Module-5
Part – 1
Joint Probability

Joint Probability Distribution

If X and Y are discrete random variables defined on the samples space S then the joint probability function of X and Y is defined by

$$P(X=x, Y=y)=f(x,y)$$

where $f(x,y)$ satisfy the conditions

(i) $f(x,y) \geq 0$ and

(ii) $\sum_x \sum_y f(x,y)=1$

Suppose $X = \{x_1, x_2, \ldots, x_m\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ then

$$P\left(X = x_i, Y = y_j\right) = f\left(x_i, y_j\right)$$

and is denoted by J_{ij}, i.e., $J_{ij} = P\left(X = x_i, Y = y_j\right) = f\left(x_i, y_j\right)$

The set of values of this function for i = 1, 2, 3, ... , m and j = 1, 2, 3, ... , n is called the joint probability distribution of X and Y.

These values are presented in the form of a table called joint probability table.

Y / X	y_1	y_2	…	y_n	Sum
x_1	J_{11}	J_{12}	…	J_{1n}	$f(x_1)$
x_2	J_{21}	J_{22}	…	J_{2n}	$f(x_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_m	J_{m1}	J_{m2}	…	J_{mn}	$f(x_m)$
Sum	$g(y_1)$	$g(y_2)$	…	$g(y_n)$	1

In the table,

$$f(x_1) = J_{11} + J_{12} + \ldots + J_{1n}; \qquad g(y_1) = J_{11} + J_{21} + \ldots + J_{m1}$$

$$f(x_2) = J_{21} + J_{22} + \ldots + J_{2n}; \qquad g(y_2) = J_{12} + J_{22} + \ldots + J_{32}$$

$$\vdots$$

$$f(x_m) = J_{m1} + J_{m2} + \ldots + J_{mn}; \qquad g(y_n) = J_{1n} + J_{2n} + \ldots + J_{mn}$$

The following tables are called marginal probability distributions of *X* and *Y*

x_i	x_1	x_2	...	x_m
$f(x_i)$	$f(x_1)$	$f(x_2)$	...	$f(x_m)$

y_j	y_1	y_2	...	y_m
$g(y_j)$	$g(y_1)$	$g(y_2)$	...	$g(y_n)$

Note: The total of all the entries in the joint probability table is equal to one.

i.e., $f(x_1)+f(x_2)+\ldots+f(x_m)=1$ and $g(y_1)+g(y_2)+\ldots+g(y_m)=1$

This is equivalent to writing

$$\sum_x \sum_y f(x_i, y_j) = \sum_x \sum_y J_{ij} = 1$$

Independent Random variables

The discrete random variables X and Y are said to be independent random variables if

$$P(X=x, Y=y) = P(X=x).P(Y=y)$$

$$\text{i.e., } P(X=x_i, Y=y_j) = P(X=x_i).P(Y=y_j)$$

This is equivalent to $f(x_i).g(y_j)=J_{ij}$ in the joint probability table.

Note: If $f(x_i).g(y_j) \neq J_{ij}$ then X and Y are dependent.

Expectation, Variance and Covariance

If X and Y are discrete random variables taking values having probability function f(x) and g (y) then

1. The Expectation of X is $E(X) = \sum_{i=1}^{m} x_i f(x_i)$
2. The Expectation of Y is $E(Y) = \sum_{j=1}^{n} y_j g(y_j)$
3. The Expectation of XY is $E(XY) = \sum_{i=1}^{m} \sum_{j=1}^{n} x_i\, y_j\, J_{ij}$

4. The variance of X is $V(X)=E(X^2)-[E(X)]^2$ where $E(X^2)=\sum_{i=1}^{m}x_i^2 f(x_i)$

5. The variance of Y is $V(Y)=E(Y^2)-[E(Y)]^2$ where $E(Y^2)=\sum_{j=1}^{n}y_j^2 g(y_j)$

6. The standard deviation of X is $\sigma_X=\sqrt{V(X)}$

7. The standard deviation of Y is $\sigma_Y=\sqrt{V(Y)}$

8. The covariance of X and Y is $COV(X,Y)=E(XY)-E(X).E(Y)$

9. The correlation of X and Y is $\rho(X,Y)=\dfrac{COV(X,Y)}{\sigma_X\,\sigma_Y}$

Note: If X and Y are independent random variables then

(i) $E(XY)=E(X).E(Y)$

(ii) $COV(X,Y)=0$ and hence $\rho(X,Y)=0$

Example 5.1.1: The joint distribution of two random variables X and Y is as follows:

$X \backslash Y$	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Find marginal probability distributions of X and Y and compute the following:

(i) E (X) and E (Y), (ii) E(XY), (iii) σ_X and σ_Y (iv) COV (X, Y) and (v) $\rho(X,Y)$

Marginal probability distributions of X and Y are got by adding all the respective row entries and the respective column entries.

x_i	1	5
$f(x_i)$	1/2	1/2

y_j	– 4	2	7
$g(y_j)$	3/8	3/8	1/4

(i) $E(X)=\sum_{i=1}^{m} x_i f(x_i)=(1)\left(\frac{1}{2}\right)+(5)\left(\frac{1}{2}\right)=3$

$E(Y)=\sum_{j=1}^{n} y_j g(y_j)=(-4)\left(\frac{3}{8}\right)+(2)\left(\frac{3}{8}\right)+(7)\left(\frac{1}{4}\right)=1$

(ii) $E(XY)=\sum_{i=1}^{m}\sum_{j=1}^{n} x_i\, y_j\, J_{ij}$

$$=(1)(-4)\left(\frac{1}{8}\right)+(1)(2)\left(\frac{1}{4}\right)+(1)(7)\left(\frac{1}{8}\right)+(5)(-4)\left(\frac{1}{4}\right)+(5)(2)\left(\frac{1}{8}\right)+(5)(7)\left(\frac{1}{8}\right)$$

$$=\frac{3}{2}$$

(iii) $E\left(X^2\right)=\sum_{i=1}^{m} x_i^2 f\left(x_i\right)=(1)^2\left(\frac{1}{2}\right)+(5)^2\left(\frac{1}{2}\right)=13$

$$E\left(Y^2\right)=\sum_{j=1}^{n} y_j^2 g\left(y_j\right)=(16)\left(\frac{3}{8}\right)+(4)\left(\frac{3}{8}\right)+(49)\left(\frac{1}{4}\right)=\frac{79}{4}$$

$$V(X)=E\left(X^2\right)-\left[E(X)\right]^2=13-3^2=4$$

$$V(Y)=E\left(Y^2\right)-\left[E(Y)\right]^2=\frac{79}{4}-1^2=\frac{75}{4}$$

$$\sigma_X=\sqrt{V(X)}=\sqrt{4}=2 \text{ and } \sigma_Y=\sqrt{V(Y)}=\sqrt{18.75}=4.33$$

(iv) $COV(X,Y)=E(XY)-E(X).E(Y)=\frac{3}{2}-(3)(1)=-\frac{3}{2}$

(v) $\rho(X,Y)=\frac{COV(X,Y)}{\sigma_X\sigma_Y}=\frac{-(3/2)}{(2)(4.33)}=-0.1732$

Example 5.1.2: The joint distribution of two random variables X and Y is as follows:

$X \backslash Y$	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find marginal probability distributions of X and Y and compute the following:

(i) E (X) and E (Y), (ii) E(XY), (iii) σ_X and σ_Y (iv) COV (X, Y) and (v) $\rho(X,Y)$

Marginal probability distributions of X and Y are got by adding all the respective row entries and the respective column entries.

x_i	1	3
$f(x_i)$	0.5	0.5

y_j	– 3	2	4
$g(y_j)$	0.4	0.3	0.3

(i) $E(X)=\sum_{i=1}^{m} x_i f(x_i)=(1)(0.5)+(3)(0.5)=2$

$$E(Y)=\sum_{j=1}^{n} y_j g(y_j)=(-3)(0.4)+(2)(0.3)+(4)(0.3)=0.6$$

(ii) $E(XY)=\sum_{i=1}^{m}\sum_{j=1}^{n} x_i\, y_j\, J_{ij}$

$$=(1)(-3)(0.1)+(1)(2)(0.2)+(1)(4)(0.2)+(3)(-3)(0.3)+(3)(2)(0.1)+(3)(4)(0.1)$$

$$=0$$

(iii) $E\left(X^2\right)=\sum_{i=1}^{m} x_i^2 f\left(x_i\right)=(1)^2(0.5)+(3)^2(0.5)=5$

$$E\left(Y^2\right)=\sum_{j=1}^{n} y_j^2 g\left(y_j\right)=(-3)^2(0.4)+(2)^2(0.3)+(4)^2(0.3)=9.6$$

$$V(X)=E\left(X^2\right)-\left[E(X)\right]^2=5-2^2=1$$

$$V(Y)=E\left(Y^2\right)-\left[E(Y)\right]^2=9.6-(0.6)^2=9.24$$

$$\sigma_X=\sqrt{V(X)}=\sqrt{1}=1 \text{ and } \sigma_Y=\sqrt{V(Y)}=\sqrt{9.24}=3.04$$

(iv) $COV(X,Y)=E(XY)-E(X).E(Y)=0-1.2=-1.2$

(v) $\rho(X,Y)=\dfrac{COV(X,Y)}{\sigma_X\sigma_Y}=\dfrac{-1.2}{(1)(3.04)}=-0.3947$

Example 5.1.3: A fair coin is tossed thrice. The random variables *X* and *Y* are defined as follows: *X* = 0 or 1 according as head or tail occurs on the first toss. *Y* = Number of heads. Determine (i) the distributions of *X* and *Y*, (ii) the joint distribution of *X* and *Y*, (iii) the expectations of *X*, *Y* and *XY*, (iv) standard deviations of *X* and *Y*, (v) covariance and correlation of *X* and *Y*.

The sample space S and the association of random variables X and Y is

S	**HHH**	**HHT**	**HTH**	**THH**	**HTT**	**THT**	**TTH**	**TTT**
X	0	0	0	1	0	1	1	1
Y	3	2	2	2	1	1	1	0

Here $X = \{0, 1\}$ and $Y = \{0, 1, 2, 3\}$

(i) $P(X=0)=\frac{4}{8}=\frac{1}{2}; \qquad P(X=1)=\frac{4}{8}=\frac{1}{2}$

$$P(Y=0)=\frac{1}{8}; \qquad P(Y=1)=\frac{3}{8}; \quad P(Y=2)=\frac{3}{8}; \quad P(Y=3)=\frac{1}{8}$$

Thus the probability distribution X and Y are

x_i	0	1
$f(x_i)$	1/2	1/2

y_j	0	1	2	3
$g(y_j)$	1/8	3/8	3/8	1/8

(ii) The joint distribution of X and Y is $J_{ij} = P(X = x_i, Y = y_j) = f(x_i, y_j)$

$J_{11} = P(X = 0, Y = 0) = 0;$ $\qquad$ $J_{12} = P(X = 0, Y = 1) = \frac{1}{8};$

$J_{13} = P(X = 0, Y = 2) = \frac{2}{8} = \frac{1}{4};$ $\qquad$ $J_{14} = P(X = 0, Y = 3) = \frac{1}{8};$

$J_{21} = P(X = 1, Y = 0) = \frac{1}{8};$ $\qquad$ $J_{22} = P(X = 1, Y = 1) = \frac{2}{8} = \frac{1}{4};$

$J_{23} = P(X = 1, Y = 2) = \frac{1}{8};$ $\qquad$ $J_{24} = P(X = 1, Y = 3) = 0;$

∴ The joint probability distribution of X and Y is

X \ Y	0	1	2	3	Sum
0	0	1/8	1/4	1/8	1/2
1	1/8	1/4	1/8	0	1/2
Sum	1/8	3/8	3/8	1/8	1

(iii) $E(X)=\sum_{i=1}^{m} x_i f(x_i)=(0)(0.5)+(1)(0.5)=\dfrac{1}{2}$

$$E(Y)=\sum_{j=1}^{n} y_j g(y_j)=(0)\left(\frac{1}{8}\right)+(1)\left(\frac{3}{8}\right)+(2)\left(\frac{3}{8}\right)+(3)\left(\frac{1}{8}\right)=\frac{3}{2}$$

$$E(XY)=\sum_{i=1}^{m}\sum_{j=1}^{n}x_i\,y_j\,J_{ij}$$

$$=(0)(0)(0)+(0)(1)\left(\frac{1}{8}\right)+(0)(2)\left(\frac{1}{4}\right)+(0)(3)\left(\frac{1}{8}\right)+(1)(0)\left(\frac{1}{8}\right)$$

$$+(1)(1)\left(\frac{1}{4}\right)+(1)(2)\left(\frac{1}{8}\right)+(1)(3)\left(\frac{1}{8}\right)$$

$$=\frac{1}{2}$$

(iv) $E\left(X^2\right)=\sum_{i=1}^{m}x_i^2 f\left(x_i\right)=(0)\left(\frac{1}{2}\right)+(1)\left(\frac{1}{2}\right)=\frac{1}{2}$

$$E\left(Y^2\right)=\sum_{j=1}^{n}y_j^2 g\left(y_j\right)=(0)\left(\frac{1}{8}\right)+(1)\left(\frac{3}{8}\right)+(4)\left(\frac{3}{8}\right)+(9)\left(\frac{1}{8}\right)=3$$

$$V(X)=E(X^2)-\left[E(X)\right]^2=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

$$V(Y)=E(Y^2)-\left[E(Y)\right]^2=3-\frac{9}{4}=\frac{3}{4}$$

$$\sigma_X=\sqrt{V(X)}=\frac{1}{2} \text{ and } \sigma_Y=\sqrt{V(Y)}=\sqrt{3/4}=\frac{\sqrt{3}}{2}$$

(v) $$COV(X,Y)=E(XY)-E(X).E(Y)=\frac{1}{2}-\frac{3}{4}=-\frac{1}{4}$$

$$\rho(X,Y)=\frac{COV(X,Y)}{\sigma_X\sigma_Y}=\frac{-(1/4)}{(1/2)\left(\sqrt{3}/2\right)}=-\frac{1}{\sqrt{3}}$$

Example 5.1.4: If the joint probability distribution of X and Y is given by $f(x,y)=c(x^2+y^2)$, for $x=-1, 0, 1, 3;\ y=-1, 2, 3$. Find (i) the value of c, (ii) $P(X=0, Y\le 2)$, (iii) $P(X\le 1, Y>2)$, (iv) $P(X\ge 2-Y)$.

(i) Since $f(x,y)$ is a probability distribution, we have

$$f(x,y)\ge 0 \Rightarrow c\ge 0$$

Also, $\sum_x \sum_y f(x,y)=1$

$$\Rightarrow \begin{aligned} &f(-1,-1)+f(-1,2)+f(-1,3)+f(0,-1)+f(0,2)+f(0,3)+f(1,-1)+f(1,2)\\ &+f(1,3)+f(3,-1)+f(3,2)+f(3,3)=1 \end{aligned}$$

$$\Rightarrow \quad c[2+5+10+1+4+9+2+5+10+10+13+18]=1$$

$$\Rightarrow \quad 89c=1\Rightarrow c=\frac{1}{89}$$

(ii) $$P(X=0, Y\le 2) = f(0,-10) + f(0,2) = \frac{1}{89}[1+4] = \frac{5}{89}$$

(iii) $$P(X\le 1, Y>2) = f(-1,3) + f(0,3) + f(1,3) = \frac{1}{89}[10+9+10] = \frac{29}{89}$$

(iv) $$P(X\ge 2-Y) = f(3,-1) + f(3,2) + f(3,3) + f(1,2) + f(1,3) + f(0,3) + f(0,2) + f(-1,3)$$

$$= \frac{1}{89}[10+13+18+5+10+9+4+10] = \frac{79}{89}$$

Example 5.1.5: Suppose *X* and *Y* are independent random variables with the following distributions:

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	– 2	5	8
$g(y_j)$	0.3	0.5	0.2

Find the joint distribution of *X* and *Y*. Show that *X* and *Y* are independent random variables and also find COV (*X*, *Y*).

$$E(X)=\sum_{i=1}^{m} x_i f(x_i)=(1)(0.7)+(2)(0.3)=1.3$$

$$E(Y)=\sum_{j=1}^{n} y_j g(y_j)=(-2)(0.3)+(5)(0.5)+(8)(0.2)=3.5$$

$$E(X).E(Y)=(1.3)(3.5)=4.55$$

Suppose X and Y are independent random variables then the joint distribution of X and Y is $J_{ij}=f(x_i).g(y_j)$

$J_{11}=(0.7)(0.3)=0.21;$ $J_{12}=(0.7)(0.5)=0.35;$ $J_{13}=(0.7)(0.2)=0.14$

$J_{21}=(0.3)(0.3)=0.09;$ $J_{22}=(0.3)(0.5)=0.15;$ $J_{23}=(0.3)(0.2)=0.06$

$\therefore$ The joint probability distribution of X and Y is

$X \backslash Y$	-2	5	8	Sum
1	0.21	0.35	0.14	0.7
2	0.09	0.15	0.06	0.3
Sum	0.3	0.5	0.2	1

$$E(XY) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_i\, y_j\, J_{ij}$$

$$= (1)(-2)(0.21) + (1)(5)(0.35) + (1)(8)(0.14) + (2)(-2)(0.09)$$

$$+ (2)(5)(0.15) + (2)(8)(0.06) = 4.55$$

$$E(XY) = E(X).E(Y)$$

$\therefore$ X and Y are independent random variables

Hence $COV(X,Y) = 0$

Example 5.1.6: The joint probability distribution of two discrete random variables *X* and *Y* is given by $f(x,y)=k(2x+y)$ for $0 \le x \le 2; 0 \le y \le 3$. Find (i) the value of *k*, (ii) the marginal distribution of X and Y, (iii) show that *X* and *Y* are dependent.

Given $X = \{0, 1, 2\}$ and $Y = \{0, 1, 2, 3\}$

$$f(x,y)=k(2x+y)$$

The joint probability distribution of *X* and *Y* is

$X \backslash Y$	0	1	2	3	Sum
0	0	k	$2k$	$3k$	$6k$
1	$2k$	$3k$	$4k$	$5k$	$14k$
2	$4k$	$5k$	$6k$	$7k$	$22k$
Sum	$6k$	$9k$	$12k$	$15k$	$42k$

(i) We must have $42k = 1$

$\Rightarrow \quad k = 1/42$

(ii) Marginal probability distribution X and Y are

x_i	0	1	2
$f(x_i)$	1/7	1/3	11/21

y_j	0	1	2	3
$g(y_j)$	1/7	3/14	2/7	5/14

(iii) Here, $f(x_i).g(y_j) \neq J_{ij}$

Hence X and Y are dependent.

Example 5.1.8: **X and Y are independent random variables. X take values 2, 5, 7 with probabilities 1/2, 1/4, 1/4 respectively. Y take values 3, 4, 5 with the probabilities 1/3, 1/3, 1/3.**

(i) Find the joint probability distribution of X and Y,
(ii) Show that the covariance of X and Y is equal to zero.
(iii) Find the probability distribution of $Z = X + Y$

Given data is as follows

x_i	2	5	7
$f(x_i)$	1/2	1/4	1/4

y_j	3	4	5
$g(y_j)$	1/3	1/3	1/3

(i) The joint distribution of X and Y is $J_{ij} = P\left(X = x_i, Y = y_j\right) = f\left(x_i, y_j\right)$

$$J_{11} = P\left(X = 2, Y = 3\right) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}; \qquad J_{12} = P\left(X = 2, Y = 4\right) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6};$$

$$J_{13} = P\left(X = 2, Y = 5\right) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}; \qquad J_{21} = P\left(X = 5, Y = 3\right) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12};$$

$$J_{22} = P\left(X = 5, Y = 4\right) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}; \qquad J_{23} = P\left(X = 5, Y = 5\right) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12};$$

$$J_{31} = P\left(X = 7, Y = 3\right) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}; \qquad J_{32} = P\left(X = 7, Y = 4\right) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12};$$

$$J_{32} = P\left(X = 7, Y = 5\right) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

∴ The joint probability distribution of X and Y is

X \ Y	3	4	5	Sum
2	1/6	1/6	1/6	1/2
5	1/12	1/12	1/12	1/4
7	1/12	1/12	1/12	1/4
Sum	1/3	1/3	1/3	1

(ii) $$E(X)=\sum_{i=1}^{m} x_i f(x_i)=(2)\left(\frac{1}{2}\right)+(5)\left(\frac{1}{4}\right)+(7)\left(\frac{1}{4}\right)=4$$

$$E(Y)=\sum_{j=1}^{n} y_j g(y_j)=(3)\left(\frac{1}{3}\right)+(4)\left(\frac{1}{3}\right)+(5)\left(\frac{1}{3}\right)=4$$

$$E(XY)=\sum_{i=1}^{m}\sum_{j=1}^{n} x_i\, y_j\, J_{ij}$$

$$=(2)(3)\left(\frac{1}{6}\right)+(2)(4)\left(\frac{1}{6}\right)+(2)(5)\left(\frac{1}{6}\right)+(5)(3)\left(\frac{1}{12}\right)+(5)(4)\left(\frac{1}{12}\right)$$
$$+(5)(5)\left(\frac{1}{12}\right)+(7)(3)\left(\frac{1}{12}\right)+(7)(4)\left(\frac{1}{12}\right)+(7)(4)\left(\frac{1}{12}\right)=16$$

$$COV(X,Y)=E(XY)-E(X).E(Y)=16-(4)(4)=0$$

(iii) $Z = X + Y$

Let $z_i = x_i + y_i$ and hence $z_i = 5, 6, 7, 8, 9, 10, 11, 12$

The corresponding probabilities are 1/6, 1/6, 1/6, 1/12, 1/12, 1/6, 1/12, 1/12

The probability distribution of $Z = X + Y$ is as follows:

Z	5	6	7	8	9	10	11	12
$P(Z)$	1/6	1/6	1/6	1/12	1/12	1/6	1/12	1/12

Example 5.1.7: If X and Y have the joint probabilities shown in the following table:

X \ Y	0	1	2	3
0	1/12	1/4	1/8	1/120
1	1/6	1/4	1/20	-
2	1/24	1/40	-	-

Find (i) $P(X = 1, Y = 2)$, (ii) $P(X = 0, 1 \le Y < 3)$,

(iii) $P(X + Y < 2)$, (iv) $P(X < Y)$

EXERCISE

1. The joint distribution of two random variables X and Y is as follows:

Y / X	1	3	6
1	1/9	1/6	1/18
3	1/6	1/4	1/12
6	1/18	1/12	1/36

Determine the marginal distribution of X and Y. Also find whether X and Y are independent or not. **(VTU 2006, 2009)**

2. The joint distribution of two random variables X and Y is as follows:

$X \backslash Y$	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find marginal probability distributions of X and Y and compute the following:

(i) E (X) and E (Y), (ii) E(XY), (iii) σ_X and σ_Y (iv) COV (X, Y) and (v) $\rho(X,Y)$

3. The joint distribution of two random variables X and Y is as follows:

$X \backslash Y$	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

Find marginal probability distributions of X and Y and compute the following:

(i) E (X) and E (Y), (ii) E(XY), (iii) σ_X and σ_Y (iv) COV (X, Y) and (v) $\rho(X,Y)$

4. A fair coin is tossed twice. The random variables X and Y are defined as follows: $X = 1$ or 0 according as head or tail occurs on the first toss, $Y = 1$ if both the tosses are head and $Y = 0$ otherwise. Determine the marginal probability distributions of X and Y and the joint distribution of X and Y and also verify that X and Y are independent or not.

5. Two cards are drawn at random from a box which contains five cards numbered 1, 1, 2, 2 and 3. Let X denotes the sum and Y denote the maximum of the two numbers drawn. Find the joint distribution of X and Y. Also compute COV (X, Y) and $\rho(X,Y)$

6. Let X be random variable with the following distribution and Y defined by X^2

$X(=x_i)$	-2	-1	1	2
$f(x_i)$	1/4	1/4	1/4	1/4

Determine; (i) the distribution of X of Y (ii) joint distribution of X and Y (iii) E (XY) **(VTU 2017)**

7. X and Y are independent random variables, X takes the values 1, 2 with probability 0.7, 0.3 and Y take the values –2, 5, 8 with probabilities 0, 3, 0.5, 0.2. Find the joint distribution of X and Y hence find Cov (X, Y).

(VTU 2016)