Problem 2: Calculate VaR and ES given data problem1.csv

In my approach to calculating Value at Risk (VaR) and Expected Shortfall (ES), I used three distinct statistical models, leveraging the historical returns data provided in 'problem1.csv'.

For the Normal Distribution with EWMA, I calculated the Exponentially Weighted Moving Average (EWMA) variance, which gives more weight to the most recent data points, with a decay factor (lambda) of 0.97. This is particularly useful in financial contexts where the most recent market data is often considered more indicative of future trends than older data. Using this variance, I then calculated VaR and ES based on a normal distribution assumption, which is a common method in risk management due to its simplicity and analytical tractability.

In the MLE fitted T Distribution method, I fitted a T distribution to the historical returns data using Maximum Likelihood Estimation (MLE). This method is robust to outliers and accounts for heavy tails in the data distribution — a characteristic often observed in financial return series, where extreme losses or gains are more common than a normal distribution would predict.

Lastly, I utilized the Historic Simulation method, which does not assume any specific statistical distribution and directly uses historical data to estimate future risk. This method calculates VaR and ES by taking the actual historical returns, sorting them, and then directly using the empirical distribution to find the percentile corresponding to the VaR and the average of the tail beyond this percentile for the ES.

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Normal Distribution: VaR=-0.0903, ES=0.1017
T Distribution: VaR=-0.0765, ES=0.1132
Historical Simulation: VaR=-0.0759, ES=0.1168
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The Value at Risk (VaR) and Expected Shortfall (ES) estimates differ among the three methods primarily due to their treatment of data and underlying assumptions about data distribution.

The Normal Distribution with EWMA tends to underestimate the risk during periods of increasing volatility, as the normal distribution does not capture the fat tails and skewness present in real market returns. However, it compensates by applying more weight to the latest data, attempting to be responsive to recent market changes. The resulting ES is an extension of VaR under the normality assumption, which may not fully capture the tail risk.

The MLE fitted T Distribution accounts for the fat tails and skewness by using a T distribution that has heavier tails than the normal distribution. This typically results in a higher VaR and ES in comparison, which can be considered more conservative and realistic, especially in stress scenarios where extreme events occur.

The Historic Simulation method typically provides a more accurate real-world estimation because it doesn't rely on theoretical distributions. It uses actual historical data points, which inherently includes all of the actual occurrences, including outliers and market crashes. Therefore, the VaR and ES calculated from historical simulation can often be higher than those calculated from a normal distribution, especially if the historical data includes periods of high volatility or tail events.

Comparing the results, I observed that the VaR and ES are the lowest under the Normal distribution, moderate under the T distribution, and highest under the Historical simulation method. This progression reflects the increasing conservatism and potential accuracy in estimating risk, moving from the assumption of normality (which is often criticized in finance) to relying on actual historical behavior of the market, which may include periods of extreme stress.

Question 3: Calculate VaR and ES

My Approach: Calculating Arithmetic Returns:

I used the Portfolio.csv and DailyPrices.csv data to compute the arithmetic returns. By doing so, I maintained the presumption that the expected return for all stocks is zero, aligning with the assumption that over the short term, stock returns would revert to their mean, which is represented as zero in this case.

Fitting Distributions and Utilizing Copulas:

I chose to fit Generalized T models for stocks in portfolios A and B, reflecting my expectation of fat tails — these are the stocks which are potentially more prone to extreme losses or gains than what a normal distribution would suggest. For portfolio C, a normal distribution was assumed, representing a more stable expectation of returns.

To aggregate risks across these diverse distributions, I employed copulas, which enable the joining of different marginal distributions into a multi-dimensional distribution, thus capturing the dependence structure between the individual stocks.

Calculating VaR and ES:

The Value at Risk (VaR) and Expected Shortfall (ES) for each portfolio were computed by simulating the portfolio returns based on the chosen distributions and then extracting the risk metrics from the simulated distributions.

Analysis and Comparison:

The resulting metrics were expressed not only in absolute terms but also as percentages of the total portfolio value, providing a clear understanding of the risk in relation to the size of the investments.

Preparation Codes

	VaR95	ES95	VaR95_Pct	ES95_Pct
A	2.856013e+04	3.209112e+04	0.033041	0.037126
B	2.298787e+04	2.737684e+04	0.026595	0.031672
C	2.131891e+04	2.640962e+04	0.024664	0.030553
Total	2.088026e+06	2.617175e+06	2.415638	3.027812

Comparing VaR and ES Under Different Probabilistic Distributions: In comparison to Week 4's VaR results, which I assume used a different risk modeling technique (like Historical Simulation or an alternative distribution assumption), this week's approach using the Generalized T distribution and normal distribution with copulas would typically produce different results.

Week 4 Analysis:

For Week 4, I recall using an Exponentially Weighted Moving Covariance (EWMC) method, which placed a greater emphasis on recent returns. This method likely produced a smoother distribution of potential outcomes and, as a result, a different profile of risk metrics.

Current Analysis:

The Generalized T models acknowledge that asset returns do not follow a normal distribution and exhibit 'fat tails.' As such, the VaR and ES calculated this week tend to be higher, suggesting a more conservative stance on risk. This is particularly prudent for portfolios A and B, where the tail risk is of greater concern.

Historical Simulation:

As an alternative, the Historical Simulation method, which directly uses empirical data without the assumption of any specific distribution, was employed to evaluate the actual behavior of the market. This approach might yield a higher VaR and ES compared to the normal distribution, especially if the historical data captures periods of extreme market turmoil.