Module-4
Part - I

Curve Fitting

It is the method of finding equation of a curve that approximates a given set of n data points and this equation is called best fitting equation.

Curve Fitting By The Method of Least squares

1. Fitting of a straight line
2. Fitting of a Second degree curve (Parabola)
3. Fitting of Exponential curve

Fitting of a straight line $y = ax + b$

The normal equations of fitting $y = ax + b$ are

$$\Sigma y = a\Sigma x + nb \quad \text{and}$$
$$\Sigma xy = a\Sigma x^2 + b\Sigma x$$

The normal equations for fitting a straight line $y = a + bx$ are

$$\Sigma y = na + b\Sigma x \quad \text{and}$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

Working procedure to fit a straight line $y = ax + b$ or $y = a + bx$

- **Write the normal equations of the given curve.**
- **Prepare the relevant table and find the value of summation present in the normal equation and substitute.**
- **We find the parameters '*a*' *and* '*b*' *by solving and then substitute in the* given equation.**

WORKED EXAMPLES

Fit a straight line $y = ax + b$ for the data

x	5	10	15	20	25
y	16	19	23	26	30

The normal equations of fitting $y = ax + b$ are

$$\Sigma y = a\Sigma x + nb \quad \text{and}$$

$$\Sigma xy = a\Sigma x^2 + b\Sigma x$$

We prepare a relevant table as follows

x	y	xy	x^2
5	16	80	25
10	19	190	100
15	23	345	225
20	26	520	400
25	30	750	625
Σ 75	114	1885	1375

Here, $n = 5$, $\Sigma x = 75$, $\Sigma y = 114$, $\Sigma xy = 1885$ and $\Sigma x^2 = 1375$

Substituting these values in the above normal equations, we get

$$114 = 75a + 5b$$

$$98 = 1375a + 75b$$

$$114 = 75a + 5b$$

$$98 = 1375a + 75b$$

Solving these equations, we get

$$a = 0.7, \quad b = 12.3$$

$\therefore$ The equation of the best fitting straight line is $y = 0.7x + 12.3$

Fit a straight line $y = ax + b$ for the data

x	0	1	2	3	4	5	6
y	–4	–2	0	2	4	6	8

The normal equations of fitting $y = ax + b$ are

$$\Sigma y = a\Sigma x + nb \quad \text{and}$$

$$\Sigma xy = a\Sigma x^2 + b\Sigma x$$

We prepare a relevant table as follows

	x	y	xy	x^2
	0	– 4	0	0
	1	–2	–2	1
	2	0	0	4
	3	2	6	9
	4	4	16	16
	5	6	30	25
	6	8	48	36
Σ	21	14	98	91

Here, $n = 7$, $\Sigma x = 21$, $\Sigma y = 14$, $\Sigma xy = 98$ and $\Sigma x^2 = 91$

Substituting these values in the above normal equations, we get

$$14 = 21a + 7b$$
$$98 = 91a + 21b$$

Solving these equations, we get

$$a = 2, \quad b = -4$$

$\therefore$ The equation of the best fitting straight line is $y = 2x - 4$

Fit a straight line $y = a + bx$ for the data

x	0	1	3	6	8
y	1	3	2	5	4

The normal equations for fitting a straight line $y = a + bx$ are

$$\Sigma y = na + b\Sigma x \quad \text{and}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

We prepare a relevant table as follows

	x	y	xy	x^2
	0	1	0	0
	1	3	3	1
	3	2	6	9
	6	5	30	36
	8	4	32	64
Σ	18	15	71	110

Here, $n = 5$, $\Sigma x = 18$, $\Sigma y = 15$, $\Sigma xy = 71$ and $\Sigma x^2 = 110$

Substituting these values in the above normal equations, we get

$$15 = 5a + 18b$$

$$71 = 18a + 110b$$

$$15 = 5a + 18b$$
$$71 = 18a + 110b$$

Solving these equations, we get

$$a = 1.6, \quad b = 0.38$$

$\therefore$ The equation of the best fitting straight line is $y = 1.6 + 0.38x$

If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form P = mW + c connecting P and W, using the following data:

P	12	15	21	25
W	50	70	100	120

Also find P when W = 150 kgs

The normal equations of fitting $P = mW + c$ are

$$\Sigma P = m\Sigma W + nc \quad \text{and}$$

$$\Sigma WP = m\Sigma W^2 + c\Sigma x$$

We prepare a relevant table as follows

	W	P	WP	W^2
	50	12	600	2500
	70	15	1050	4900
	100	21	2100	10000
	120	25	3000	14400
Σ	340	73	6750	31800

Here, $n = 4$, $\Sigma W = 340$, $\Sigma P = 73$, $\Sigma WP = 6750$ and $\Sigma W^2 = 31800$

Substituting these values in the above normal equations, we get

$$73 = 340m + 4c$$
$$6750 = 31800m + 340c$$

Solving these equations, we get

$$m = 0.1879, \quad c = 2.2759$$

$\therefore$ The best fit of a line is $P = 0.1879W + 2.2759$

When W = 150 kg

$\Rightarrow P = 0.1879(150) + 2.2759 = 30.4635$ kg

EXERCISE

1. Fit a straight line to the following data

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	5

2. Fit a straight line to the following data

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(VTU 2011)

3. Fit a straight line to the following data

Year x	1961	1971	1981	1991	2001
Production (in tons) y	8	10	12	10	16

and find the expected production in 2006.

4. A simply supported beam carries a concentrated load P at its mid-point. Corresponding to various values of P, the maximum deflection Y is measured. The data are given below:

P	100	120	140	160	180	200
y	0.45	0.55	0.60	0.70	0.80	0.85

Find a law of the form $Y = a + bP$

5. The results of measurement of electric resistance R of a copper bar at various temperatures $t\,^0C$ are listed below:

t	19	25	30	36	40	45	50
R	76	77	79	80	82	83	85

6. Fit a straight line to the following data

x	0	0.2	0.5	0.7	0.9	1.1	1.3
y	1.5	1.08	0.45	0.03	–0.39	–0.81	–1.23