Pendulum project -- lab report 1

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1. Introduction

The main purpose of this lab is to examine the relationship between period and angle, and the relationship between period and length, and analyzing Q factors and their relationship with period and length.

The apparatus used in this lab is a home made pendulum. The mass of the weight is 15.10 $\pm$0.05 g, the mass of a 30.4 $\pm$0.1 cm string is 0.20$\pm$0.05g. The weight of the string is much more smaller than the weight of the pendulum. The length of this pendulum is adjustable.

The relationship between the period and the amplitude of pendulum was examined in lab one:

$$T = T_0(1 + B\theta_0 + C\,\theta_0^2 + \ldots) \tag{1}$$

Equation (1) [1] is used to find the period and the symmetry of the pendulum as T_0 represent the period and B indicate the symmetry. When B is experimentally zero, the pendulum is symmetric. C represents the quadratic factor. We find $T_0 \approx 1.130 \pm 0.002$ s. For this experiment, B $\approx$ 0.001 $\pm$ 0.002 rad^{-1}. B is experimentally zero since the uncertainty is bigger than the value. We find an experimental value of C of 0.053 $\pm$ 0.003 rad^{-2}. C is not experimentally zero, since its uncertainty is smaller than the value.

$$u_{(t)} > T_0 C\theta^2 \tag{2}$$

Equation (2) [2] is used to find the small angle approximation by computing the C value from Equation (1) [1], the period, and uncertainty. θ = 14.0$\pm$0.4 degrees, by computing these values from lab one. The period is independent of small angles only when the angles are in the range of 14 degrees. When the angles are more than 14 degrees, the period increases as a quadratic as the angle increases.

$$\theta(t) = \theta_0 e^{-\frac{t}{\tau}} \tag{3}$$

Equation (3) [1] is an experimental function. τ can be calculated by fitting the amplitude vs time decaying graph into this equation. The experimental τ is 51 $\pm$ 3 s, $\theta_0 = 0.16 \pm 0.01$ rad calculated by Python using fit_black_box.py (Wilson).

$$Q = \pi\frac{\tau}{T} \tag{4}$$

Equation (4) [1] is the definition function of Q. The Q factor measures how slow an oscillation dies out.

$$\text{Amount of oscillation} = \frac{Q}{2} \tag{5}$$

Equation (5) [1], is used to calculate Q as the second method, which count the number of oscillations until it decays to 20%.

In lab one, the Q from calculating τ is Q =143 $\pm$ 8, while the Q from counting oscillation is 148 $\pm$ 6. Two Q are matching since the differences between their values are smaller than the uncertainty.

The relationship between the period and the length of the pendulum was examined in lab two:

$$T = kL^n \tag{6}$$

Equation (6) [1] is the power law function for period and length. The prediction is k= 2 $\frac{s}{m^n}$, n= 0.5. The experimental parameter is k = 2.11$\pm$0.06 $\frac{s}{m^n}$, n = 0.503 $\pm$ 0.007, obtained by fitting in Equation 5 using curve_fit function from SciPy [3]. Our data support this prediction well, since the difference between experimental and predicted values is smaller than the uncertainty for both values.

$$T \approx 2\sqrt{L} \tag{7}$$

The unit of Equation 7 [1] does not match.

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{8}$$

Equation (8) [4] illustrates the relationship between period and length with the right unit. According to parameters from Equation 6 [1], n=0.5, k=2 $\frac{s}{m^n}$. By calculation, we found that $\frac{2\pi}{\sqrt{g}}$ is $2.007\sqrt{s^2/m}$. This value is within two error bars from the experimental k (2.11±0.06 $\frac{s}{m^n}$).

$$Q = aL^2 + bL + c \tag{9}$$

The Q factor vs. Length fit the best in the quadratic formula as Equation 9 [4]. The parameter is a = $-3000 \pm 1000\ m^{-2}$, b = $1700 \pm 400\ m^{-1}$, c = 44 ± 40. Figure 7 shows that the change in Q factor is greater than the uncertainty, illustrating the dependency between Q factor and length.

1. Methods and Procedures

1) The experimental setup of the pendulum

The pendulum was set up as follows.

Firstly, A needle was tightly restrained on the desk as the pivot point of the pendulum. A pencil and a coffee stirring rod were not found work well as the pivot point. The pencil was too thick, which sharply increases the uncertainty of measuring the angle, while a coffee stirring rod is weak, so it kept bouncing without providing a fixed pivot point. A needle worked the best.

Secondly, a piece of tape was wrapped around the outer side of the needle as shown in Figure 2, constraining the string of the pendulum from moving back and forth more than 3 mm. This tape decreased the uncertainty of measuring period from this pendulum by preventing the string from sliding off from its original place, hence reducing the pendulum from oscillating in elliptical motion.

Thirdly, a light cotton string was used to hang the weight on the needle. It can adjust the length of the string by taking out knots. Three coins stuck together were selected as the weight since the center of the rotation of disk shape is more uniform than other shapes, which theoretically reduced the uncertainty of measuring angles and period by reduced wobbling. The mass is 15.10 $\pm$0.05 g.

Fourthly, there was a horizontal line taped on two sides of the table, preventing the pendulum from elliptical motion. An image of a protractor was fixed behind the pendulum, making the pendulum perfectly in the same direction as the 90 degrees mark of the protractor when it at rest. Total set up as Figure 1.

Figure 1

Three coins were tied to a light cotton string and hang on a fixed needle. The length of this cotton string was 30.4cm $\pm$0.1cm. The fixed needle was stuck perpendicular with the edge of the table. A protractor was behind the pendulum. There was a second string hanging horizontally to restrain the string from elliptical motion.

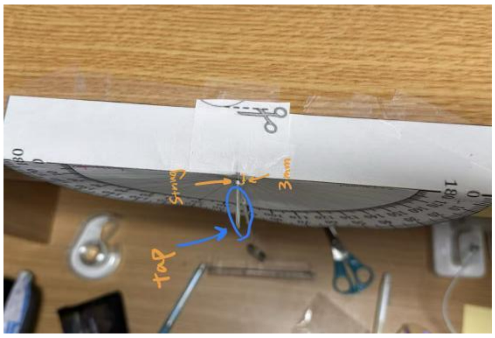

Figure 2: This was the top view of the pendulum with tap wrapped on the outer side of needle to prevent the string from sliding off and reduce the uncertainty might be caused be ellipsical motion.

2) Procedure for lab 1

The purpose of lab 1 is to find the dependency between period on angle, and measure Q factor of the home made pendulum. The length of the cotton string is 30.4cm $\pm$0.1cm, the mass of the object is 15.10 $\pm$0.05 g.

The period and angle dependency was firstly investigated. To get the graph showing this relationship, the angle starts to be released at 10 degrees. The angle was increased 15 degrees for each trial, up to 60 degree. One trail was conducted twice to take the average. The period of pendulum was calculated by $T_{ave} = \frac{T_{total}}{\text{amount of oscillation}}$. In this case, amount of oscillations were selected to be 5 to reduce the type A uncertainty.

The angle, corresponding average period, type A and type B uncertainty for both period and length was recorded. The bigger one was selected as the main source contribute to the uncertainty. The data was plotted in to python code, while angle as the dependent variable, period as the independent variable to get the period and angle graph. Secondly, the relationship between time and the decaying of angle was investigated. To get time and angle decaying graph, pendulum was released from 0.16 rad to get τ. The video until pendulum decay to the 20% from it original was recorded. This video is putted into tracker, to get each corresponding data about time and angle. Then,python code was used to get the time and angle decaying graph and τ was calculated by equation 5 [1]. The pendulum was decided to always be released from 0.175rad for lab 2. More explanation in detailed at lab 1 data analysis section.

3). Procedure for lab 2

The purpose of lab 2 is examine how the pendulum's length impacts both the period and the Q factor.

To determine the relationship between period and length. Pendulum length of 0.010m, 0.15m, 0.20m, 025m, 0.30m were used for each trial. The pendulum was released from 14 degree for every trial. The time for 8 oscillations was recorded, and the period for each length was calculated by dividing this time by 8. Each length was tested in two trials and the average period was calculated to reduce the random error. The period and length was plotted into Equation 6 [1] to get the pendulum period vs. length graph. To determine the relationship between Q factor and the length. Pendulum length of 0.010m, 0.15m, 0.20m, 025m, 0.30m were used as length. Two trails were used to calculate the average for each length to reduce random error. Pendulum was released from 14 degree, the amount of oscillation were counted until it decays to 20%. Equation (5) [1] was used to calculated Q. Q and lengths were plotted in the best fit Equation (9) [4] to get the pendulum Q factor vs. Length graph.

Result and Data Analysis for lab 1

1. period and Angle

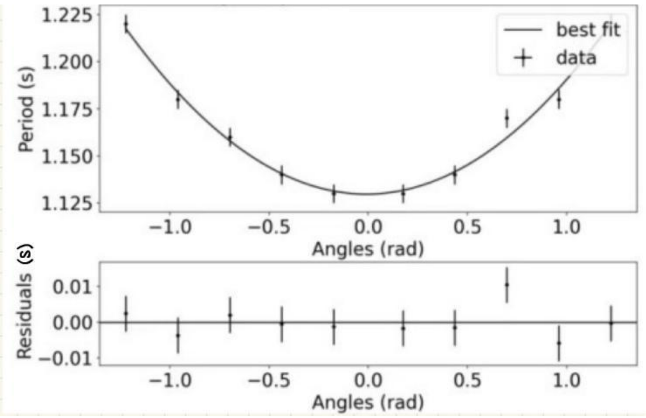

Figure 3: This graph shows the relationship between the angles and period. The best fit function of the data is quadratic function.

Figure 3 shows that, periods are only independent from the amplitude within a small range of angle.

1) . Uncertainties

Since the scale of the x-axis are greater than the horizontal uncertainty, so the horizontal uncertainty is not really visible. For both horizontal and vertical, both Type B uncertainty is greater. The vertical Type B uncertainty of measuring period which is always ± 0.005 s. The vertical type A uncertainty is 0.003 s, which is smaller than Type B. The horizontal type B uncertainty is about the firm limit of precision of the protractor. The smallest measurement of protractor is 0.02 rad, after divide by 2 and round it. The uncertainty is ± 0.009 rad for all angles. The horizontal Type A uncertainty is 0.003 rad, which is smaller than Type B. For both horizontal and vertical, both Type B uncertainty is greater. Since type B uncertainty dominate, for next time, the most important to fix uncertainty is use the equipment with less uncertainty.

2). Prediction one failed

According to the small-angle series expansion of the pendulum series, Equation(1) [1]:

$$T = T_0(1 + B\theta_0 + C\,\theta_0^2 + \ldots)$$

Prediction one state that both B and C should be experimentally zero. We get $T_0 \approx 1.130 \pm 0.002$s, B $\approx$ 0.001 ± 0.002 rad^{-1}, and C $\approx$ 0.053 ± 0.003 rad^{-2} by subbing the data from Figure 3 in to Equation (1) [1]. B is experimentally zero and thus mostly symmetric, because B is smaller than its uncertainty. C is not experimentally zero, since C is much bigger than its uncertainty by more than two error bar. This is arbitrary with the prediction. The data can not support with the prediction very well.

3) period dependency

The period is independent from the amplitude while in the range of 14.0 $\pm$ 0.4 degree.

We got $\theta < 0.289 \pm 0.008$ rad, by Sub in data in Figure 3 to Equation (2)[2], that $\sqrt{\frac{0.005}{(0.053 \pm 0.003)\times(1.130 \pm 0.002s)}} > \theta$. In Figure 3, the error bar is greater than the change in periods within 14.0 ± 0.4 degrees, so this change can be explained by error, which does not show the dependency. For lab 2, pendulum should be released from 14 degree.

2. Using Time and Amplitude to calculate Q factor.

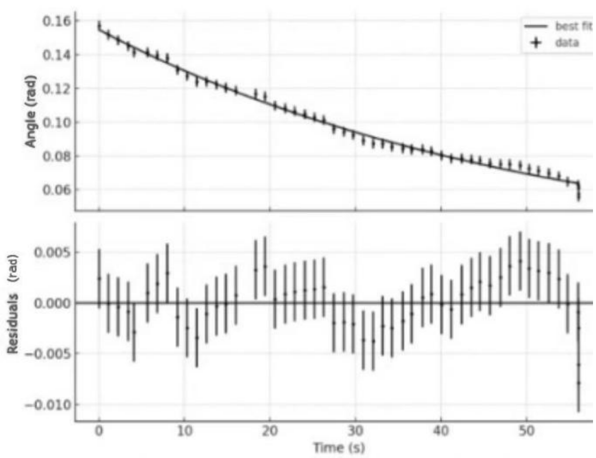

Figure 4: This graph shows the relationship between angle and time for the decay of pendulum. We obtained τ is 50$\pm$3 s, $\theta_0 =$ 0.16 ± 0.01 rad, by fitting the data in Equation (3) [1].

The residuals are randomly distributed about zero, indicating good fit.

(1) Measure the Q factor in both way:

The first way is to find the τ(time constant of the decay) and measure the period. We got τ is 50$\pm$3 s, and $T_0 \approx 1.130 \pm 0.002$ s (period for very small angles). After calculating using Equation (4) [1]:

$$Q = \pi\frac{\tau}{T}$$

$Q = \pi\frac{51.3 \pm 3}{1.130 \pm 0.002s}$ = 143 $\pm$ 8, since $\frac{0.002}{1.13} < \frac{51.3}{3}$, so the uncertainty is 143 $\times \frac{3}{8.33}$. The uncertainty is 8 after rounded to one Significant Figures.

The second way of finding the Q factor is count numbers of oscillations. Count the amount of oscillation before pendulum decay to 0.56 degree. The amount of oscillations are 74 $\pm$ 3,when the pendulum decay to 20% from the original, which is 0.28 degree in this experiment. By using Equation 5 [1], Q = (74 ± 3)×2=148± 6.Since the differences between 143 ± 8 and 148 ± 6 is smaller than the uncertainty, these two Q are matching with each other.

(2) Uncertainties

The horizontal Type B uncertainty is 0.005s, since the minimum distinguishable time of phone camera is 0.01s. The type B uncertainty is 0.01/2 s, which is 0.005 s. The horizontal Type A uncertainty is 0.02 s. Type A is considered as the biggest uncertainty horizontally. The horizontal error bar seems to be small because it is very small compared with the scale of x-axis. The vertical average Type A uncertainty is 0.005 rad. The vertical Type B uncertainty is 0.004 rad. Since the smallest measurement of angle is 0.0089 rad, 0.0089 rad divide by 2 and then rounded is 0.004 rad. Therefore, Type A uncertainty is considered as the greatest uncertainty. Overall, Type A uncertainty is the one considered as the biggest error and reflecting in the error bar for both horizontal and vertical. For next time, run more trials and calculate for the average to reduce Type A which is random error.

Result and Data Analysis for lab 2

Period vs. Pendulum Length and Q factor vs. Pendulum length

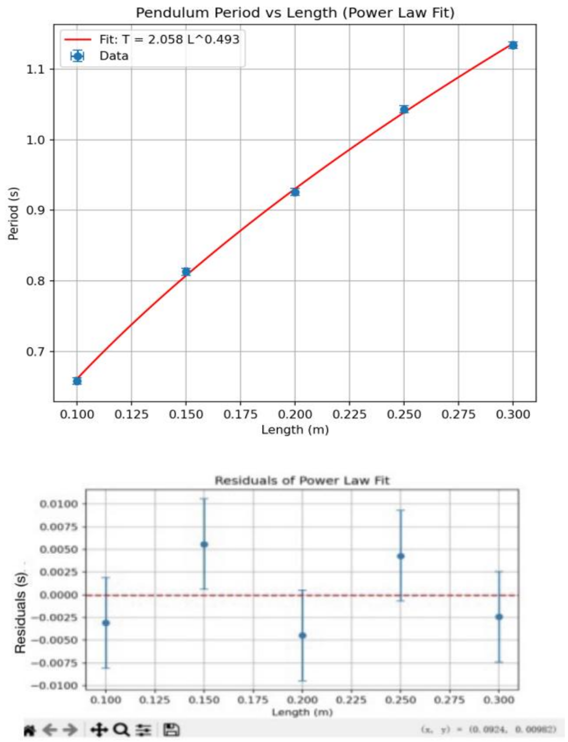

Figure 5: Period vs length data is plotted and fit in to the power function in equation. The length of the pendulum is adjusted from 0.1m to 0.3m. Each trial adjust 0.05m more. The red curve is the best fit line of Equation (6) [1], T = kL^n. The parameters are k =2.06 $\pm$ 0.06$\frac{\mathrm{s}}{\mathrm{m^n}}$, n = 0.500 $\pm$ 0.007.

The parameter from Figure 5 is able to justify equation (8) [4]. According to parameters from Equation (6) [1] about Figure 5, n=0.5, $\frac{2\pi}{\sqrt{g}}$ = 2.007$\sqrt{s^2/m}$. This value is within two error bars from the the experimental k (2.11±0.06 $\frac{\mathrm{s}}{\mathrm{m^n}}$).

1) Uncertainty

The horizontal type B uncertainty is 0.005 m. Because, the smallest unit the meter stick can measure is 1cm (0.01m). The type B uncertainty of the meter stick 0.01/2 m, is 0.005 m. The horizontal error bar seems to be small because it is very small compared with the scale of x-axis. The vertical type A uncertainty is 0.07 s, the vertical type B uncertainty is 0.005 s. The type A is the major uncertainty. Next time should getting average from conducting more trials to reduce the random error.

In Figure 5, since 4/5 of the result is within 1 standard deviation of the means, so this error bar is reasonable small. The measured period does not perfectly followed the normal distribution, which might due to the small data size. Next time, adjust length for more time to get a greater size of data.

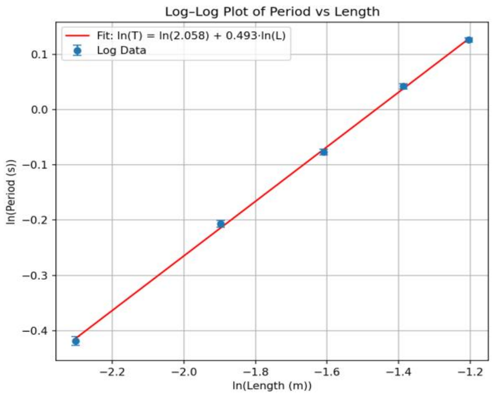

Figure 6: Plot the data into log-log plot that the slope is n and the intercept tells the k. k =2.06 $\pm$ 0.06$\frac{\mathrm{s}}{\mathrm{m^n}}$, n= 0.500 $\pm$ 0.007. The residuals from log-log fit of period vs. Length, which illustrates how the uncertainties are randomly distributed around zero.

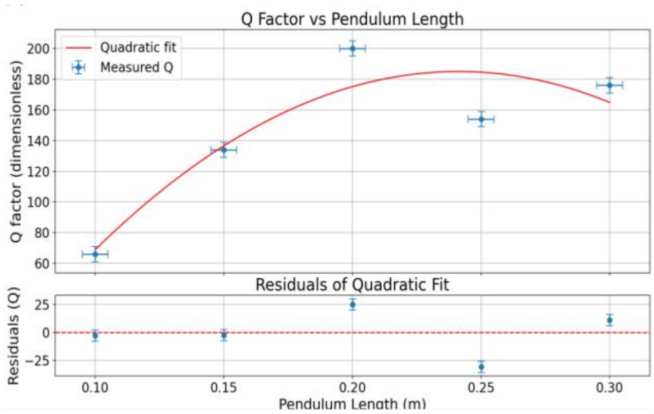

Figure 7: This figure illustrates how Q factor depends on the length of pendulum. The Q factor vs. Pendulum length fit the best in the quadratic function. Three functions were tried to describe the relationship between pendulum length and Q factor: linear, power-law, and quadratic. The linear (m = 480 ± 200, b = 50 ± 50) and power-law (A = 339 ± 200, k = 0.5 ± 0.4) fits both had very large relative uncertainties, which around 50-80%, indicating poor fit. The quadratic fit according to Equation (9) [4] (a = −3229 ± 1000 m^{-2}, b = 1760 ± 400 m^{-1}, c = 44 ± 40) shows smallest relative uncertainties around 25-30%.

2) Uncertainty

The horizontal type B uncertainty is 0.005, which is the same as Figure 5. The vertical type A uncertainty is ± 5, which is the random error when counting the amount of oscillations. The vertical type B is 0.5, since human reaction time might cause 1 oscillation missed. The uncertainty is 1/2. Type A uncertainty is the major uncertainty compared with type B. Each trials should be conduct for more times to get an average. In Figure 7, 2/5 of the error bar is on the residual line, illustrating that this data is not fully consistent with the normal distribution, making the data less reliable.

3) How to measure Q factor:

Counting the number of oscillations until it decays to 20%. Counting method works better when oscillate more than 50 oscillations (Q is relatively high). Using tracker will increases error since it is hard to detect small amplitude changes. In addition, any slight shake of the camera can cause uncertainty, while the uncertainty from counting method can be reduced by counting for multiple times.

4) What effect pendulum's length has on the Q factor:

Since the difference of Q factor in different length is not smaller than the uncertainty, illustrating that the Q factor does depend on length. The relationship between length and Q factor is non-linear, it is quadratic. Q factor increases as the pendulum length increases from 0.10m to the length between 0.20m and 0.25m. After that threshold, Q factor starts decreases as the length increases.

5). Whether C is important:

Lab 2 should be released at 14 degree according to the result from lab 1. C is important. According to Equation(1) [1], $C \approx 0.053 \pm 0.003$ rad^{-2}, which is not experimentally zero. So when the amplitude increases, the period increases. Q affects by energy lose. The effect from C should try to be minimized, meaning period and Q factor can be analyzed independently. So change in angle only have a minor effect on energy loss and damping.Therefore, any decay in amplitude and therefore Q mainly comes from energy lose.

4. Conclusion

In conclusion, The period is independent from the amplitude while in a small range, in this experiment is 14.0$\pm$ 0.4 degree. After the amplitude is bigger than 14 degrees, period starts increases significantly when the amplitude increases.

The amplitude decay exponentially, and two Q factors getting from different measure methods are matching. We got Q =143 $\pm$ 8 from using Equation (3)[1] and (4)[1]. We got Q is 148 $\pm$ 6 from counting method and Equation (5) [1]. The differences between these two Q is smaller than its uncertainty.

In lab two, Equation (8) [4] can be justified by the result. According to parameters from Equation 6 [1], n=0.5, k=2 $\frac{s}{m^n}$. We found that $\frac{2\pi}{\sqrt{g}}$ is $2.007\sqrt{\frac{s^2}{m}}$ from calculation. This value is within two error bars from the the experimental k (2.11±0.06 $\frac{s}{m^n}$). The

relationship between length and Q factor is non-linear, it is quadratic. The parameters are a = -3000 ± 1000 m^{-2}, b = 1700 ± 400 m^{-1}, c = 44 ± 40 when data fit in Equation (9) [4]. Q factor increases as the pendulum length increases from 0.10m to the length between 0.20m and 0.25m. After that threshold, Q factor starts decreases as the length increases.

To improve this experiment. Firstly, change the object to a metal ball, so it is less easy to rotate around the string. Because three coins sticks together can not keep itself perfectly from wobbling, and since their width is narrow relative with their diameter, so they rotate around the string at a few beginning's oscillation, which might cause extra lose of energy. This lower the Q factor and causing slight derivation when measuring period. A metal ball is very stable, theoretically will minimal the rotation. Secondly, for both two labs, each trial should be conduct for more times, and take the average to reduce random error.

Reference

[1] Brian Wilson. “PHY180H1F ALL SECTIONS 20259:Classical Mechanic.” https://q.utoronto.ca/courses/411727/files/39071655?module_item_id=7122439.Sep.30th.2025

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[4]R. W. Chabay and B. A. Sherwood, *Matter and Interactions, Volume 1*. John Wiley & Sons, 2018.

Chat GPT generates python code for Figure 5,6,7.