

### Module 3

## CONTEXT FREE GRAMMER

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Context Free Grammar (CFG):

$$G = (V, T, P, S)$$

The context Free Grammar is defined by quadruple or 4-tuple where  $G = (V, T, P, S)$  where

V - set of variables or non-terminals

T - set of terminals

P - set of productions or rules and

S - start variable.

All productions are of the form  $A \rightarrow \alpha$  where  $\alpha \in (V \cup T)^*$  & A is non-terminal

The languages of this CFG are called context free languages (CFL) which are accepted by push down Automata (PDA)

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1. Write CFG to generate string consisting of any no of a's

$$\Sigma = \{a\}$$

String is  $a^*$

$$G = (V, T, P, S)$$

$$V = \{S\} \quad P: S \rightarrow aS$$

$$T = \{a\} \quad S \rightarrow \epsilon$$

S = Start Variable

$w = aaaa$

$S \rightarrow aS$

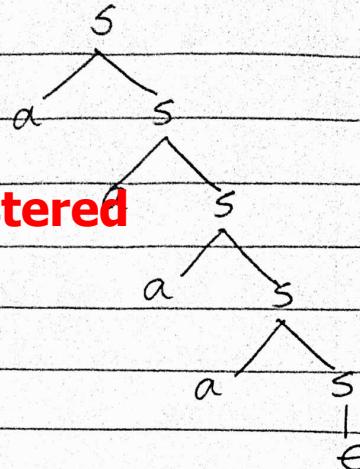
$\rightarrow aaaS$

$\rightarrow aaas$

$\rightarrow aaaas$

$\rightarrow aaaa$

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2. Strings with a's & b's  $\Sigma = \{a, b\}$

$L = \{ab, aa, \dots\}$

$(a+b)^*$

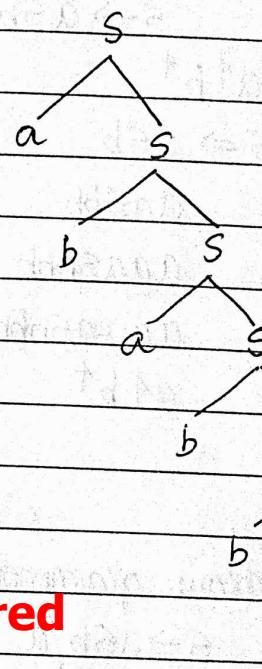
$G = (V, T, P, S)$

$V = \{S\}$

$T = \{a, b\}$

$S = \text{start variable}$

$S \rightarrow aS / bS / \epsilon$



$w = ababb$

$S \rightarrow aS$

$\rightarrow abS$

$\rightarrow abas$

$\rightarrow ababs$

$\rightarrow ababbs$

$\rightarrow ababb$

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3. Obtain CFG to generate even number of a's  $\Sigma = \{a\}$

$L = \{aa, aaaa, \dots\}$

$(aa)^*$

$S \rightarrow aas / \epsilon$

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4 Obtain CFG for the language  $L = \{a^n b^n \mid n \geq 0\}$

$$L = \{ab, aabb, \dots\}$$

$$G = (V, T, P, S)$$

$$T = \{a, b\}$$

$$S \rightarrow aSb / \epsilon$$

$$a^4 b^4$$

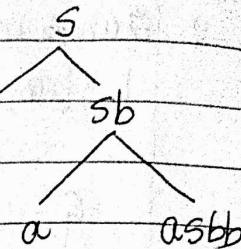
$$S \Rightarrow aSb$$

$$aasbb$$

$$aaasbbb$$

$$aaaasbbbb$$

$$a^4 b^4$$



5 Obtain grammar for  $L = \{a^{n+1} b^n \mid n \geq 0\}$

$$S \rightarrow aSb/a$$

$$a^3 b^2$$

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$$S \rightarrow aSb$$

$$aasbb$$

$$aaabb$$

$$L = \{a^n b^{n+1} \mid n \geq 0\}$$

$$S \rightarrow aSb/b$$

6. Obtain grammar  $L = \{a^n b^{2n} : n \geq 0\}$

$$S \rightarrow aSbb/E$$

$$a^2 b^4$$

$$S \rightarrow aSbb$$

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$$\rightarrow aaSbbbb$$

$$\rightarrow aabbba$$

4/18/21

7. Write grammar to generate set of palindromes over  $\Sigma = \{a, b\}$

$$L = \{aa, bb, abba, bba, abbbba, \dots\}$$

$$G = (V, T, P, S) \quad T = \{a, b\}$$

$$S \rightarrow aSa / bSb / a / b / E$$

$$L = abba$$

$$S \rightarrow aba$$

$$S \rightarrow aSa$$

$$absba \quad (S \rightarrow bSb)$$

$$abba \quad (S \rightarrow E)$$

8. Obtain CFG for the language given below

$$L = \{ww^R \mid w \in \{a, b\}^*\}$$

$$L = \{abba, bba, ab, abbbba, \dots\}$$

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All strings of  $ww^R$  are palindrome but all palindrome are not  $ww^R$

$$S \rightarrow aSa / bSb / E$$

$$L = abba$$

$$S \rightarrow aSa$$

$$\rightarrow absba \quad (S \rightarrow bSb)$$

$$\rightarrow abbsbba \quad (S \rightarrow bsb)$$

$$\rightarrow abbbba \quad (S \rightarrow E)$$

9. obtain a grammar to generate the following language.

$$L = \{ 0^m 1^n 2^n \mid m \geq 1 \text{ and } n \geq 0 \}$$

$$L = \{ 01, 00112, \dots \}$$

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$$S \rightarrow 0S0 \mid 1S1 \mid 2S2 \mid \epsilon$$

$$00112$$

$$S \rightarrow 0S0$$

$$00S$$

$$001S$$

$$0011S$$

$$00112S$$

$$S \rightarrow A^m B^n \mid \epsilon$$

$$A \rightarrow 0A1 \mid 01$$

$$B \rightarrow 2B \mid \epsilon$$

$$S \rightarrow AB$$

$$\rightarrow 0A1B \quad (BA \rightarrow 0A1)$$

$$\rightarrow 0011B \quad (A \rightarrow 01)$$

$$\rightarrow 00112B \quad (B \rightarrow 2B)$$

$$\rightarrow 00112 \quad (B \rightarrow \epsilon)$$

10. obtain CFG for the language

$$L = \{ w \mid n_a(w) = n_b(w) \} \quad \Sigma = \{ a, b \}$$

$$L = \{ aa, bb, ab, ba, aabb, \dots \}$$

$$S \rightarrow aSb \mid bSa \mid \epsilon$$

$$S \rightarrow aSb \mid bSa \mid \epsilon$$

$$aab$$

$$S \rightarrow aSb$$

$$\rightarrow aaSbb$$

$$\rightarrow aabb$$

$$abba$$

$$S \rightarrow aSb$$

$$\rightarrow$$

$$S \rightarrow aS$$

$$abS$$

$$abbs$$

$$abbaS$$

$$abbaE$$

$$aS \rightarrow aS$$

$$aas$$

$$aabs$$

$$aabS$$

$$aabbe$$

$$S \rightarrow SS$$

$$S \rightarrow aSbS$$

$$\rightarrow abSabs$$

$$\rightarrow ab$$

$$\rightarrow abs$$

$$\rightarrow abbS$$

$$\rightarrow abber$$

$$aababb$$

$$S \rightarrow aSb$$

$$\rightarrow aaSbb$$

$$\rightarrow abSabb$$

$$\rightarrow ababb$$

11. Obtain CFG to generate string of balanced parentheses.
- $$T = \{(), \{(), \}, [(), ]\}$$
- $$L = \{(), (\{()\}), ((\{()\})), [\{()\}], [(\{()\})], \dots \}$$

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$$S \rightarrow (S) / \{S\} / [S] / \epsilon / SS$$

$$S \rightarrow (S)$$

$$\rightarrow ((S))$$

$$\rightarrow ((\{S\}))$$

$$\rightarrow (([\{S\}]))$$

~~$$S \rightarrow SS$$~~

~~$$\rightarrow (S)S$$~~

~~$$\rightarrow ((S))S$$~~

~~$$\rightarrow ((\{S\}))S$$~~

$$S \rightarrow SS$$

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$$S(S) \rightarrow (SS)$$

$$S() \rightarrow (S)S$$

$$\rightarrow ((S))S$$

$$\rightarrow ((())\{S\})$$

$$\rightarrow ((())\{[S]\})$$

$$\rightarrow ((())\{[J]\})$$

Prove  $W = ((())())$

$$S \rightarrow (SS) / \epsilon$$

$$\rightarrow ((SS))$$

$$\rightarrow ((())S)$$

OS1 | D | D

$$\rightarrow ((())\{S\})$$

$$\rightarrow ((())())$$

$$S \rightarrow OS1$$

$$\rightarrow OOS1$$

$$\rightarrow OOOS1$$

$$\rightarrow OOOOS1$$

12. Chain grammar to generate the language

$$L = \{0^i 1^j \mid i \neq j, i \geq 0, j \geq 0\}$$

$$L = \{011, 001, 00111, \dots \}$$

$$S \rightarrow OS1 | 1 | 0$$

$$S \rightarrow OS1$$

$$\rightarrow 011$$

$$S \rightarrow OS1$$

$$\rightarrow 00S11$$

$$\rightarrow 00111$$

$S \rightarrow OS1$

$A \rightarrow A/B$

$A \rightarrow OA/O$

$B \rightarrow IB/I$

00011

$S \rightarrow OS1$

00511

00A11

01111

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6/12/21

13. Obtain a grammar to generate a language  $L = \{a^n b^{n-3} \mid n \geq 3\}$

$$L = \{a^3, a^4 b, a^5 b^2, \dots\}$$

$a^4 b$

$S \rightarrow aaas | asb |$

$S \rightarrow aaaA$

$A \rightarrow a^4 b / \epsilon$

$a^5 b^2$

$S \rightarrow aaaA$

$aaaasb$

$aaaaasbb$

$aaaaabb$

$S \rightarrow aaas$   
 $aaasb$

$aaaab$

$a^5 b^2$

$S \rightarrow aaas$   
 $aaaasb$   
 $aaaaasbb$   
 $a^5 b^2 \epsilon$

14. Obtain a grammar to generate a language  $L = \{a^n b^m \mid n \geq 0, m > n\}$

$$L = \{b, ab^2, a^2b^3, a^3b^4, \dots\}$$

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$S \rightarrow asb / bs / b$

$S \rightarrow asb$

$aasbb$

$a^2b^3$

$S \rightarrow AB$

$A \rightarrow aAb / \epsilon$

$B \rightarrow bB / b$

$s \rightarrow asb / bs / b$

$\rightarrow asb$

$absb$

$abb$

$\rightarrow asb$

$aaasbb$

$aaasbbb$

$aaabbhhh$

$asb$

$ab$

$abb$

$aashb$

$aashhh$

$S \rightarrow aSb/E$

$aab$   
 $S \rightarrow AOb$   
 $aasbb$   
 $aaasbb$   
 $aaabb$

M	T	W	T	F	S	S
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15 Obtain a grammar to generate no of a's greater than no of b's

$$L = \{ w \mid n_a(w) > n_b(w) \}$$

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$$L = \{ a^2b, ba, b^2a, ab^2, ab, ba, aab, bab \}$$

$$\approx L = \{ ab, ab^2, \dots \}$$

$S \rightarrow aA$

$A \rightarrow aAb / E / bAa$

$S \rightarrow aA$   
 $S \rightarrow aab abbaa$   
 $S \rightarrow$

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16 Obtain a grammar to generate integers

$$L = \{ 123, -123, +2, -1, 0, \dots \}$$

Show the derivation of unsigned integer

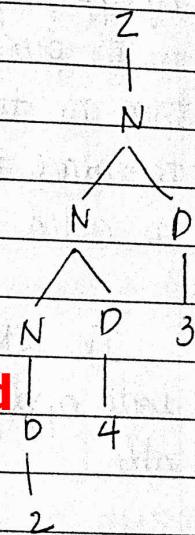
$$D \rightarrow 0/1/2/3/4/5/6/7/8/9$$

$$I \rightarrow N/SN$$

$$S \rightarrow +/-/E$$

$$N \rightarrow ND/D$$

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17 What is the language generated by a grammar given below

$$S \rightarrow OA/E \quad W = \{0, 1\}$$

$$A \rightarrow 1S$$

$$L = \{ 01, 0101, 010101, \dots \}$$

$$L = \{ (01)^n \mid n \geq 0 \}$$

$$S \rightarrow OA$$

010101S

$$01S$$

010101

$$010A$$

$\Rightarrow (0 \cdot 1)^*$

$$0101S$$

$$01010A$$

18 Write the language generated by the grammar  
 $S \rightarrow aSa \mid bSb \mid e$

$L = \{www\}^*$  **PDFZilla - Unregistered**

Derivation Tree (Parse Tree):

Let  $G = (V, T, P, S)$  be a CFG, then we have two types of derivation:

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→ left most derivation (LMD)

→ Right most derivation (RMD)

Derivation is a process of deriving a string  $w$  by applying rules or productions start with start variable  $S$ , in the process of derivation if we apply on RHS side if there are two or more non terminal variables than we try to replace the leftmost variable by its production, then we called derivation as left most derivation.

In RMD, in the process of derivation at each step we will apply rules or productions on the right most variable.

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2/12/21

1. Consider the grammar  $E \rightarrow +EE \mid *EE \mid -EE \mid x \mid y$

Find LMD and RMD  $w = + * - xy my$  and write parse tree

$T = \{+, -, *, x, y\}$

$V = \{E\}$

$E \rightarrow +EE$

$+ * EEE$

$+ * - EEEE$

Page No. 1  
Date 10/10/2019  
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$\rightarrow +\star-xyxNy$

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RMD:

$E \rightarrow +EE$

$+E\star EEE$

$+EE$

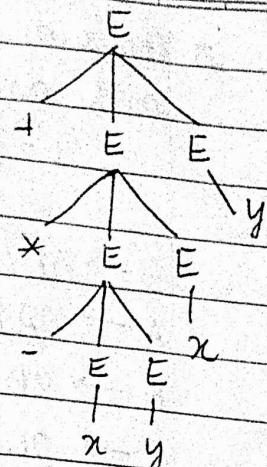
$+Ey$

$+\star EEy$

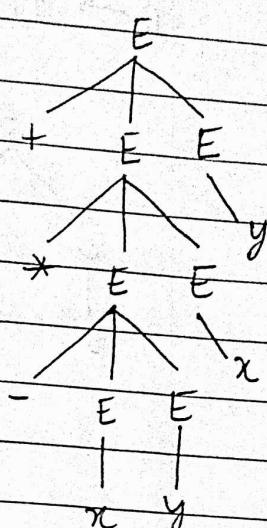
$+\star EExy$

$+\star -EEExy$

$+\star -xyxNy$



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Q. Construct CFG for the language given below

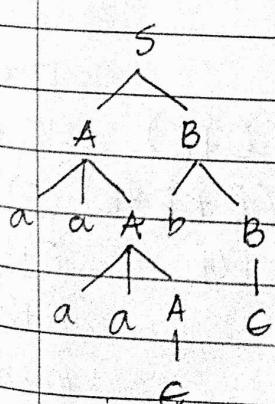
a.  $L = \{a^{2n} b^m \mid n \geq 0, m \geq 0\}$  w = aaaab

Generate LMP **PDFZilla - Unregistered**

$L = \{a^3b, aabb, aaaab, aaabb, \dots\}$

<u>LMD</u>	<u>RMD</u>
$S \rightarrow AB$	$S \rightarrow AB$
$A \rightarrow aaA/G$	$aaAB$
$B \rightarrow bB/G$	$aaaaAB$

$S$	$S \rightarrow AB$	$S \rightarrow AB$
$A$	$aaAB$	$A bb$
$B$	$aaaaAB$	$Ab$
	$aaaabB$	$aaAb$
	$aaaab$	$aaaab$



3 Construct CFG for the language  $L = \{0^i 1^j 2^k \mid i=j \text{ or } j=k\}$   
and generate LMD for the string 01122

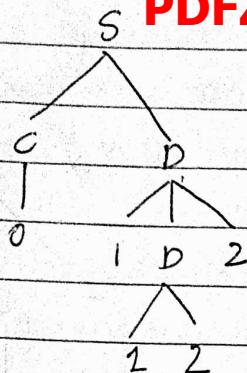
$$L = \{0122, 01122, 00112, 0111222, \dots\}$$

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$$\begin{aligned} S &\rightarrow AB/CD \\ A &\rightarrow 0A1/01 \\ B &\rightarrow 2B/02 \\ C &\rightarrow 0C/0 \\ D &\rightarrow 1D2/02 \end{aligned}$$

$$\begin{aligned} S &\rightarrow AB/CD \\ 0A1B &\quad 0CD \\ 011B &\quad 01D2 \\ 0112B &\quad 011D22 \\ 01122 &\quad 01122 \end{aligned}$$

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4 Consider the grammar  $E \rightarrow E+E/E-E$

$$E \rightarrow E * E / E / E$$

$$E \rightarrow (E)$$

$$E \rightarrow a/b/c$$

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LMD for  $(a+b)*c$

RMD for  $(a+b)*c$

$$T = \{+, -, *, /, (, ), a, b, c\}$$

$$V = \{E\}$$

$$E \rightarrow E * E$$

LMD

$$E \rightarrow (E)$$

$$(E+E)$$

$$(a+E)$$

$$(a+E * E)$$

$$(a+b*c)$$

RMD

$$E \rightarrow (E)$$

$$(EAE) (E+TE)$$

$$(a+FE) (E+TE*E) (E)*C$$

$$(a+ E+A)$$

$$(E+E)*C$$

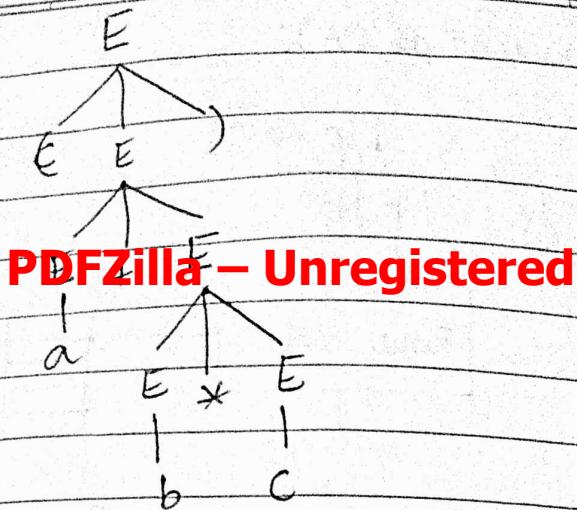
$$E * E$$

$$E * C$$

$$(E)*C$$

$$(E+b)*C$$

$$(a+b)*C$$



### Ambiguous Grammar **PDFZilla - Unregistered**

Let  $(G = (V, T, P, S))$  be a CFG. A grammar  $G$  is ambiguous if there exist one string  $w \in T^*$  for which two or more different parse trees exists by applying either the LRD or the RMD.

Ex:

$a+b*c$

$E \rightarrow E+E$

$a+E*E$

$a+b*E$

$a+b*c$

$E \rightarrow E * E$

$E+E*E$

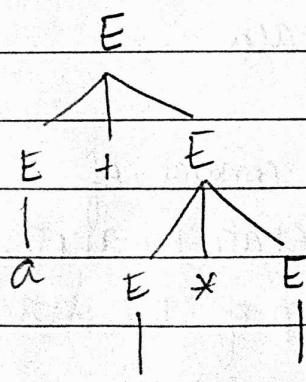
$a+E*E$

$a+b*E$

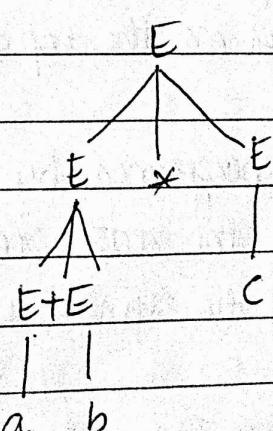
$a+b*c$

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$$\begin{aligned} a &= 2 \\ b &= 3 \\ c &= 4 \end{aligned}$$



14



20

Techniques to reduce Ambiguity:

No general purpose algorithm exist to check for ambiguity in a grammar or to remove when it is found. We often reduce ambiguity by eliminating.

1. E rules like  $S \rightarrow S + S$ ,  $E \rightarrow E * E$

2. Rules with symbolic right-hand sides  $S \rightarrow SS$ ,  $E \rightarrow E + E$

3. Rules that lead to ambiguous attachment of optional postfix

1. Show that the following grammar is ambiguous & find unambiguous grammar  $E \rightarrow E + E / E - E / E * E / E / E / (E) / id$   $w = id + id * id$

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$$E \rightarrow E + E$$

$$id + E * E$$

$$id + id * E$$

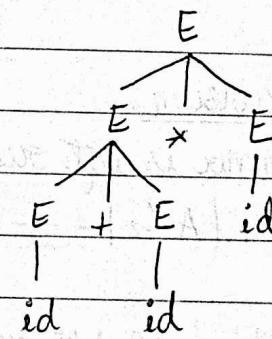
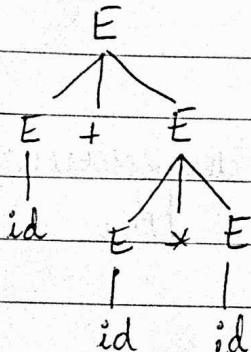
$$id + id * id$$

$$E \rightarrow E * E$$

$$id * E + E * E$$

$$id * id + E * E$$

$$id + id * id$$



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$$E \rightarrow E + T / E - T / T$$

$$T \rightarrow T * F / T / F / F$$

$$F \rightarrow (E) / id$$

$$id + id * id$$

$$E \rightarrow E + T$$

$$\rightarrow T + T$$

$$E \rightarrow id + T$$

$$\rightarrow F + T$$

$$E \rightarrow id + F * F$$

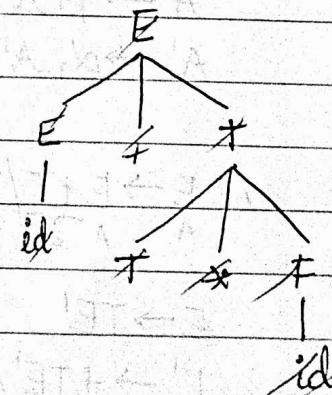
$$\rightarrow id + T$$

$$E \rightarrow id + T * F$$

$$\rightarrow id + id * F$$

$$E \rightarrow id + id * id$$

$$\rightarrow id + F * F$$



$$\rightarrow id + id * F \quad \rightarrow id + id * id$$

& ST the following grammar is ambiguous & find the unambiguous grammar

grammar  $S \rightarrow i \text{ct}S / i \text{ct} + S \text{S} / a \quad C \rightarrow b \quad w = \text{ibtibtaea}$

$S \rightarrow i \text{ct}S$

$i \text{ct}S$

$i \text{bt}i \text{ct}S \text{S}$

$i \text{bt}i \text{bt}a \text{ea}$

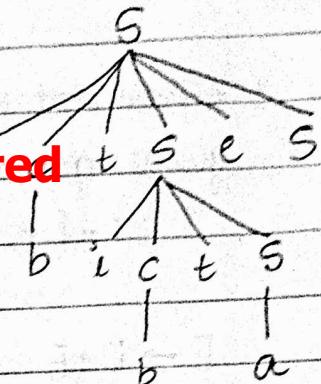
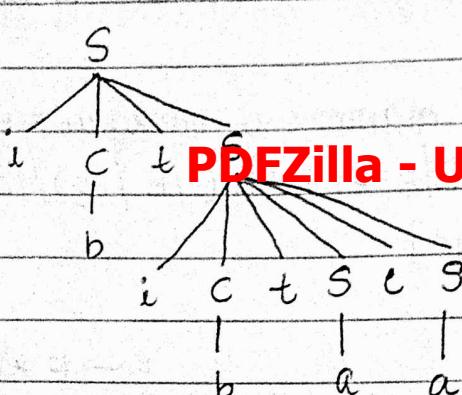
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$S \rightarrow i \text{ct} + S \text{S}$

$i \text{bt}S \text{S}$

$i \text{bt}i \text{ct}S \text{S}$

$i \text{bt}i \text{bt}a \text{ea}$



$S \rightarrow i \text{ct}SM / M$

$M \rightarrow i \text{ct}M \text{e}M / a$

$C \rightarrow b$

left Recursion:

A grammar is left recursive if it is of the following form

$A \rightarrow A\alpha_1 | A\alpha_2 | \dots | A\alpha_n | B_1 | B_2 | B_3 | \dots | B_m$

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Elimination of left recursion:

$A \rightarrow B_1 A' | B_2 A' | B_3 A' | \dots | B_m A'$

$A' \rightarrow \alpha_1 A' | \alpha_2 A' | \dots | \alpha_n A' | \epsilon$

ex:  $E \rightarrow E + T / T$   
 $\quad \quad \quad \overbrace{A \quad A}^{\sim} \alpha_1 B_1$

$T \rightarrow T * F / F$

$E \rightarrow TE'$

$E' \rightarrow +TE' / \epsilon$

$T \rightarrow TF'$

$F' \rightarrow *FT' / \epsilon$

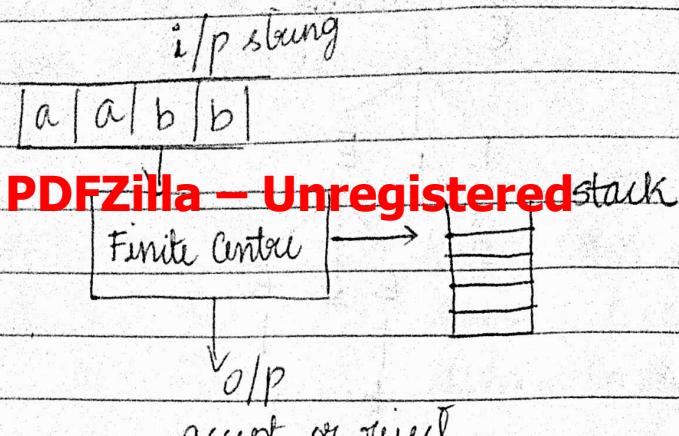
Eliminate left recursion for the following grammar  
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$$\begin{aligned} E &\rightarrow E + T / T \quad | \quad E \rightarrow + TE' \quad E \rightarrow + TE' / E \\ T &\rightarrow T * F / F \quad | \quad T \rightarrow * FT' \quad T' \rightarrow * FT' / E \\ T &\rightarrow (E) / id \quad | \quad F \rightarrow (E) / id \end{aligned}$$

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Pushdown Automata (PDA):



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PDA is defined as  $M = (Q, \Sigma, \Gamma, S, q_0, F)$ , where

$Q$  - set of finite states

$\Sigma$  - input alphabet which contains set of finite i/p symbols

$\Gamma$  - stack alphabet which contains stack symbols.

$S$  - transition function defined by  $S: Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma^* \rightarrow Q \times \Gamma^*$

$q_0$  - start state

$F$  - set of final states

The transitions performed by PDA depends on

1. Current state
2. Input symbol
3. The symbol on top of the stack

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The action performed by a PDA consist of

1. Changing the state from one state to another
2. Replacing the symbol on the top of the stack.

Instantaneous Description (ID):

e.g.

$$(q_0, a, z_0) + (q_1, z_0, A)$$

$$(q_1, b, A) + (q_2, G)$$

PDA accepts context free languages which are defined by context free grammars.

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1 Construct PDA to accept the language  $L = \{a^n b^n / n \geq 1\}$

$$M = (Q, \Sigma, \Gamma, S, q_0, F)$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{z, A\}$$

		a	b	z	
		Push A, q <sub>0</sub>	X	X	
$q_0$	A	Push A, q <sub>0</sub>	Pop A, q <sub>1</sub>	X	
$q_1$	z	X	X	Pop Z, q <sub>2</sub>	
	A	X	Pop A, q <sub>1</sub>		

$$1) S(q_0, a, z) = (q_0, Az)$$

$$2) S(q_0, a, A) = (q_1, AA)$$

$$3) S(q_0, b, A) = (q_1, E)$$

$$4) S(q_1, b, A) = (q_1, E)$$

$$5) S(q_1, E, z) = (q_2, E)$$

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ID's:

$$(q_0, aabb, z) \neq (q_0, abb, Az)$$

$$\neq (q_1, bb, AAz)$$

$$\neq (q_1, b, Az)$$

$$\neq (q_1, G, z)$$

$$\neq (q_1, E, E)$$

$$w = aabb$$

not accepted

$$w = a^4 b^3$$

$$(q_0, aaaabb, z) + (q_0, aaabbb, A)$$

$$\neq (q_0, aabb, AAz)$$

$$\neq (q_0, abbb, AAAz)$$

$$\neq (q_0, bb, AAAz)$$

$$\neq (q_1, b, AAz)$$

$$\neq (q_1, E, Az)$$

11/12/21

2. Design PDA to accept the language  $L = \{w c w^R / w \in \{a, b\}^*\}$

$$M = (Q, \Sigma, \Gamma, S, q_0, F)$$

$$L = \{abbCDBca, bacab, \dots\}$$

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	a	b	c	e
$q_0$	$\lambda$	push A, $q_0$	push B, $q_0$	X
	A	push A, $q_0$	push B, $q_0$	$q_1, A$
	B	push A, $q_0$	push B, $q_0$	$q_1, B$
$q_1$	$\lambda$	X	X	pop Z, $q_2$
	A	pop A, $q_1$	X	X
	B	X	pop B, $q_1$	X

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$$1) S(q_0, a, \lambda) = (q_0, (q_0, Az))$$

$$2) S(q_0, b, \lambda) = (q_0, Bz)$$

$$3) S(q_0, a, A) = (q_0, AA)$$

$$4) S(q_0, b, A) = (q_0, BA)$$

$$5) S(q_0, a, B) = (q_0, AB)$$

$$6) S(q_0, b, B) = (q_0, BAB)$$

$$7) S(q_0, \epsilon, A) = (q_1, A)$$

$$8) S(q_0, C, B) = (q_1, B)$$

$$9) S(q_1, \lambda, R) = (q_1, \epsilon)$$

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$$10) S(q_1, b, B) = (q_1, \epsilon)$$

$$11) S(q_1, \epsilon, \lambda) = (q_2, \epsilon)$$

ID's :  $w = abcbacab$

$w = abcb$

$$(q_0, bacab) \vdash (q_0, acab, Bz)$$

$$(q_0, abcb) \vdash (q_0, bcb, Az)$$

$$\vdash (q_0, cab, ABz)$$

$$\vdash (q_0, cb, ABz)$$

$$\vdash (q_0, ab, ABz)$$

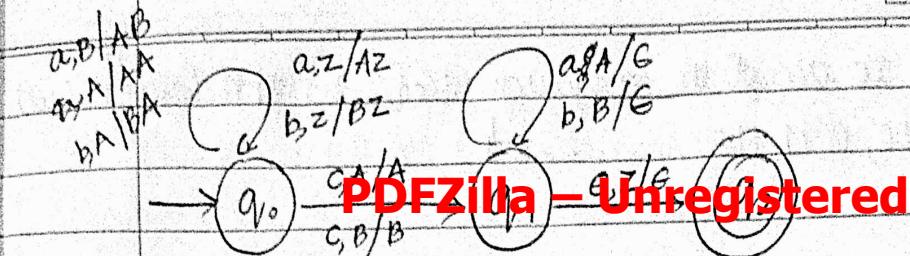
\*  $(q_0, b, Bz)$

$$\vdash (q_1, b, Bz)$$

Not accepted

$$\vdash (q_1, \epsilon, z)$$

$$\vdash (q_2, \epsilon)$$



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3 Design NPDA for the language  $L = \{ww^R / w \in \{a, b\}^*\}$

		a	b	$\epsilon$
$q_0$	$a$	push $A, q_0$	push $B, q_0$	
	$b$	push $A, q_0$	push $B, q_0$	
	$\epsilon$	pop $B, q_1$		
$q_1$	$a$	pop $A, q_1$		pop $Z, q_2$
	$b$		pop $B, q_1$	
	$\epsilon$			

1)  $S(q_0, a, z)$

Same as Q2 problem except 7, 8 and add change

$$3) S(q_0, a, A) = \{(q_0, AA), (q_1, \epsilon)\}$$

$$6) S(q_0, b, B) = \{(q_0, BB), (q_1, \epsilon)\}$$

ID's

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$(q_0, abba, z) \vdash (q_0, bba, Az)$

$\vdash (q_0, ba, BAZ)$

$\vdash (q_0, a, AZ)$

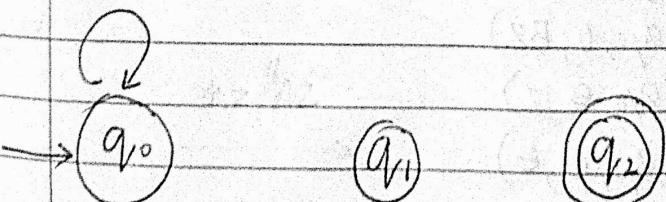
$\vdash (q_1, \epsilon, Z)$

$\vdash (q_2, \epsilon, \epsilon)$

$(q_0, aba, z) \vdash (q_0, ba, Az)$

$\vdash (q_0, a, BAZ)$

not accepted



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Date:

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4 Design PDA to accept the language  $L \in G(0, 1^*) \# n_0(w) = n_1(w)$

$$L = \{01, 10, 0011, 1100, \dots\}$$

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		0	1	$\epsilon$
$q_0$	Z	push A, $q_0$	push B, $q_0$	pop Z, $q_1$
	A	push A, $q_0$	pop A, $q_0$	
	B	pop B, $q_0$	push B, $q_0$	
$q_1$	Z			
	A			
	B			

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1.  $S(q_0, 0, Z) = \{(q_0, AZ)\}$
2.  $S(q_0, 1, Z) = \{(q_0, BZ)\}$
3.  $S(q_0, 0, A) = \{(q_0, AA)\}$
4.  $S(q_0, 1, A) = \{(q_0, \epsilon)\}$
5.  $S(q_0, 0, B) = \{(q_0, \epsilon)\}$
6.  $S(q_0, 1, B) = \{(q_0, BB)\}$
7.  $S(q_0, \epsilon, Z) = \{(q_1, \epsilon)\}$

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ID's :

 $w = 010110$ 

$$\begin{aligned}
 (q_0, 010110, Z) &\vdash (q_0, 10110, AZ) \\
 &\vdash (q_0, 0110, Z) \\
 &\vdash (q_0, 110, AZ) \\
 &\vdash (q_0, 10, Z) \\
 &\vdash (q_0, 0, BZ) \\
 &\vdash (q_0, \epsilon, Z) \\
 &\vdash (q_1, \epsilon, \epsilon)
 \end{aligned}$$

$w = 11010$  $(q_0, 11010, z) \vdash (q_0, 1010, Bz)$  $\vdash (q_0, 010, BBz)$ **PDFZilla - Unregistered** $\vdash (q_0, 10, Bz)$  $\vdash (q_0, 0, BBz)$  $\vdash (q_0, \epsilon, Bz)$ 

not accepted

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5. Design PDA to accept balanced parenthesis

$Bal = \{ w \in \{(), C\}^* : \text{the parenthesis are balanced} \}$

$L = \{ (C(C)), (C)(C), C \}$

	(	)	$\epsilon$	
$q_0$	$z$	push A, $q_0$	X	$(q_1, \epsilon)$
$q_0$	A	push A, $q_0$	pop A, $q_0$	
	B	pop X	X	

$$1. S(q_0, C, z) = \{ q_0, Az \}$$

$$2. S(q_0, C, A) = \{ q_0, AA \}$$

$$3. S(q_0, (), A) = \{ q_0, () \}$$

$$4. S(q_0, \epsilon, z) = \{ q_1, \epsilon \}$$

ID's

 $w = (( ))$  $(q_0, (( )), z) \vdash (q_0, ( )), Az$  $\vdash (q_0, ), AAZ$  $\vdash (q_0, ), Az$  $\vdash (q_0, \epsilon, z)$  $\vdash (q_1, \epsilon)$  $w = (( ))$  $(q_0, (( )), z) \vdash (q_0, ( )), Az$  $\vdash (q_0, ), AAZ$  $\vdash (q_0, \epsilon, Az)$ 

not accepted

6. Design PDA to accept string of well formed parentheses

or input symbol  $\Sigma = \{ (, ) , [ , ] , \{ , \} \}$

$L = \{ [( \{ \} )] , [ [ ] ] \{ ( ) \} , \dots \}$

C-A,

[ - B, { - C

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	$=$	push $(, q_0$	push $\{, q_0$	push $[, q_0$
$q_0$	$)$	push $(, q_0$ pop $(, q_0$	push $\{, q_0$	push $[, q_0$
	$[$	push $[, q_0$	push $\{, q_0$	push $[, q_0$
	$\}$	push $\{, q_0$	push $\}, q_0$ pop $\{, q_0$	push $[, q_0$

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pop  $Z, q_1$

pop  $[, q_0$

$w = [( \{ \} )]$

$(q_0, [( \{ \} )], z) \rightarrow (q_0, \{ \})$

$$1. S(q_0, (, z) = (q_1, Az)$$

$$2. S(q_0, (, A) = (q_1, AA)$$

$$3. S(q_0, (, B) = (q_1, AB)$$

$$4. S(q_0, (, C) = (q_1, AC)$$

$$5. S(q_0, ), A) = (q_1, G)$$

$$6. S(q_0, \{, z) = (q_1, Bz)$$

$$7. S(q_0, \{, A) = (q_1, BA)$$

$$8. S(q_0, \{, B) = (q_1, BB)$$

$$9. S(q_0, \{, C) = (q_1, BC)$$

$$10. S(q_0, \{, C) = (q_1, \epsilon)$$

$$11. S(q_0, [, z) = (q_1, CZ)$$

$$12. S(q_0, [, A) = (q_1, CA)$$

$$13. S(q_0, [, B) = (q_1, CB)$$

$$14. S(q_0, [, C) = (q_1, CC)$$

$$15. S(q_0, ], C) = (q_1, \epsilon)$$

$$16. S(q_0, \epsilon, z) = (q_1, \epsilon)$$

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ID's:

- (q<sub>10</sub>, ( ) { [ ] }, Z) ⊢ (q<sub>10</sub>, ( ) { [ ] } , A<sub>Z</sub>)  
    F (q<sub>10</sub>, ) { [ ] } , A<sub>Z</sub>)  
    F (q<sub>10</sub>, & [ ] Y, Z)  
    F (q<sub>10</sub>, [ ] Y, B<sub>Z</sub>)  
    F (q<sub>10</sub>, ] Y, CB<sub>Z</sub>)  
    F (q<sub>10</sub>, Y, B<sub>Z</sub>)  
    F (q<sub>10</sub>, E, E)  
**PDFZilla - Unregistered**  
    F (q<sub>11</sub>, E, E)  
accepted

**PDFZilla - Unregistered**