Module-4
Part - 2

Fitting of a Second degree curve (Parabola) $y = ax^2 + bx + c$

The normal equations of fitting $y = ax^2 + bx + c$ are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x \quad \text{and}$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

The normal equations for fitting a straight line $y = a + bx + cx^2$ are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{and}$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

Working procedure to fit a second degree curve $y = ax^2 + bx + c$ or $y = a + bx + cx^2$

- **Write the normal equations of the given curve.**
- **Prepare the relevant table and find the value of summation present in the normal equation and substitute.**
- **We find the parameters *'a', 'b' and 'c' by solving and then substitute in the* given equation.**

WORKED EXAMPLES

Fit a second degree curve $y = ax^2 + bx + c$ for the data

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

The normal equations of fitting $y = ax^2 + bx + c$ are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x \quad \text{and}$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

We prepare a relevant table as follows

x	y	xy	x^2	x^3	x^4	x^2y
1.0	1.1	1.1	1	1	1	1.1
1.5	1.3	1.95	2.25	3.375	5.0625	2.925
2.0	1.6	3.2	4.0	8.0	16.0	6.4
2.5	2.0	5.0	6.25	15.625	39.0625	12.5
3.0	2.7	8.1	9.0	27.0	81.0	24.3
3.5	3.4	11.9	12.25	42.875	150.0625	41.65
4.0	4.1	16.4	16	64	256	65.6
Σ 17.5	16.2	47.65	50.75	161.875	548.1875	154.475

Here, $n = 7$, $\Sigma x = 17.5$, $\Sigma y = 16.2$, $\Sigma xy = 47.65$, $\Sigma x^2 = 50.75$, $\Sigma x^3 = 161.875$, $\Sigma x^4 = 548.1875$ and $\Sigma x^2y = 154.475$

Substituting these values in the above normal equations, we get

$$16.2 = 50.75a + 17.5b + 7c$$

$$47.65 = 161.875a + 50.75b + 17.5c$$

$$154.475 = 548.1875a + 161.875b + 50.75c$$

$$16.2 = 50.75a + 17.5b + 7c$$
$$47.65 = 161.875a + 50.75b + 17.5c$$
$$154.475 = 548.1875a + 161.875b + 50.75c$$

Solving these equations, we get

$a = 0.243$, $b = -0.193$ and $c = 1.04$

$\therefore$ The equation of the best fitting curve is $y = 0.243x^2 - 0.193x + 1.04$

Fit a second degree curve $y = a + bx + cx^2$ for the data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

The normal equations for fitting a straight line $y = a + bx + cx^2$ are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{and}$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

We prepare a relevant table as follows

	x	y	xy	x^2	x^3	x^4	x^2y
	0	1	0	0	0	0	0
	1	1.8	1.8	1	1	1	1.8
	2	1.3	2.6	4	8	16	5.2
	3	2.5	7.5	9	27	81	22.5
	4	6.3	25.2	16	64	256	100.8
Σ	10	12.9	37.1	30	100	354	130.3

Here, $n = 5$, $\Sigma x = 10$, $\Sigma y = 12.9$, $\Sigma xy = 37.1$, $\Sigma x^2 = 30$, $\Sigma x^3 = 100$, $\Sigma x^4 = 354$ and $\Sigma x^2 y = 130.3$

Substituting these values in the above normal equations, we get

$$12.9 = 4a + 10b + 30c$$
$$37.1 = 10a + 30b + 100c$$
$$130.3 = 30a + 100b + 354c$$

Solving these equations, we get

$$a = 1.42,\quad b = -1.07 \text{ and } c = 0.55$$

$\therefore$ The equation of the best fitting curve is $y = 1.42 - 1.07x + 0.55\,x^2$

Fit a second degree curve $y = a + bx + cx^2$ for the data

x	-3	-2	-1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

The normal equations for fitting a straight line $y = a + bx + cx^2$ are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{and}$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

We prepare a relevant table as follows

	x	y	xy	x^2	x^3	x^4	x^2y
	–3	4.63	–13.89	9	–27	81	41.67
	–2	2.11	– 4.22	4	–8	16	8.44
	–1	0.67	– 0.67	1	–1	1	0.67
	0	0.09	0	0	0	0	0
	1	0.63	0.63	1	1	1	0.63
	2	2.15	4.3	4	8	16	8.6
	3	4.58	13.74	9	27	81	41.22
Σ	0	14.86	– 0.11	28	0	196	101.23

Here, $n = 7$, $\Sigma x = 0$, $\Sigma y = 14.86$, $\Sigma xy = -0.11$, $\Sigma x^2 = 28$, $\Sigma x^3 = 0$, $\Sigma x^4 = 196$ and $\Sigma x^2y = 101.23$

Substituting these values in the above normal equations, we get

$$14.86 = 7a + 0b + 28c$$

$$-0.11 = 0a + 28b + 0c$$

$$101.23 = 28a + 0b + 196c$$

$$14.86 = 7a + 0b + 28c$$
$$-0.11 = 0a + 28b + 0c$$
$$101.23 = 28a + 0b + 196c$$

Solving these equations, we get

$a = 0.1329$, $b = -0.00393$ and $c = 0.4975$

$\therefore$ The equation of the best fitting curve is

$$y = 0.1329 - 0.00393x + 0.4975x^2$$

Example Find the best values of a, b and c if the equation $y = a + bx + cx^2$ is to fit most closely to the following observations:

x	-2	-1	0	1	2
y	-3.150	-1.390	0.620	2.880	5.378

The normal equations for fitting a straight line $y = a + bx + cx^2$ are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{and}$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

We prepare a relevant table as follows

	x	y	xy	x^2	x^3	x^4	x^2y
	-2	-3.150	6.3	4	-8	16	-12.6
	-1	-1.390	1.39	1	-1	1	-1.39
	0	0.620	0	0	0	0	0
	1	2.880	2.880	1	1	1	2.880
	2	5.378	10.756	4	8	16	21.512
Σ	0	4.328	21.326	10	0	34	10.402

Here, $n = 5$, $\Sigma x = 0$, $\Sigma y = 4.328$, $\Sigma xy = 21.326$, $\Sigma x^2 = 10$, $\Sigma x^3 = 0$, $\Sigma x^4 = 34$ and $\Sigma x^2 y = 10.402$

Substituting these values in the above normal equations, we get

$$4.338 = 5a + 0b + 10c$$

$$21.326 = 0a + 10b + 0c$$

$$10.402 = 10a + 0b + 34c$$

$$4.338 = 5a + 0b + 10c$$
$$21.326 = 0a + 10b + 0c$$
$$10.402 = 10a + 0b + 34c$$

Solving these equations, we get

$$a = 0.621,\ \ b = 2.1326 \text{ and } c = 0.1233$$

$\therefore$ The best values of a, b and c for fitting of $\boldsymbol{y = a + bx + cx^2}$ are

$$a = 0.621,\ \ b = 2.1326 \text{ and } c = 0.1233$$

Fit a second degree curve $y = ax^2 + bx + c$ for the data

x	1	2	3	4	5
y	10	12	13	16	19

The normal equations for fitting a straight line $y = a + bx + cx^2$ are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{and}$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

We prepare a relevant table as follows

	x	y	xy	x^2	x^3	x^4	x^2y
	1	10	10	1	1	1	10
	2	12	24	4	8	16	48
	3	13	39	9	27	81	117
	4	16	64	16	64	256	256
	5	19	95	25	125	625	475
Σ	15	61	232	55	225	979	906

Here, $n = 5$, $\Sigma x = 15$, $\Sigma y = 61$, $\Sigma xy = 232$, $\Sigma x^2 = 55$, $\Sigma x^3 = 225$, $\Sigma x^4 = 979$ and

$\Sigma x^2 y = 906$

Substituting these values in the above normal equations, we get

$$61 = 55a + 15b + 5c$$
$$232 = 225a + 55b + 15c$$
$$906 = 979a + 225b + 55c$$

$$61 = 55a + 15b + 5c$$
$$232 = 225a + 55b + 15c$$
$$906 = 979a + 225b + 55c$$

Solving these equations, we get

$$a = 0.29, \quad b = 0.46 \text{ and } c = 9.43$$

$\therefore$ The equation of the best fitting curve is $y = 0.29x^2 + 0.46x + 9.43$

Fit a second degree Parabola for the following data

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

The normal equations of fitting $y = ax^2 + bx + c$ are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x \quad \text{and}$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

We prepare a relevant table as follows

	x	y	xy	x^2	x^3	x^4	x^2y
	1	2	2	1	1	1	2
	2	6	12	4	8	16	24
	3	7	21	9	27	81	63
	4	8	32	16	64	256	128
	5	10	50	25	125	625	250
	6	11	66	36	216	1296	396
	7	11	77	49	343	2401	539
	8	10	80	64	512	4096	640
	9	9	81	81	729	6561	729
Σ	45	74	421	285	2025	15333	2771

Here, $n = 9$, $\Sigma x = 45$, $\Sigma y = 74$, $\Sigma xy = 421$, $\Sigma x^2 = 285$, $\Sigma x^3 = 2025$, $\Sigma x^4 = 15333$ and $\Sigma x^2 y = 2771$

Substituting these values in the above normal equations, we get

$$74 = 285a + 45b + 9c$$

$$421 = 2025a + 285b + 45c$$

$$2771 = 15333a + 2025b + 285c$$

Solving these equations, we get

$$a = -0.2673, \quad b = 3.523 \text{ and } c = -0.9282$$

$\therefore$ The equation of the best fitting curve is

$$y = -0.2673x^2 + 3.523x - 0.9282$$

EXERCISE

1. Fit a second degree Parabola to the following data:

x	1	2	3	4	5	6	7	8	9	10
y	124	129	140	159	228	289	315	302	263	210

2. Fit a second degree Parabola to the following data:

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

(VTU 2011)

3. Find the best values of a, b, c if the equation $y = a + bx + cx^2$ is to fit most closely to the following observation.

x	1	2	3	4	5
y	10	12	13	16	19

(VTU 2013)

4. The velocity V of a liquid is known to vary with temperature according to a quadratic law $V = a + bT + cT^2$. Find the best values of a, b and c for the data:

T	1	2	3	4	5	6	7
V	2.31	2.01	3.80	1.66	1.55	1.47	1.41

5. The following table gives the results of the measurements of train resistances, V is the velocity in miles per hour, R is the resistance in pounds per ton:

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

If R is related to V by the relation $R = a + bV + cV^2$, find a, b and c.

6. Fit a second degree Parabola $y = ax^2 + bx + c$ to the following data:

x	10	20	30	40	50	60
y	157	179	210	252	302	361