)\,dx + \int_{0}^{2} e^{-x}\,dx$$

$$= 0 + \int_{0}^{2} e^{-x}\,dx$$

$$= \left[\frac{e^{-x}}{-1}\right]_{0}^{2}$$

$$= -\left[e^{-2} - e^{0}\right] = 0.865$$

EXERCISE

1. If a function $f(x)$ defined by $f(x)=\begin{cases}\left(\dfrac{x+1}{8}\right), & 2<x<4 \\ 0, & \text{otherwise}\end{cases}$

   (i) Prove that $f(x)$ is a p.d.f

   (ii) Find $P(X < 3.5)$ and $P(X \geq 3.5)$

2. Find the value of k such that $f(x)=\begin{cases}\dfrac{x}{6}+k, & 0\leq x\leq 3 \\ 0, & \text{otherwise}\end{cases}$

   is a p.d.f. Also find $P(1 \leq X \leq 2)$.

3. Find the value of k such that $f(x)=\begin{cases}k\,x\,e^{-x}, & 0<x<1 \\ 0, & \text{otherwise}\end{cases}$

   is a p.d.f. Also find its mean.

4. Find the value of k such that $f(x)=\begin{cases}k\,e^{-3x}, & x>0 \\ 0, & \text{otherwise}\end{cases}$

   is a p.d.f. Also find $P(0.5 \leq X \leq 1)$ and $P(-2 \leq X \leq 1.5)$

Module-3
Part – 3
Binomial Distribution

Binomial Distribution

Trials of a random experiment are called **Bernoulli trials**, if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes: **success or failure.**

(iv) The probability of success remains the same in each trial.

If p is the probability of success and q is the probability of failure then the probability distribution of number of successes in an experiment consisting of n Bernoulli trials may be obtained by the binomial expansion of $(q+p)^n$.

Hence, this distribution of number of successes ***X*** can be written as

X	0	1	2	...	r	...	n
$P(X)$	${}^nC_0 q^n$	${}^nC_1 q^{n-1} p$	${}^nC_2 q^{n-2} p^2$	...	${}^nC_r q^{n-r} p^r$		${}^nC_n p^n$

The above probability distribution is known as binomial distribution with parameters n and p, because for given values of n and p, we can find the complete probability distribution. A binomial distribution with n-Bernoulli trials and probability of success in each trial as p, is denoted by $B(n, p)$.

If ***p* is the probability of success and *q* is the probability of failure** then the probability of ***r* successes out of *n* trails** is given by

$$P(X=r)=P(r)={}^{n}C_{r}\,q^{n-r}\,p^{r},$$

$r = 0, 1, \ldots, n$ and $q = 1 - p$.

Mean and variance of Binomial distribution

The mean and variance of the binomial distribution are given by

Mean $\mu = np$

Variance $V = npq$

Standard deviation $\sigma = \sqrt{V} = \sqrt{npq}$

Example **The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that (i) exactly two will be defective, (ii) at least two will be defective, (iii) none will be defective.**

Here $n = 12$ pens are manufactured

Let p = Probability of defective = $\frac{1}{10}$

Hence, $q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

We have, $P(r) = {}^{n}C_{r}\, q^{n-r} p^{r}$

$$\Rightarrow \quad P(r) = {}^{12}C_{r}\left(\frac{9}{10}\right)^{12-r}\left(\frac{1}{10}\right)^{r}$$

$$P(r) = {}^{12}C_r\left(\frac{9}{10}\right)^{12-r}\left(\frac{1}{10}\right)^{r}$$

(i) P (exactly two will be defective) $= P(r=2)$

$$= {}^{12}C_2\left(\frac{9}{10}\right)^{12-2}\left(\frac{1}{10}\right)^{2} \quad = (66)\left(\frac{9}{10}\right)^{10}\left(\frac{1}{10}\right)^{2}$$

$$= 0.2301$$

(ii) P (at least two will be defective) $= P(r \geq 2)$

$$= 1 - P(r < 2)$$

$$= 1 - \{P(r=0) + P(r=1)\}$$

$$=1-\left\{{}^{12}C_0\left(\frac{9}{10}\right)^{12-0}\left(\frac{1}{10}\right)^0+{}^{12}C_1\left(\frac{9}{10}\right)^{12-1}\left(\frac{1}{10}\right)^1\right\}$$

$$=1-\left\{(1)\left(\frac{9}{10}\right)^{12}(1)+(12)\left(\frac{9}{10}\right)^{11}\left(\frac{1}{10}\right)\right\}$$

$$=1-\left(\frac{9}{10}\right)^{11}\left(\frac{21}{10}\right)=0.3412$$

(iii) P (none will be defective) $=P(r=0)$

$$={}^{12}C_0\left(\frac{9}{10}\right)^{12-0}\left(\frac{1}{10}\right)^0 \quad =(1)\left(\frac{9}{10}\right)^{12}(1)=0.2824$$

Example The number of telephone lines busy at an instant of time is a binomial variate with probability 0.2. If at an instant 10 lines are chosen at random, find the probability that (i) 5 lines are busy, (ii) at most 2 lines are busy, (iii) all lines are busy.

Here n = 10 lines are chosen

Let p = Probability of busy = 0.2

Hence, $q = 1 - p = 1 - 0.2 = 0.8$

We have, $P(r) = {}^{n}C_{r}\, q^{n-r} p^{r}$

$$\Rightarrow \quad P(r) = {}^{10}C_{r}\,(0.8)^{10-r}(0.2)^{r}$$

$$P(r) = {}^{10}C_r (0.8)^{10-r} (0.2)^r$$

(i) P (5 lines are busy) $= P(r=5)$

$$= {}^{10}C_5 (0.8)^{10-5} (0.2)^5 = (252)(0.8)^5 (0.2)^5$$

$$= 0.02642$$

(ii) P (at most 2 lines are busy) $= P(r \le 2)$

$$= P(r=0) + P(r=1) + P(r=2)$$

$$= {}^{10}C_0 (0.8)^{10-0} (0.2)^0 + {}^{10}C_1 (0.8)^{10-1} (0.2)^1 + {}^{10}C_2 (0.8)^{10-2} (0.2)^2$$

$$= 0.6778$$

$$P(r) = {}^{10}C_r\,(0.8)^{10-r}\,(0.2)^r$$

(iii) P (all lines are busy) $= P(r=10)$

$$= {}^{10}C_{10}\,(0.8)^{10-10}\,(0.2)^{10}$$

$$= (1)(1)(0.2)^{10} = \frac{1}{5^{10}}$$

Example The probability of a newly generated virus attacked to the computer will corrupt the 4 files out of 20 files opened in an hour. If 12 files are opened in an hour, find the probability that (i) at least 10 files are corrupted, (ii) exactly 3 files are corrupted, (iii) all the files are corrupted, (iv) all the files are safe, (v) more than 2 but not more than 5 files are corrupted.

Here $n = 12$ files are opened in an hour

Let p = Probability of corrupted files = $\dfrac{4}{20} = \dfrac{1}{5}$

Hence, $q = 1 - p = 1 - \dfrac{1}{5} = \dfrac{4}{5}$

We have, $P(r) = {}^nC_r\, q^{n-r} p^r$

$$\Rightarrow \qquad P(r) = {}^{12}C_r \left(\frac{4}{5}\right)^{12-r} \left(\frac{1}{5}\right)^r$$

$$P(r) = {}^{12}C_r\left(\frac{4}{5}\right)^{12-r}\left(\frac{1}{5}\right)^r$$

(i) P (at least 10 files are corrupted) $= P(r \geq 10)$

$$= P(r=10) + P(r=11) + P(r=12)$$

$$= {}^{12}C_{10}\left(\frac{4}{5}\right)^{12-10}\left(\frac{1}{5}\right)^{10} + {}^{12}C_{11}\left(\frac{4}{5}\right)^{12-11}\left(\frac{1}{5}\right)^{11} + {}^{12}C_{12}\left(\frac{4}{5}\right)^{12-12}\left(\frac{1}{5}\right)^{12}$$

$= 0.000004526$

(ii) P (3 files are corrupted) $= P(r=3)$

$$= {}^{12}C_3\left(\frac{4}{5}\right)^{12-3}\left(\frac{1}{5}\right)^3 = 0.001074$$

$$P(r) = {}^{12}C_r \left(\frac{4}{5}\right)^{12-r} \left(\frac{1}{5}\right)^r$$

(iii) P (all files are corrupted) $= P(r = 12)$

$$= {}^{12}C_{12} \left(\frac{4}{5}\right)^{12-12} \left(\frac{1}{5}\right)^{12} \quad = \left(\frac{1}{5}\right)^{12}$$

(iv) P (all files are safe) = 1 – P (all files are corrupted)

$$= 1 - \left(\frac{1}{5}\right)^{12}$$

(v) P (more than 2 files but not more than 5 are corrupted)

$$= P(2 < r \leq 5)$$

$$= P(r = 3) + P(r = 4) + P(r = 5)$$

$$= {}^{12}C_3\left(\frac{4}{5}\right)^{12-3}\left(\frac{1}{5}\right)^3 + {}^{12}C_4\left(\frac{4}{5}\right)^{12-4}\left(\frac{1}{5}\right)^4 + {}^{12}C_5\left(\frac{4}{5}\right)^{12-5}\left(\frac{1}{5}\right)^5$$

$$= 0.42225$$

Example **A box contains 100 transistors, 20 of which are defective and 10 are selected at random, find the probability that (i) all are defective, (ii) at least one is defective, (iii) all are good, (iv) at most 3 are defective.**

Here $n = 10$ are selected at random

Let p = Probability of defective = $\frac{20}{100} = \frac{1}{5}$

Hence, $q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$

We have, $P(r) = {}^{n}C_r\, q^{n-r}\, p^r$

$$\Rightarrow \quad P(r) = {}^{10}C_r \left(\frac{4}{5}\right)^{10-r} \left(\frac{1}{5}\right)^{r}$$

(i) P (all are defective) $= P(r = 10)$

$$= {}^{10}C_{10} \left(\frac{4}{5}\right)^{10-10} \left(\frac{1}{5}\right)^{10} = (1)(1)\left(\frac{1}{5}\right)^{10} = \frac{1}{5^{10}}$$

(ii) P (at least one is defective) $= P(r \geq 1)$

$$= 1 - P(r < 1) = 1 - P(r = 0)$$

$$=1-{}^{10}C_0\left(\frac{4}{5}\right)^{10-0}\left(\frac{1}{5}\right)^0 \qquad =1-\left(\frac{4}{5}\right)^{10}$$

(iii) P (all are good) = P (none is defective) $= P(r=0)$

$$= {}^{10}C_0\left(\frac{4}{5}\right)^{10-0}\left(\frac{1}{5}\right)^0 = (1)\left(\frac{4}{5}\right)^{10}(1) = \left(\frac{4}{5}\right)^{10}$$

(iv) P (at most 3 are defective) $= P(r \le 3)$

$$= P(r=0)+P(r=1)+P(r=2)+P(r=3)$$

$$= {}^{10}C_0\left(\frac{4}{5}\right)^{10-0}\left(\frac{1}{5}\right)^{0} + {}^{10}C_1\left(\frac{4}{5}\right)^{10-1}\left(\frac{1}{5}\right)^{1} + {}^{10}C_2\left(\frac{4}{5}\right)^{10-2}\left(\frac{1}{5}\right)^{2} + {}^{10}C_3\left(\frac{4}{5}\right)^{10-3}\left(\frac{1}{5}\right)^{3}$$

$$=(1)\left(\frac{4}{5}\right)^{10}(1)+(10)\left(\frac{4}{5}\right)^{9}\left(\frac{1}{5}\right)+(45)\left(\frac{4}{5}\right)^{8}\left(\frac{1}{5}\right)^{2}+(120)\left(\frac{4}{5}\right)^{7}\left(\frac{1}{5}\right)^{3}$$

$$=\left(\frac{4}{5}\right)^{7}\left[\left(\frac{4}{5}\right)^{3}+2\left(\frac{4}{5}\right)^{2}+\left(\frac{4}{5}\right)\left(\frac{9}{5}\right)+(120)\left(\frac{1}{5}\right)^{3}\right]$$

$$=\left(\frac{4}{5}\right)^{7}\left[\frac{64}{125}+\frac{32}{25}+\frac{36}{25}+\frac{120}{125}\right]=\left(\frac{4}{5}\right)^{7}\left(\frac{524}{125}\right)$$

Example **Out of 800 families with 5 children each, how many would you expect to have (i) 3 boys, (ii) 5 girls, (iii) either 2 or 3 boys? (Assume equal probabilities for boys and girls)**

Here $n = 5$ children

Let p = Probability = $\frac{1}{2}$

Hence, $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

We have, $P(r) = {}^{n}C_{r}\, q^{n-r} p^{r}$

$$\Rightarrow \qquad P(r) = {}^{5}C_{r} \left(\frac{1}{2}\right)^{5-r} \left(\frac{1}{2}\right)^{r}$$

$$P(r) = {}^5C_r \left(\frac{1}{2}\right)^5$$

(i) $P\,(3\text{ boys}) = P(r=3)$

$$= {}^5C_3 \left(\frac{1}{2}\right)^5 = \frac{5}{16}$$

$\therefore$ For 800 families, the probability of number families having 3 boys is

$$= \frac{5}{16} \times 800 = 250$$

(ii) $P\,(5\text{ girls}) = P\,(\text{no boys}) = P(r=0)$

$$= {}^5C_0 \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$\therefore$ For 800 families, the probability of number families having 5 girls is

$$= \frac{1}{32} \times 800 = 25$$

(iii) P (either 2 or 3 boys) $= P(2 \leq r \leq 3)$

$$= P(2) + P(3)$$

$$= \frac{1}{2^5}\left\{{}^5C_2 + {}^5C_3\right\} = \frac{5}{8}$$

$\therefore$ For 800 families, the probability of number families having either 2 or 3 boys is $\frac{5}{8} \times 800 = 500$

Example The probability that a man aged 60 will live upto 70 is 0.65. Out of 10 men, now at the age of 60, find the probability that (i) at least 7 will live upto 70, (ii) exactly 9 will live upto 70, (iii) at most 9 will live upto 70.

Here n = 10 men

Let p = Probability that a man aged 60 will live upto 70 = 0.65

Hence, $q = 1 - p = 1 - 0.65 = 0.35$

We have, $P(r) = {}^{n}C_{r}\, q^{n-r} p^{r}$

$\Rightarrow$ $P(r) = {}^{10}C_{r}\,(0.35)^{10-r}(0.65)^{r}$

$$P(r) = {}^{10}C_r (0.35)^{10-r} (0.65)^r$$

(i) P (at least 7 will live upto 70) $= P(r \geq 7)$

$$= P(r=7) + P(r=8) + P(r=9) + P(r=10)$$

$$= {}^{10}C_7 (0.35)^{10-7} (0.65)^7 + {}^{10}C_8 (0.35)^{10-8} (0.65)^8$$
$$+ {}^{10}C_9 (0.35)^{10-9} (0.65)^9 + {}^{10}C_{10} (0.35)^{10-10} (0.65)^{10}$$

$= 0.5138$

(ii) P (exactly 9 will live upto 70) $= P(r=9)$

$$= {}^{10}C_9 (0.35)^{10-9} (0.65)^9 \quad = 0.07249$$

(iii) P (at most 9 will live upto 70) $= P(r \le 9)$

$$= 1 - P(r > 9)$$

$$= 1 - P(r = 10)$$

$$= 1 - {}^{10}C_{10}(0.35)^{10-10}(0.65)^{10}$$

$$= 0.9865$$

Example In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts.

Here $n = 20$

Given, Mean = 2

We have, Mean $\mu = np$

$$\Rightarrow \qquad p = \frac{\mu}{n} = \frac{2}{20} = 0.1$$

Hence, $q = 1 - p = 1 - 0.1 = 0.9$

We have, $P(r) = {}^{n}C_{r}\, q^{n-r}\, p^{r}$

$$\Rightarrow \qquad P(r) = {}^{20}C_{r}\,(0.9)^{20-r}\,(0.1)^{r}$$

P (at least 3 defective parts) $= P(r \geq 3)$

$$= 1 - P(r < 3)$$

$$= 1 - \{P(r=0) + P(r=1) + P(r=2)\}$$

$$= 1 - \left\{ {}^{20}C_{0}\,(0.9)^{20-0}\,(0.1)^{0} + {}^{20}C_{1}\,(0.9)^{20-1}\,(0.1)^{1} + {}^{20}C_{2}\,(0.9)^{20-2}\,(0.1)^{2} \right\}$$

$$= 0.323$$

$\therefore$ For 1000 such samples, $P(r \geq 3) = 1000 \times 0.323 = 323$

Example **Fit a binomial distribution for the data**

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Here $n = 5$ [since 6 values of x_i are given, $n + 1 = 6$]

$$\sum f_i = 2 + 14 + 20 + 34 + 22 + 8 = 100$$

$$\sum x_i f_i = 0 + 14 + 40 + 102 + 88 + 40 = 284$$

$$\therefore \text{ Mean } \mu = \frac{\sum x_i f_i}{\sum f_i} = \frac{284}{100} = 2.84$$

We have, Mean $\mu = np$

$$\Rightarrow \quad p = \frac{\mu}{n} = \frac{2.84}{5} = 0.568$$

Hence, $q = 1 - p = 1 - 0.568 = 0.432$

We have, $P(r) = {}^{n}C_{r}\, q^{n-r}\, p^{r}$

$$P(r) = {}^{5}C_{r}\,(0.432)^{5-r}\,(0.568)^{r}$$

Now, $F(x_i) = \left(\sum f_i\right) P(x_i)$

$$F(0) = (100)P(0) = (100)\left[{}^{5}C_{0}\,(0.432)^{5-0}\,(0.568)^{0}\right] = 1.505$$

$$F(1) = (100)P(1) = (100)\left[{}^{5}C_{1}\,(0.432)^{5-1}\,(0.568)^{1}\right] = 9.891$$

$$F(2) = (100)P(2) = (100)\left[{}^{5}C_{2}\,(0.432)^{5-2}\,(0.568)^{2}\right] = 26.01$$

$$F(3) = (100)P(3) = (100)\left[{}^{5}C_{3}\,(0.432)^{5-3}\,(0.568)^{3}\right] = 34.199$$

$$F(4)=(100)P(4)=(100)\left[{}^5C_4(0.432)^{5-4}(0.568)^4\right]=22.483$$

$$F(5)=(100)P(5)=(100)\left[{}^5C_5(0.432)^{5-5}(0.568)^5\right]=5.912$$

$\therefore$ The theoretical frequencies are

x	0	1	2	3	4	5
f	1.505	9.89	26.01	34.199	22.483	5.912

Fit a binomial distribution to the following frequency distribution:

x	0	1	3	4
f	28	62	10	4

Fit a binomial distribution to the following frequency distribution:

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

EXERCISE

1. Determine the binomial distribution for which mean is 2 and mean + variance = 3. Also find $P(X \leq 3)$.
2. If the probability that a new born child is a male is 0.6, find the probability that in a family of 5 children there are exactly 3 boys.
3. A die is thrown five times. If getting an odd number is a success, find the probability of getting at least four successes?
4. The probability that a bomb dropped from a plane will strike the target is 1/5. If six bombs are dropped, find the probability that (i) exactly two will strike the target, (ii) at least two will strike the target.
5. If 10% of the rivets produced by a machine are defective, find the probability that out of 5 rivets chosen at random (i) none will be defective, (ii) one will be defective and (iii) at least two will be defective.
6. A product is 0.5% defective and is packed in cartons of 100. What percentage contains not more than 3 defectives?
7. If in a lot of 500 solenoids 25 are defective, find the probability of 0, 1, 2, 3 defective solenoids in a random sample of 20 solenoids.

8. A sortie of 20 aeroplanes is sent on an operational flight. The chance that an aeroplane fails to return is 5%. Find the probability that (i) one plane does not return, (ii) at the most 5 planes do not return.
9. If on an average one vessel in every 10 is wrecked, find the probability that out of 5 vessels expected to arrive, at least 4 will arrive safely.
10. Five hundred articles were selected at random out of a batch containing 10,000 articles and 30 were found to be defective. How many defective articles would you reasonably expect to have in the whole batch?
11. Fit a binomial distribution to the following frequency distribution:

x	0	1	3	4
f	28	62	10	4

12. Fit a binomial distribution to the following frequency distribution:

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

Module-3
Part – 4
Poisson Distribution

Poisson Distribution

In a binomial distribution, if the following axioms hold:

(i) the number of trails $n \to \infty$,

(ii) the probability of success $p \to 0$ and

(iii) $np = \lambda$, is a finite number

then the probability distribution reduces to

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!},\ r = 0, 1, 2, \ldots$$

This probability distribution is called Poisson distribution.

Mean and variance of Poisson distribution

The mean and variance of the Poisson distribution are given by

Mean $\mu = \lambda$

Variance $V = \lambda$

Standard deviation $\sigma = \sqrt{V} = \sqrt{\lambda}$

Example Assume that the probability of an individual coalminer being killed in a mine accident during a year is $\frac{1}{2400}$. Use Poisson distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year.

Here $n = 200$

Let p = Probability of killed = $\frac{1}{2400}$

We have, $\lambda = np$

$$\lambda = (200)\left(\frac{1}{2400}\right) = 0.0833$$

We have, $$P(r)=\frac{\lambda^r e^{-\lambda}}{r!},\ r=0,1,2,\ldots$$

$$\Rightarrow \quad P(r)=\frac{(0.0833)^r e^{-0.0833}}{r!},\ r=0,1,2,\ldots$$

P (at least one fatal accident) $= P(r\geq 1)$

$$=1-P(r<1)$$

$$=1-P(r=0)$$

$$=1-\frac{(0.0833)^0 e^{-0.0833}}{0!}$$

$$=0.08$$

Example In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing (i) no defective, (ii) one defective, (iii) two defective blades respectively in a consignment of 10,000 packets.

Here $n = 10$

Let p = Probability of defective = 0.002

We have, $\lambda = np$

$$\lambda = (10)(0.002) = 0.02$$

We have, $$P(r)=\frac{\lambda^r e^{-\lambda}}{r!},\ r=0,1,2,\ldots$$

$\Rightarrow$ $$P(r)=\frac{(0.02)^r e^{-0.02}}{r!},\ r=0,1,2,\ldots$$

(i) P (no defective) $=P(r=0)$

$$=\frac{(0.02)^0 e^{-0.02}}{0!}\quad =0.9802$$

$\therefore$ Number of packets containing no defective blade is

$$10000 \times 0.9802 = 9802$$

$$P(r) = \frac{(0.02)^r e^{-0.02}}{r!}, \quad r = 0, 1, 2, \ldots$$

(ii) P (one defective) $= P(r = 1)$

$$= \frac{(0.02)^1 e^{-0.02}}{1!} = 0.0196$$

$\therefore$ Number of packets containing one defective blade is

$$10000 \times 0.0196 = 196$$

$$P(r) = \frac{(0.02)^r e^{-0.02}}{r!}, \quad r = 0, 1, 2, \ldots$$

(iii) P (two defective) $= P(r = 2)$

$$= \frac{(0.02)^2 e^{-0.02}}{2!} \cdot = 0.000196$$

$\therefore$ Number of packets containing two defective blades is

$$10000 \times 0.000196 = 1.96 \approx 2$$

Example If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.

Here $n = 2000$

Let p = Probability of bad reaction = 0.001

We have, $\lambda = np$

$$\lambda = (2000)(0.001) = 2$$

We have, $$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}, \ r = 0, 1, 2, \ldots$$

$$\Rightarrow \quad P(r) = \frac{2^r e^{-2}}{r!}, \ r = 0, 1, 2, \ldots$$

$$P(r)=\frac{2^r e^{-2}}{r!},\ r=0,1,2,\ldots$$

P (more than two will get a bad reaction) $=P(r>2)$

$$=1-P(r\leq 2)$$

$$=1-\{P(r=0)+P(r=1)+P(r=2)\}$$

$$=1-\left\{\frac{2^0 e^{-2}}{0!}+\frac{2^1 e^{-2}}{1!}+\frac{2^2 e^{-2}}{2!}\right\}$$

$$=0.32$$

Example Given that 2% of the fuses manufactured by a firm are defective. Find by using Poisson distribution, the probability that a box containing 200 fuses has (i) no defective fuses, (ii) 3 or more defective fuses, (iii) at least one defective fuse.

Here $n = 200$

Let p = Probability of defective = 0.02

We have, $\lambda = np$

$$\lambda = (200)(0.02) = 4$$

We have, $$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}, \quad r = 0, 1, 2, \ldots$$

$$\Rightarrow \quad P(r) = \frac{(4)^r e^{-4}}{r!}, \quad r = 0, 1, 2, \ldots$$

$$P(r)=\frac{(4)^r e^{-4}}{r!},\ r=0,1,2,\ldots$$

(i) P (no defective fuse) $=P(r=0)$

$$=\frac{(4)^0 e^{-4}}{0!} \quad =0.01832$$

(ii) P (3 or more defective fuses) $=P(r\geq 3)$

$$=1-P(r<3)$$

$$=1-\{P(r=0)+P(r=1)+P(r=2)\}$$

$$=1-\left\{\frac{4^0e^{-4}}{0!}+\frac{4^1e^{-4}}{1!}+\frac{4^2e^{-4}}{2!}\right\}$$

$$=0.762$$

$$P(r)=\frac{(4)^r e^{-4}}{r!}, \ r=0, 1, 2, \dots$$

(iii) P (at least one defective fuse) $=P(r\geq 1)$

$$=1-P(r<1)$$

$$=1-P(r=0)$$

$$=1-\frac{4^0 e^{-4}}{0!}$$

$$=0.982$$

Example A certain screw making machine produces on an average two defectives out of 100 and packs them in boxes of 500. Find by using Poisson distribution, the probability that a box containing (i) 3 defectives, (ii) at least one defective, (iii) between two and four defectives.

Here $n = 500$

Let p = Probability of defective = $\frac{2}{100} = 0.02$

We have, $\lambda = np$

$$\lambda = (500)(0.02) = 10$$

We have, $P(r) = \frac{\lambda^r e^{-\lambda}}{r!}, \ r = 0, 1, 2, \ldots$

$\Rightarrow$ $P(r) = \frac{(10)^r e^{-10}}{r!}, \ r = 0, 1, 2, \ldots$

$$P(r) = \frac{(10)^r e^{-10}}{r!}, \quad r = 0, 1, 2, \ldots$$

(i) $P\text{ (3 defectives)} = P(r = 3)$

$$= \frac{(10)^3 e^{-10}}{3!} \quad = 0.00757$$

(ii) $P\text{ (at least one defective)} = P(r \geq 1)$

$$= 1 - P(r < 1) \quad = 1 - P(r = 0)$$

$$= 1 - \frac{10^0 e^{-10}}{0!}$$

$$= 0.9999546 \approx 1$$

$$P(r) = \frac{(10)^r e^{-10}}{r!}, \ r = 0, 1, 2, \ldots$$

(iii) P (between two and four defectives) $= P(2 < r < 4)$

$$= P(r = 3)$$

$$= \frac{(10)^3 e^{-10}}{3!}$$

$$= 0.00757$$

Example **Fit a Poisson distribution for the data**

x	0	1	2	3	4
f	46	38	22	9	1

$$\sum f_i = 46 + 38 + 22 + 9 + 1 = 116$$

$$\sum x_i f_i = 0 + 38 + 44 + 27 + 4 = 113$$

$$\therefore \text{ Mean } \mu = \frac{\sum x_i f_i}{\sum f_i} = \frac{113}{116} = 0.9741$$

We have, Mean $\mu = \lambda$

$$\Rightarrow \qquad \lambda = 0.9741$$

We have, $$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}, \quad r = 0, 1, 2, \ldots$$

$\Rightarrow$ $$P(r) = \frac{(0.9741)^r e^{-0.9741}}{r!}, \quad r = 0, 1, 2, \ldots$$

Now, $$F(x_i) = \left(\sum f_i\right) P(x_i)$$

$$F(0) = (116)P(0) = (116)\left[\frac{(0.9741)^0 e^{-0.9741}}{0!}\right] = 43.8$$

$$F(1) = (116)P(1) = (116)\left[\frac{(0.9741)^1 e^{-0.9741}}{1!}\right] = 42.66$$

$$F(2) = (116)P(2) = (116)\left[\frac{(0.9741)^2 e^{-0.9741}}{2!}\right] = 20.78$$

$$F(3) = (116)P(3) = (116)\left[\frac{(0.9741)^3 e^{-0.9741}}{3!}\right] = 6.75$$

$$F(4) = (116)P(4) = (116)\left[\frac{(0.9741)^4 e^{-0.9741}}{4!}\right] = 1.64$$

$\therefore$ The theoretical frequencies are

x	0	1	2	3	4
f	43.8	42.66	20.78	6.75	1.64

Fit a Poisson distribution for the data

x	0	1	2	3	4
f	180	92	24	3	1

EXERCISE

1. If a random variable has a Poisson distribution such that $P(1)=P(2)$, find (i) mean of the distribution, (ii) $P(4)$.
2. The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is probability that in a group of 7, five or more will suffer from it?
3. X is a Poisson variable and it is found that the probability that $X = 2$ is two-thirds of the probability that $X = 1$. Find the probability that $X = 0$ and the probability that $X = 3$. What is the probability that X exceeds 3?
4. Using Poisson distribution, find the probability that all of the spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trails.
5. A manufacturer knows that the condensers he makes contain on the average 1% defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 3 or more faulty condensers?