Module-3
Part – 4
Poisson Distribution

Poisson Distribution

In a binomial distribution, if the following axioms hold:

(i) the number of trails $n \to \infty$,

(ii) the probability of success $p \to 0$ and

(iii) $np = \lambda,$ is a finite number

then the probability distribution reduces to

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}, \; r = 0, 1, 2, \ldots$$

This probability distribution is called Poisson distribution.

Mean and variance of Poisson distribution

The mean and variance of the Poisson distribution are given by

Mean $\mu = \lambda$

Variance $V = \lambda$

Standard deviation $\sigma = \sqrt{V} = \sqrt{\lambda}$

Example Assume that the probability of an individual coalminer being killed in a mine accident during a year is $\frac{1}{2400}$. Use Poisson distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year.

Here $n = 200$

Let p = Probability of killed = $\frac{1}{2400}$

We have, $\lambda = np$

$$\lambda = (200)\left(\frac{1}{2400}\right) = 0.0833$$

We have, $$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}, \quad r = 0, 1, 2, \ldots$$

$$\Rightarrow \quad P(r) = \frac{(0.0833)^r e^{-0.0833}}{r!}, \quad r = 0, 1, 2, \ldots$$

P (at least one fatal accident) $= P(r \geq 1)$

$$= 1 - P(r < 1)$$

$$= 1 - P(r = 0)$$

$$= 1 - \frac{(0.0833)^0 e^{-0.0833}}{0!}$$

$$= 0.08$$

Example In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing (i) no defective, (ii) one defective, (iii) two defective blades respectively in a consignment of 10,000 packets.

Here $n = 10$

Let p = Probability of defective = 0.002

We have, $\lambda = np$

$$\lambda = (10)(0.002) = 0.02$$

We have, $$P(r)=\frac{\lambda^r e^{-\lambda}}{r!},\ r=0,1,2,\ldots$$

$$\Rightarrow \quad P(r)=\frac{(0.02)^r e^{-0.02}}{r!},\ r=0,1,2,\ldots$$

(i) P (no defective) $= P(r=0)$

$$=\frac{(0.02)^0 e^{-0.02}}{0!} = 0.9802$$

$\therefore$ Number of packets containing no defective blade is

$$10000 \times 0.9802 = 9802$$

$$P(r)=\frac{(0.02)^r e^{-0.02}}{r!},\ r=0,1,2,\ldots$$

(ii) P (one defective) $=P(r=1)$

$$=\frac{(0.02)^1 e^{-0.02}}{1!}\ =0.0196$$

$\therefore$ Number of packets containing one defective blade is

$$10000 \times 0.0196 = 196$$

$$P(r) = \frac{(0.02)^r e^{-0.02}}{r!}, \quad r = 0, 1, 2, \ldots$$

(iii) P (two defective) $= P(r = 2)$

$$= \frac{(0.02)^2 e^{-0.02}}{2!} \cdot = 0.000196$$

$\therefore$ Number of packets containing two defective blades is

$$10000 \times 0.000196 = 1.96 \approx 2$$

Example If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.

Here $n = 2000$

Let p = Probability of bad reaction = 0.001

We have, $\lambda = np$

$$\lambda = (2000)(0.001) = 2$$

We have, $$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}, \; r = 0, 1, 2, \ldots$$

$$\Rightarrow \quad P(r) = \frac{2^r e^{-2}}{r!}, \; r = 0, 1, 2, \ldots$$

$$P(r) = \frac{2^r e^{-2}}{r!}, \ r = 0, 1, 2, \ldots$$

P (more than two will get a bad reaction) $= P(r > 2)$

$$= 1 - P(r \le 2)$$

$$= 1 - \{P(r = 0) + P(r = 1) + P(r = 2)\}$$

$$= 1 - \left\{ \frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} \right\}$$

$$= 0.32$$

Example Given that 2% of the fuses manufactured by a firm are defective. Find by using Poisson distribution, the probability that a box containing 200 fuses has (i) no defective fuses, (ii) 3 or more defective fuses, (iii) at least one defective fuse.

Here $n = 200$

Let p = Probability of defective = 0.02

We have, $\lambda = np$

$$\lambda = (200)(0.02) = 4$$

We have, $P(r) = \dfrac{\lambda^r e^{-\lambda}}{r!}, \ r = 0, 1, 2, \ldots$

$\Rightarrow$ $P(r) = \dfrac{(4)^r e^{-4}}{r!}, \ r = 0, 1, 2, \ldots$

$$P(r)=\frac{(4)^r e^{-4}}{r!},\ r=0,1,2,\ldots$$

(i) P (no defective fuse) $=P(r=0)$

$$=\frac{(4)^0 e^{-4}}{0!} \quad =0.01832$$

(ii) P (3 or more defective fuses) $=P(r\geq 3)$

$$=1-P(r<3)$$

$$=1-\{P(r=0)+P(r=1)+P(r=2)\}$$

$$=1-\left\{\frac{4^0 e^{-4}}{0!}+\frac{4^1 e^{-4}}{1!}+\frac{4^2 e^{-4}}{2!}\right\}$$

$$=0.762$$

$$P(r)=\frac{(4)^r e^{-4}}{r!},\ r=0, 1, 2, \ldots$$

(iii) P (at least one defective fuse) $=P(r\geq 1)$

$$=1-P(r<1)$$

$$=1-P(r=0)$$

$$=1-\frac{4^0 e^{-4}}{0!}$$

$$=0.982$$

Example A certain screw making machine produces on an average two defectives out of 100 and packs them in boxes of 500. Find by using Poisson distribution, the probability that a box containing (i) 3 defectives, (ii) at least one defective, (iii) between two and four defectives.

Here $n = 500$

Let p = Probability of defective = $\frac{2}{100} = 0.02$

We have, $\lambda = np$

$$\lambda = (500)(0.02) = 10$$

We have, $P(r) = \frac{\lambda^r e^{-\lambda}}{r!}, \; r = 0, 1, 2, \ldots$

$$\Rightarrow \quad P(r) = \frac{(10)^r e^{-10}}{r!}, \; r = 0, 1, 2, \ldots$$

$$P(r)=\frac{(10)^r e^{-10}}{r!},\ r=0,1,2,\ldots$$

(i) P (3 defectives) $=P(r=3)$

$$=\frac{(10)^3 e^{-10}}{3!} \quad =0.00757$$

(ii) P (at least one defective) $=P(r\geq 1)$

$$=1-P(r<1) \quad =1-P(r=0)$$

$$=1-\frac{10^0 e^{-10}}{0!}$$

$$=0.9999546\approx 1$$

$$P(r) = \frac{(10)^r e^{-10}}{r!}, \; r = 0, 1, 2, \ldots$$

(iii) P (between two and four defectives) $= P(2 < r < 4)$

$$= P(r = 3)$$

$$= \frac{(10)^3 e^{-10}}{3!}$$

$$= 0.00757$$

Example Fit a Poisson distribution for the data

x	0	1	2	3	4
f	46	38	22	9	1

$$\sum f_i = 46 + 38 + 22 + 9 + 1 = 116$$

$$\sum x_i f_i = 0 + 38 + 44 + 27 + 4 = 113$$

$$\therefore \text{ Mean } \mu = \frac{\sum x_i f_i}{\sum f_i} = \frac{113}{116} = 0.9741$$

We have, Mean $\mu = \lambda$

$$\Rightarrow \qquad \lambda = 0.9741$$

We have, $$P(r)=\frac{\lambda^r e^{-\lambda}}{r!},\ r=0,1,2,\ldots$$

$$\Rightarrow \quad P(r)=\frac{(0.9741)^r e^{-0.9741}}{r!},\ r=0,1,2,\ldots$$

Now, $$F(x_i)=\left(\sum f_i\right)P(x_i)$$

$$F(0)=(116)P(0)=(116)\left[\frac{(0.9741)^0 e^{-0.9741}}{0!}\right]=43.8$$

$$F(1)=(116)P(1)=(116)\left[\frac{(0.9741)^1 e^{-0.9741}}{1!}\right]=42.66$$

$$F(2)=(116)P(2)=(116)\left[\frac{(0.9741)^2 e^{-0.9741}}{2!}\right]=20.78$$

$$F(3)=(116)P(3)=(116)\left[\frac{(0.9741)^3 e^{-0.9741}}{3!}\right]=6.75$$

$$F(4)=(116)P(4)=(116)\left[\frac{(0.9741)^4 e^{-0.9741}}{4!}\right]=1.64$$

$\therefore$ The theoretical frequencies are

x	0	1	2	3	4
f	43.8	42.66	20.78	6.75	1.64

Fit a Poisson distribution for the data

x	0	1	2	3	4
f	180	92	24	3	1

EXERCISE

1. If a random variable has a Poisson distribution such that $P(1)=P(2)$, find (i) mean of the distribution, (ii) $P(4)$.
2. The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is probability that in a group of 7, five or more will suffer from it?
3. X is a Poisson variable and it is found that the probability that $X = 2$ is two-thirds of the probability that $X = 1$. Find the probability that $X = 0$ and the probability that $X = 3$. What is the probability that X exceeds 3?
4. Using Poisson distribution, find the probability that all of the spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trails.
5. A manufacturer knows that the condensers he makes contain on the average 1% defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 3 or more faulty condensers?