Assume $\varepsilon \sim N(0, \sigma^2 I_n)$, using MLE, derive estimator for β and σ^2.

$$Y = X\beta + \varepsilon$$

$$\frac{\partial}{\partial \hat{\beta}} (y - X\hat{\beta})^T (y - X\hat{\beta}) = \frac{\partial}{\partial \hat{\beta}} (y^T y - y^T X \hat{\beta} - \hat{\beta}^T X^T y + \hat{\beta}^T X^T X \hat{\beta})$$

$$= -y^T X - y^T X + 2\hat{\beta}^T X^T X$$

$$= -2y^T X + 2\hat{\beta}^T X^T X$$

$$= -2X^T y + 2X^T X \hat{\beta}$$

Provided that $X^T X$ is invertible:

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

$$\frac{\partial L(\beta, \sigma^2 | Y)}{\partial \sigma^2} = -\frac{n}{2}\left(\frac{1}{\sigma^2}\right) + \frac{1}{2(\sigma^2)^2} (Y - X\beta)^T (Y - X\beta) = 0$$

solve for MLE:

$$\frac{n}{2}\left(\frac{1}{\hat{\sigma}^2}\right) = \frac{1}{2(\hat{\sigma}^2)^2} (Y - X\hat{\beta})^T (Y - X\hat{\beta})$$

$$n\hat{\sigma}^2 = (Y - X\hat{\beta})^T (Y - X\hat{\beta})$$

$$\hat{\sigma}^2 = \frac{(Y - X\hat{\beta})^T (Y - X\hat{\beta})}{n} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \frac{SSR}{n}.$$

Thus,

$$\begin{cases} \hat{\beta} = (X^T X)^{-1} X^T y \\ \hat{\sigma}^2 = \dfrac{(Y - X\hat{\beta})^T (Y - X\hat{\beta})}{n} \end{cases}$$