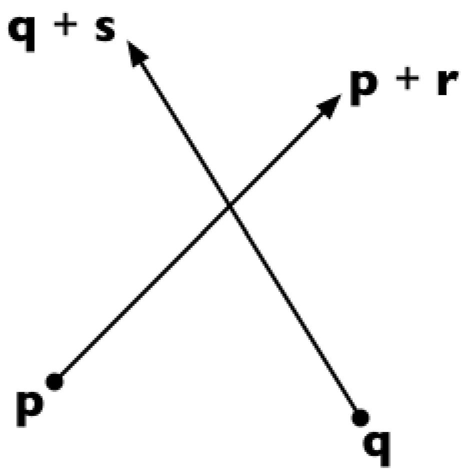
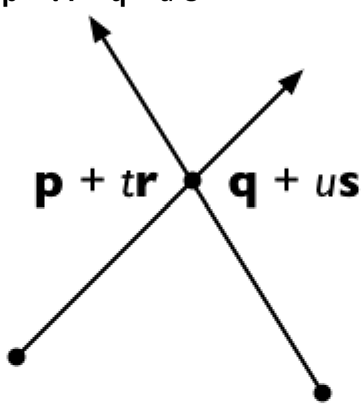
There's a nice approach to this problem that uses vector cross products. Define the 2-dimensional vector cross product **v** × **w** to be $\mathbf{v}_x \mathbf{w}_y - \mathbf{v}_y \mathbf{w}_x$ (this is the magnitude of the 3-dimensional cross product).
Suppose the two line segments run from **p** to **p** + **r** and from **q** to **q** + **s**. Then any point on the first line is representable as **p** + *t* **r** (for a scalar parameter *t*) and any point on the second line as **q** + *u* **s** (for a scalar parameter *u*).



The two lines intersect if we can find *t* and *u* such that:
p + *t* **r** = **q** + *u* **s**



Cross both sides with **s**, getting
(**p** + *t* **r**) × **s** = (**q** + *u* **s**) × **s**
And since **s** × **s** = 0, this means
t (**r** × **s**) = (**q** − **p**) × **s**
And therefore, solving for *t*:
t = (**q** − **p**) × **s** / (**r** × **s**)
In the same way, we can solve for *u*:
(**p** + *t* **r**) × **r** = (**q** + *u* **s**) × **r**
u (**s** × **r**) = (**p** − **q**) × **r**
u = (**p** − **q**) × **r** / (**s** × **r**)
To reduce the number of computation steps, it's convenient to rewrite this as follows (remembering that **s** × **r** = − **r** × **s**):
u = (**q** − **p**) × **r** / (**r** × **s**)

Now there are five cases:

1. If $\mathbf{r} \times \mathbf{s} = 0$ and $(\mathbf{q} - \mathbf{p}) \times \mathbf{r} = 0$, then the two lines are collinear. If in addition, either $0 \le (\mathbf{q} - \mathbf{p}) \cdot \mathbf{r} \le \mathbf{r} \cdot \mathbf{r}$ or $0 \le (\mathbf{p} - \mathbf{q}) \cdot \mathbf{s} \le \mathbf{s} \cdot \mathbf{s}$, then the two lines are overlapping.
2. If $\mathbf{r} \times \mathbf{s} = 0$ and $(\mathbf{q} - \mathbf{p}) \times \mathbf{r} = 0$, but neither $0 \le (\mathbf{q} - \mathbf{p}) \cdot \mathbf{r} \le \mathbf{r} \cdot \mathbf{r}$ nor $0 \le (\mathbf{p} - \mathbf{q}) \cdot \mathbf{s} \le \mathbf{s} \cdot \mathbf{s}$, then the two lines are collinear but disjoint.
3. If $\mathbf{r} \times \mathbf{s} = 0$ and $(\mathbf{q} - \mathbf{p}) \times \mathbf{r} \ne 0$, then the two lines are parallel and non-intersecting.
4. If $\mathbf{r} \times \mathbf{s} \ne 0$ and $0 \le t \le 1$ and $0 \le u \le 1$, the two line segments meet at the point $\mathbf{p} + t\,\mathbf{r} = \mathbf{q} + u\,\mathbf{s}$.
5. Otherwise, the two line segments are not parallel but do not intersect.

(Credit: this method is the 2-dimensional specialization of the 3D line intersection algorithm from the article "Intersection of two lines in three-space" by Ronald Goldman, published in *Graphics Gems*, page 304. In three dimensions, the usual case is that the lines are skew (neither parallel nor intersecting) in which case the method gives the points of closest approach of the two lines.)