SECT Discrete Structure Assignment 2 (Sem 1)

Group Number: 1 Angela Ngu Xin Yi (A24CS0226)
                2. Cheong Yi Shiun (A24S0056)
                3. Tan Xin Tian (A24CS0198)
                4. Tioh Xin Wei (A24CS0307)

1. (a) $7 \times 9 = 63$

(b) (i) $8! = 40320$

(ii) $^8P_5 = 6720$

(iii) $6! = 720$

(c) (i) $4! \times 5! = 2880$

(ii) $(9-1)! \times 2! = 80640$

(iii) $5! \times 5! = 14400$

2. (a) at least 3 women

$\underline{\quad W \quad M \quad}$

$3 + 2 \quad = 5$

$4 + 1 \quad = 5$

$5 + 0 \quad = 5$

$(^8C_3 \times {}^6C_3) +$

$(^8C_1 \times {}^6C_4) +$

$(^8C_0 \times {}^6C_5) = 560 + 120 + 6 = 686$ ways

(b) at least 1 boy

$\underline{\quad G \quad B \quad}$

$3 + 1 = 4$

$2 + 2 = 4$

$1 + 3 = 4$

$0 + 4 = 4$

$(^{10}C_3 \times {}^{10}C_1) +$

$(^{10}C_2 \times {}^{10}C_2) +$

$(^{10}C_1 \times {}^{10}C_3) +$

$(^{10}C_0 \times {}^{10}C_4) = 1200 + 2025 + 1200 + 210$

$= 4635$ ways

3. a) i. $(5-1)! = 24$ ways

    ii. $(3-1)! \times 3! = 12$ ways

b) $5! - (4! \times 2!) = 120 - 48$

                      $= 72$

c) i. $C(10+6-1, 6) = \dfrac{15!}{6!(10-1)!}$

                       $= 5005$ ways

   ii. $C(9+2-1, 2) = \dfrac{10!}{2!(8!)}$ $45 + 9 + 1 = 55$ ways

                    $= 45$ ways

      $C(9+1-1, 1) = \dfrac{9!}{1!(8!)}$

                 $= 9$ ways

      $^9C_0 = 1$ way

   iii. $C(10, 6) = \dfrac{10!}{6!(10-6)!}$

              $= 210$ ways

d) i. $C(13, 11) = {}^{13}C_{11} = 78$ ways

   ii. $P(13, 11) = {}^{13}P_{11} = 3\ 113\ 510\ 400$

   iii. $^{13}C_{11} = 78$ ways

        10M 1W

        $C(3,1) \times C(10,10) = \dfrac{3!}{1!(3-1)!} \times \dfrac{10!}{10!(10-10)}$

                              $= 3 \times 1$

                              $= 3$ ways

        9M 2W

        $C(3,2) \times C(10,9) = \dfrac{3!}{2!(3-2)!} \times \dfrac{10!}{9!(10-9)!}$

                              $= 3 \times 10$

                              $= 30$ ways

8M3W

$$C(3,3) \times C(10,8) = \frac{3!}{3!(3-3)!} \times \frac{10!}{8!(10-8)!}$$

$$= 1 \times 45$$

$$= 45 \text{ ways}$$

Total = 3 + 30 + 45

= 78 ways

Question 4

a)

$n = 3$

$k = n + 1$

$k = 3 + 1$

$k = 4$

At least 4 balls must be taken from the box to get two balls of same colour.

b)

pegion, $n = 10 \times 8 = 80$

pegionhole, $m = 30 + 2 = 32$

$$k = \left\lceil \frac{n}{m} \right\rceil$$

$$= \left\lceil \frac{80}{32} \right\rceil$$

$$= 2.5$$

$$\approx 3$$

This show that in among ten cheesecakes cut into each cakes eight pieces, divided among thirty students and two teachers, each of them can have at least three pieces of cheesecakes.

c) Pair of forming sum of $10 = \{(2,8), (3,7), (4,6)\}$

pegionhole, $m = 3$

pegion > pegionhole

pegion $= 4$

The smallest integer that must be chosen is 4 so that at least one pair of them has a sum of 10.

Question 4

d) pegion, $n = ?$

    pegionhole, $m = 5$

        $k = 6$

    $n = m(k-1) + 1$

    $n = 5(6-1) + 1$

    $n = 26$

    minimum number of student required $= 26$

e) pegion, $n = 6$

    pegionhole, $m = 5$

        $k = \lceil \frac{n}{m} \rceil$

          $= \lceil \frac{6}{5} \rceil$

          $= 1.2$

        ≈ 2

This show that among six computer, each computer is directly connected to zero or more of the other computers, at least two computer are directly connected to the same number of other computers.