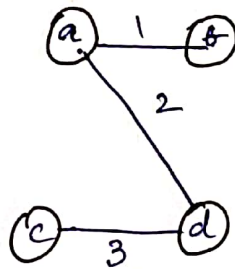
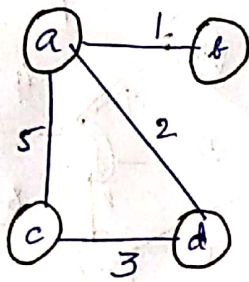


Prim's Algorithm:

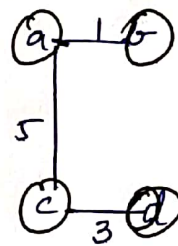
Spanning tree: A spanning tree of a connected graph is its connected acyclic sub-graph that contains all the vertices of the graph.

Minimum spanning tree:

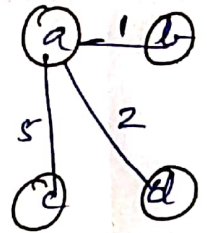
Minimum spanning tree of a weighted connected graph is its spanning tree of the smallest weight, where the weight of a tree is defined as the sum of the weights on all its edges.



$$w(T_1) = 6$$



$$w(T_2) = 9$$



$$w(T_3) = 8$$

Figure: Graph & its spanning trees
 T_1 is the minimum spanning tree.

Prim's Algorithm:

Prim's algorithm constructs a minimum spanning tree through a sequence of expanding sub-trees.

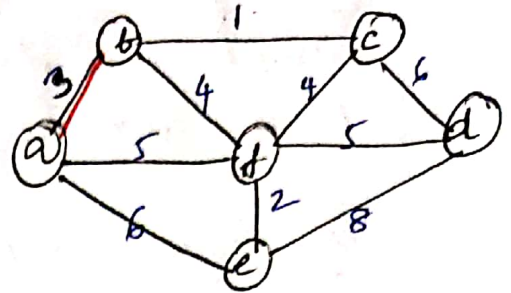
- Initial subtree consists of a single vertex selected arbitrarily from the set V of the graph's vertices.

Tree vertices

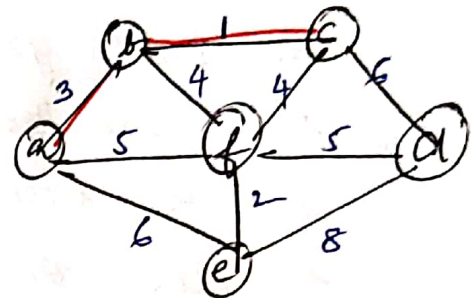
Remaining vertices

Illustration

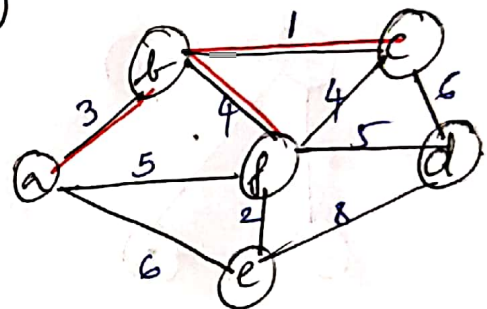
1.

 $a(-, -)$ $b(a, 3)$ $c(-, \infty)$ $d(1, \infty)$ $e(a, 6)$ $f(a, 5)$ 

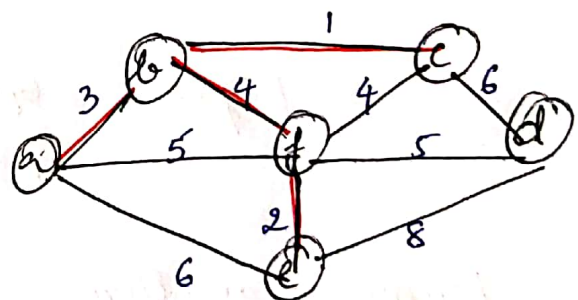
2.

 $b(a, 3)$ $c(b, 1)$ $d(-, \infty)$ $e(a, 6)$ $f(b, 4)$ 

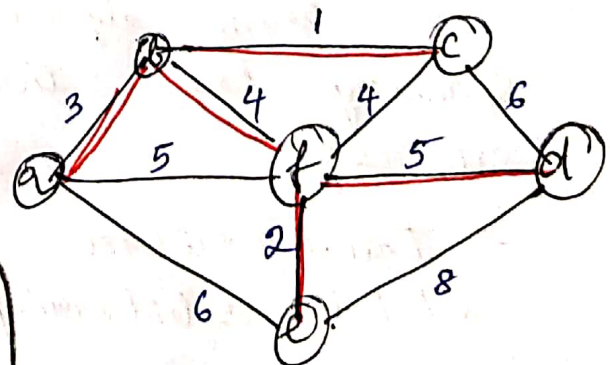
3.

 $c(b, 1)$ $d(c, 6)$ $e(a, 6)$ $f(b, 4)$ 

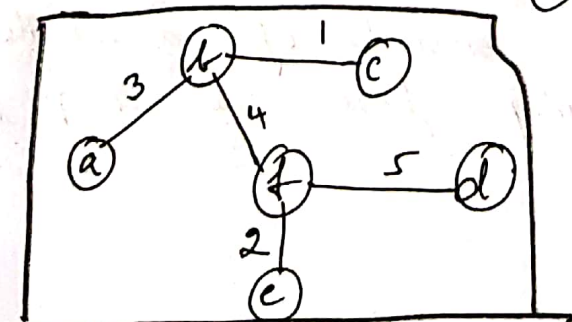
4.

 $f(b, 4)$ $d(f, 5)$ $e(f, 2)$ 

5.

 $e(f, 2)$ $d(f, 5)$ 

6.

 $d(f, 5)$ 

Algorithm $\text{prim}(G)$

// Prim's algorithm for constructing minimum spanning tree

// Input: A weighted connected graph $G = (V, E)$

// Output: E_T , the set of edges composing a minimum spanning tree of G

$V_T \leftarrow \{v_0\}$

$E_T \leftarrow \emptyset$

for $i \leftarrow 1$ to $|V| - 1$ do

 find a minimum weight edge

$e^* = (v^*, u^*)$ among all the edges
 (u, v) such that v is in V_T and
 u is in $V - V_T$

$V_T \leftarrow V_T \cup \{u^*\}$

$E_T \leftarrow E_T \cup \{e^*\}$

return E_T

• Find minimum spanning tree using prim's Algorithm:

