Computational Astrophysics

Exercises 5

April 7, 2019

1. Consider the Lagrangian interpolation of the function $f(x) = \frac{1}{25x^2+1}$ for $n = 6$, $n = 8$, $n = 10$ and $n = 12$ done in Exercises 4. Now, discretize the domain with $m = 100$ equally spaced points in the interval $[-1, 1]$. Compute the Error-Norm-2 (EN2), defined as

$$\mathrm{EN2} = \frac{1}{m}\sqrt{\sum_{i=1}^{m}\left(\frac{p(x) - f(x)}{f(x)}\right)^2},$$

   for the 4 cases $n = \{6, 8, 10, 12\}$.

2. Now discretize the same function with $m_2 = 50$ equally spaced points in the interval $[-1, 1]$. Implement a routine that interpolates $f(x)$ piecewise linearly between these m_2 data points and evaluate EN2 at the $m = 100$ points used above. Compare your result to the results of both exercises.

3. Consider once more the function $f(x) = \frac{1}{25x^2+1}$. Discretize the domain with $m = 21$ equally spaced points in the interval $[-1, 1]$ and evaluate numerically its first derivative (centered finite difference inside the interval and one-side derivative on the boundaries).
   Using this information, implement a routine that generates a piecewise cubic Hermite interpolating polynomial in the interval.
   Plot the function and the interpolating polynomial.

Happy Coding !!