Mathematical Models and Methods for Image Processing

Spring 2022

What is this course about?

What is this course about?

It is about **algorithms** *for processing* **images** *and solving image-related problems.*

Dabov, K., Foi, A., Katkovnik, V., Egiazarian, K. *"Image denoising by sparse 3-D transform-domain collaborative filtering"* TIP 2007

What is this course about?

It is about ***algorithms*** *for processing* ***images*** *and solving image-related problems.*

..like denoising

Dabov, K., Foi, A., Katkovnik, V., Egiazarian, K. *"Image denoising by sparse 3-D transform-domain collaborative filtering"* TIP 2007

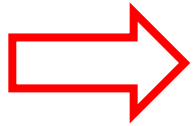
Who cares about images?

Who cares about images?

Everybody!

We will see algorithms solving problems customarily addressed in our phones,

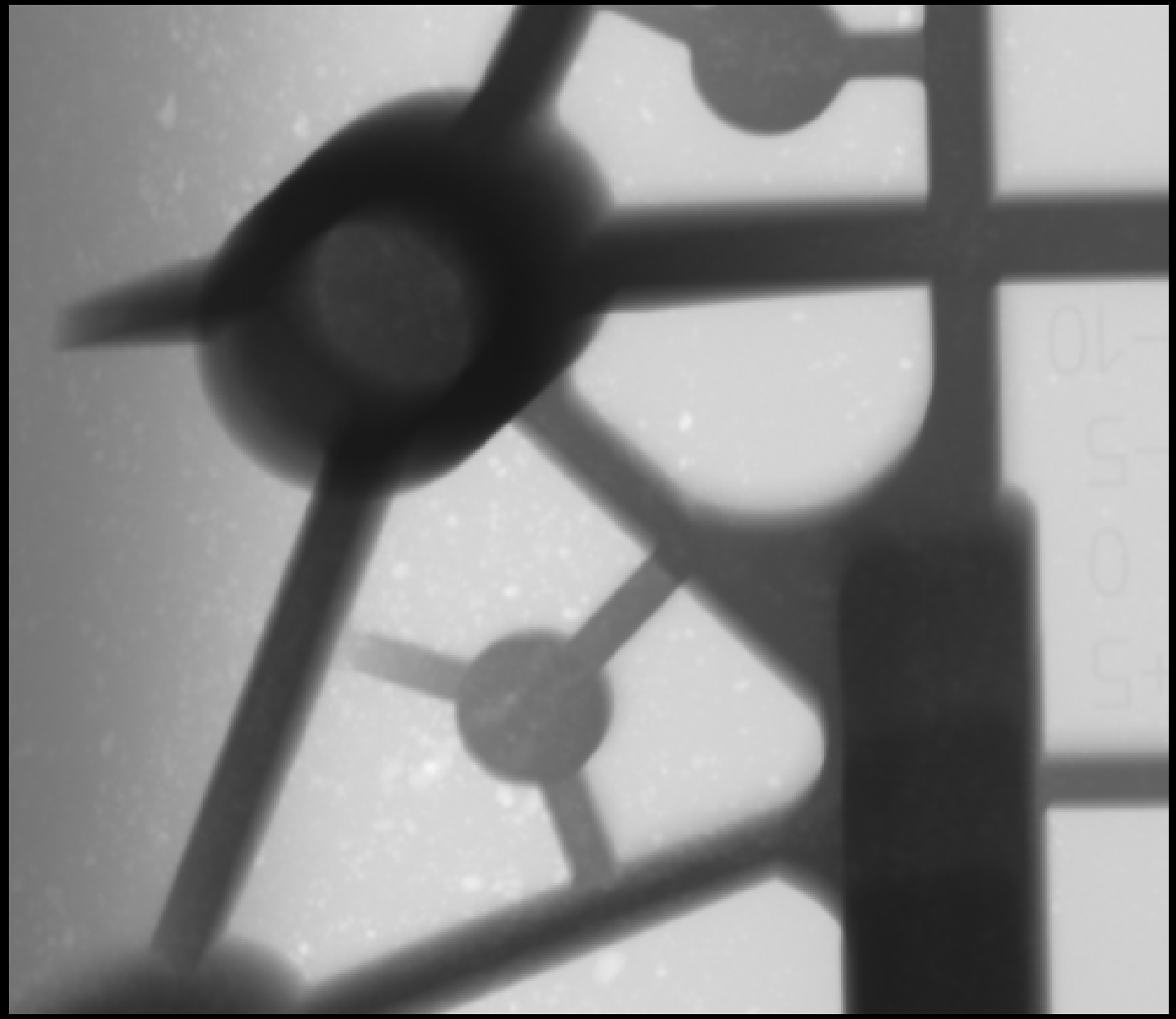
$z = y + \eta\,, \qquad \eta \sim \mathcal{N}(0, \sigma^2)$



$\hat{y} \approx y$

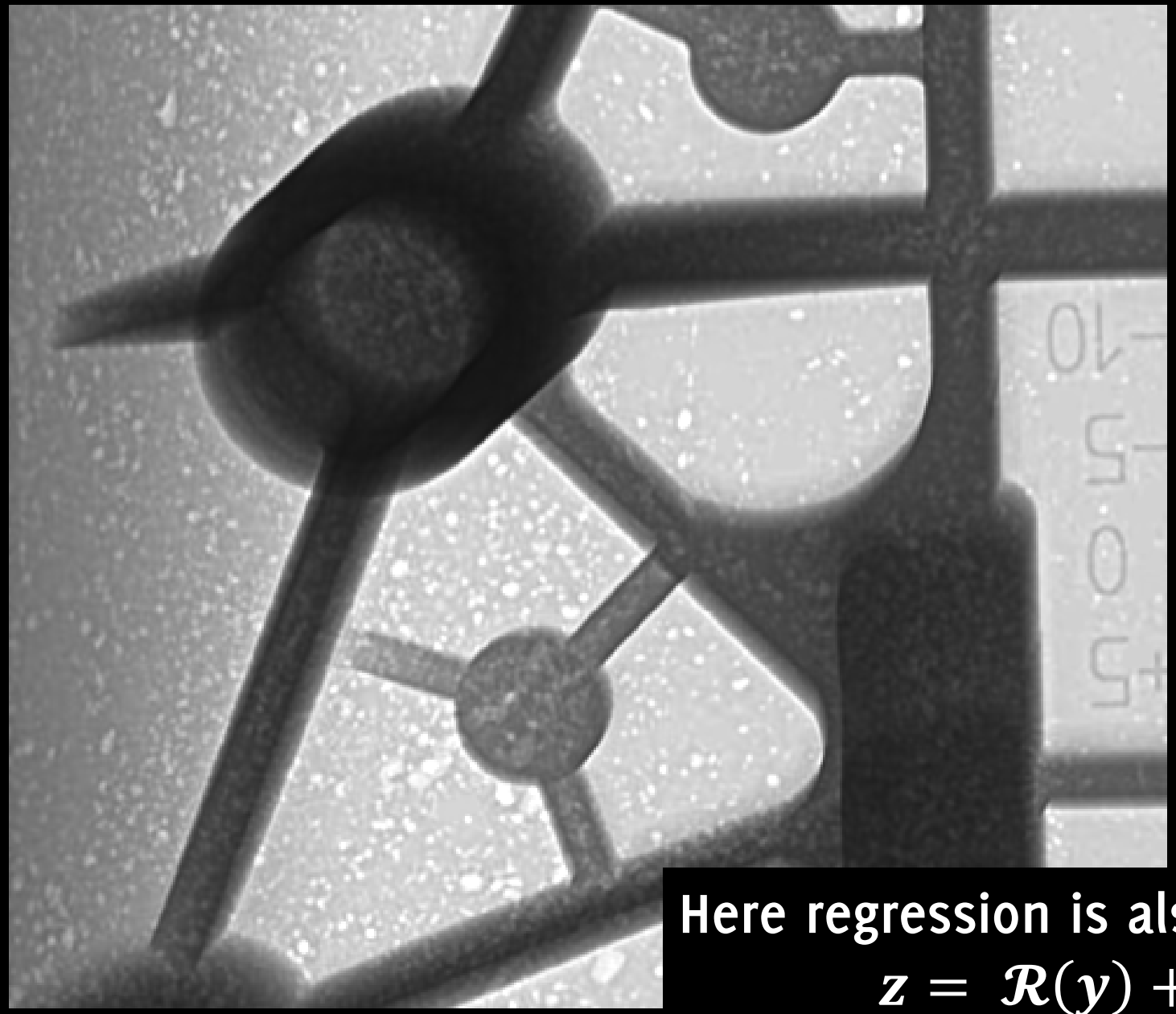
Denoising is a regression problem: given the noisy z, estimate $\hat{y}$ close to the unknown y

Who cares about images? Quality Inspection



G. Boracchi

Who cares about images? Quality Inspection

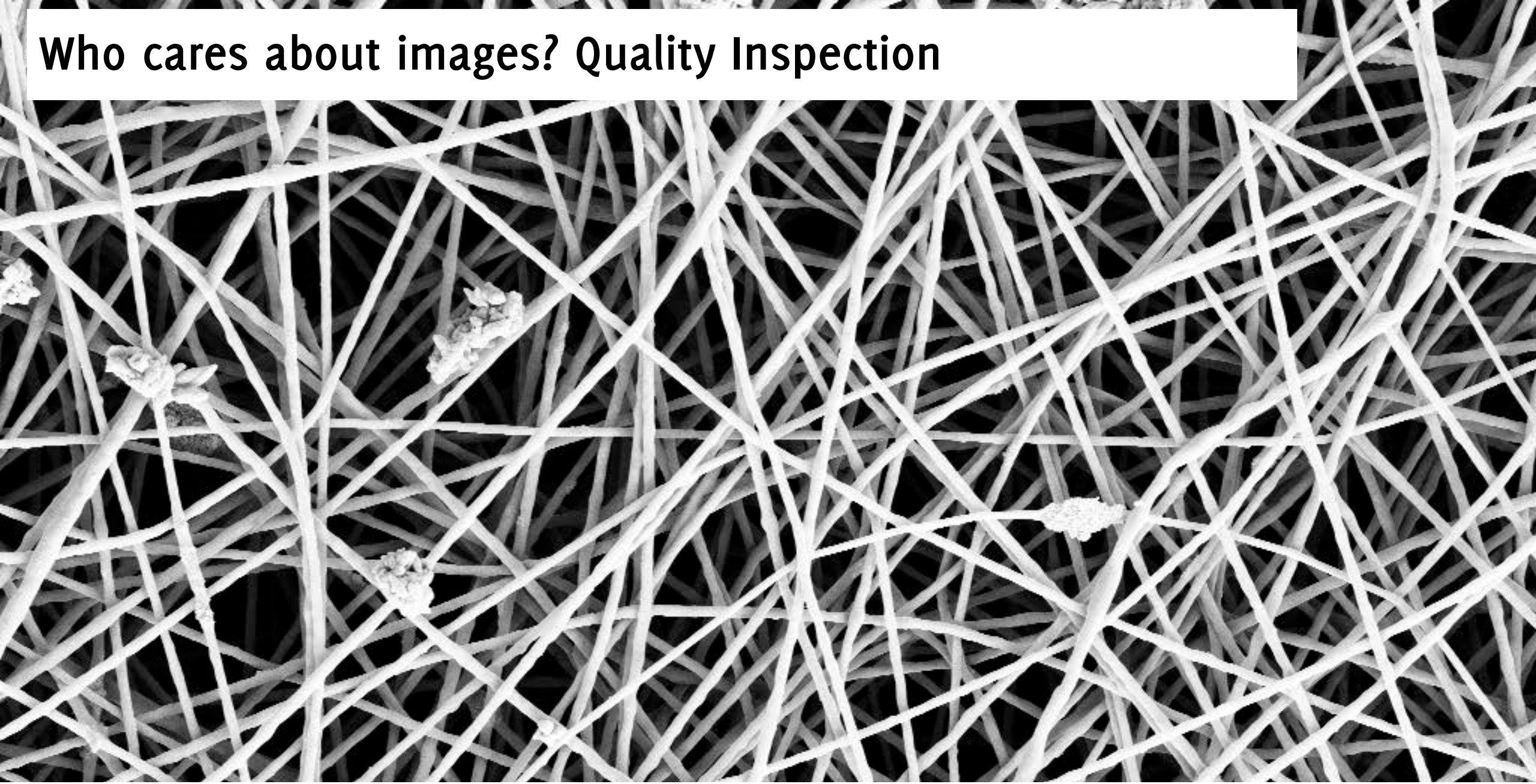


Here regression is also crucial

$$z = \mathcal{R}(y) + \eta$$

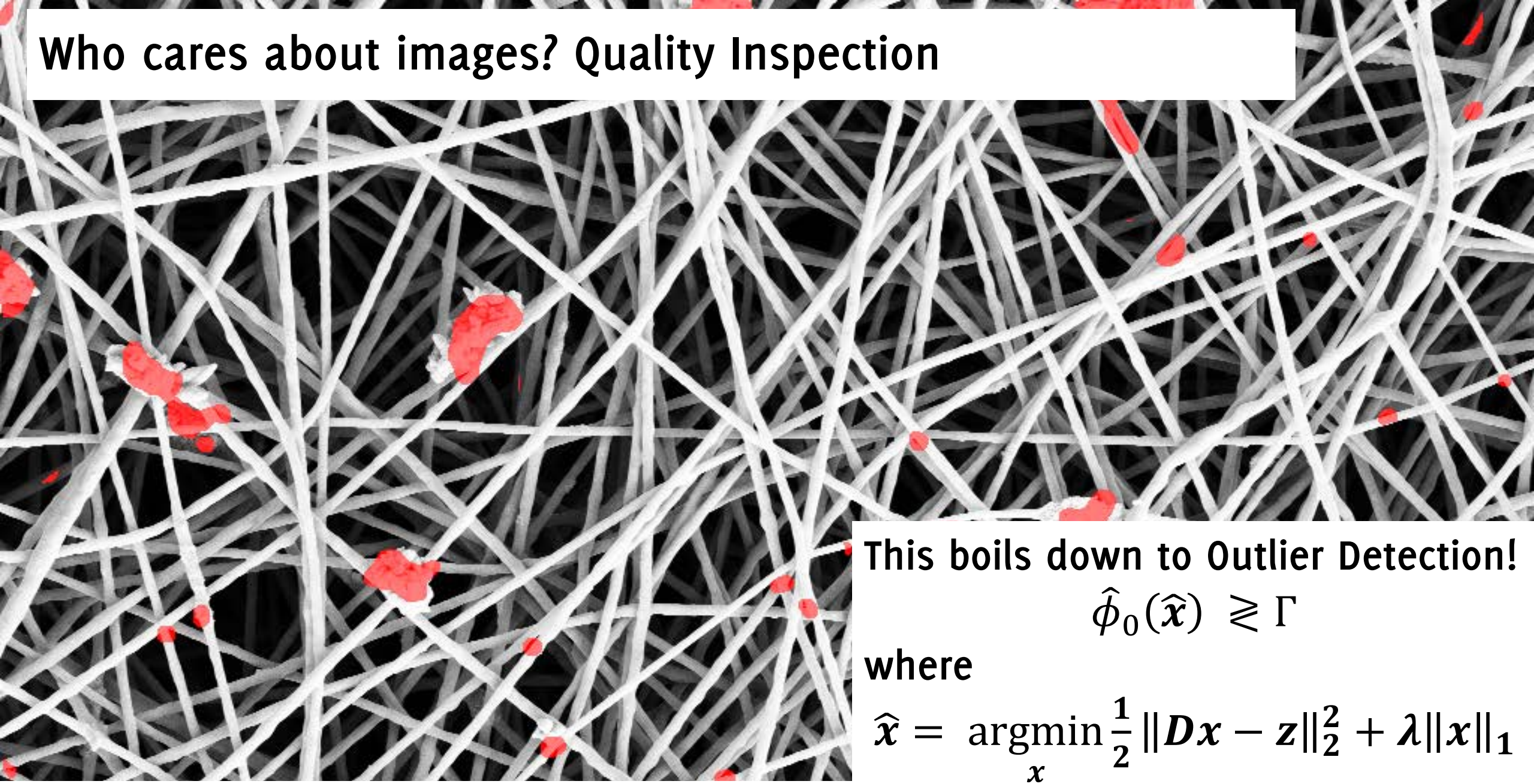
G. Boracchi

Who cares about images? Quality Inspection



Carrera D., Manganini F., Boracchi G., Lanzarone E. *"Defect Detection in SEM Images of Nanofibrous Materials"*, IEEE Transactions on Industrial Informatics 2017, 11 pages, doi:10.1109/TII.2016.2641472

Who cares about images? Quality Inspection



This boils down to Outlier Detection!

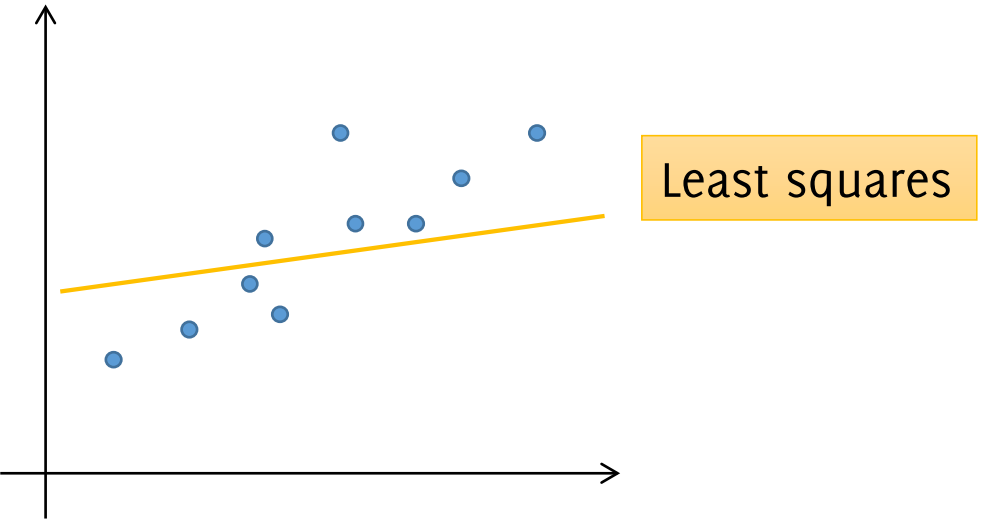
$$\hat{\phi}_0(\hat{\boldsymbol{x}}) \gtrless \Gamma$$

where

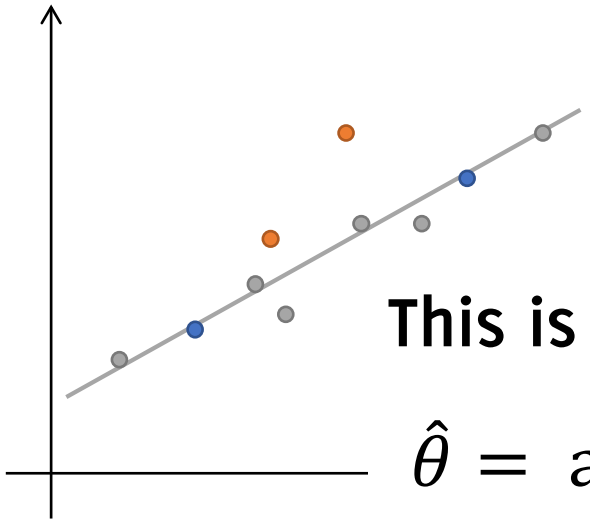
$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \frac{1}{2} \|\boldsymbol{D}\boldsymbol{x} - \boldsymbol{z}\|_2^2 + \boldsymbol{\lambda}\|\boldsymbol{x}\|_1$$

Carrera D., Manganini F., Boracchi G., Lanzarone E. *"Defect Detection in SEM Images of Nanofibrous Materials"*, IEEE Transactions on Industrial Informatics 2017, 11 pages, doi:10.1109/TII.2016.2641472

Who cares about images? *visual recognition systems*



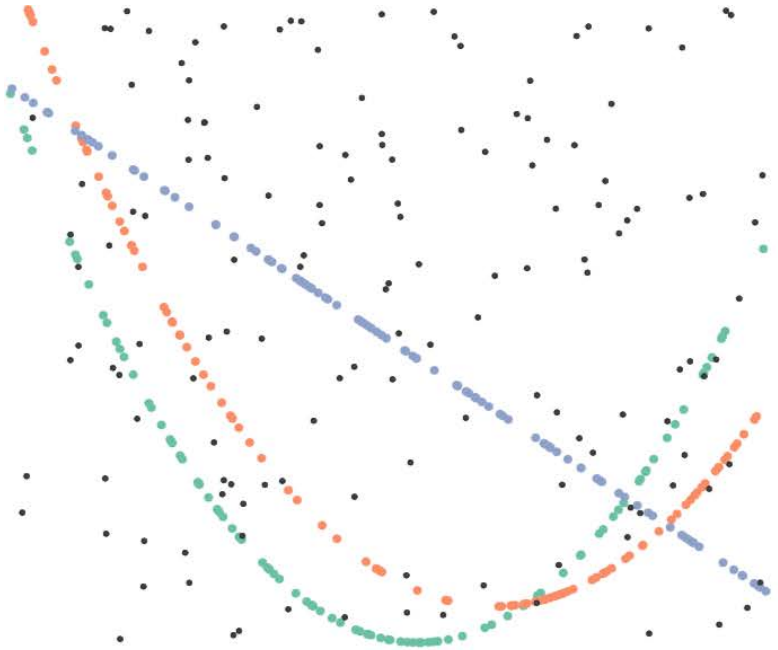
Least squares

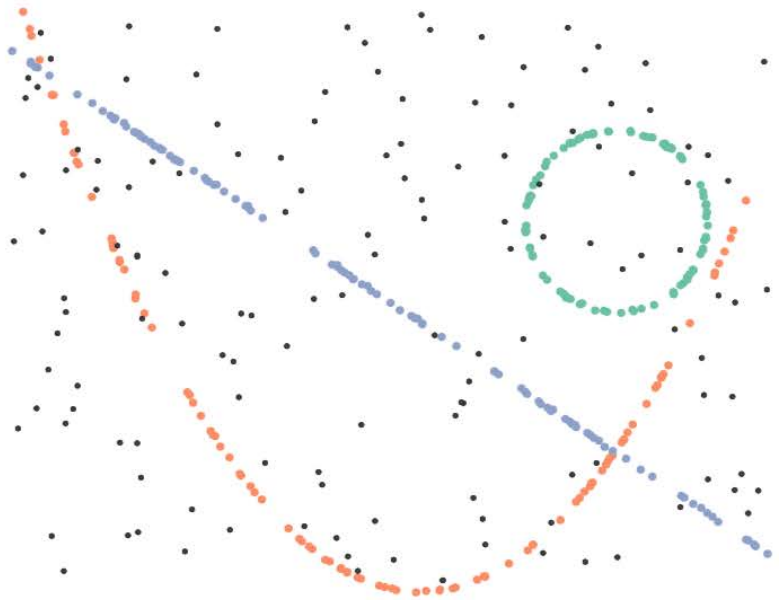


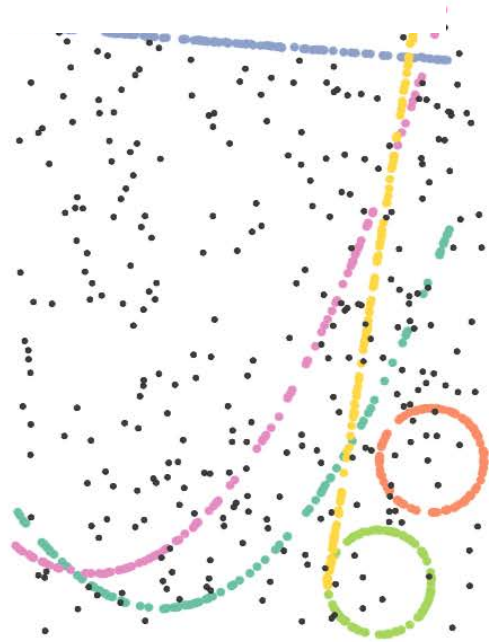
Robust Fit
(RANSAC)

This is a (robust) fitting problem

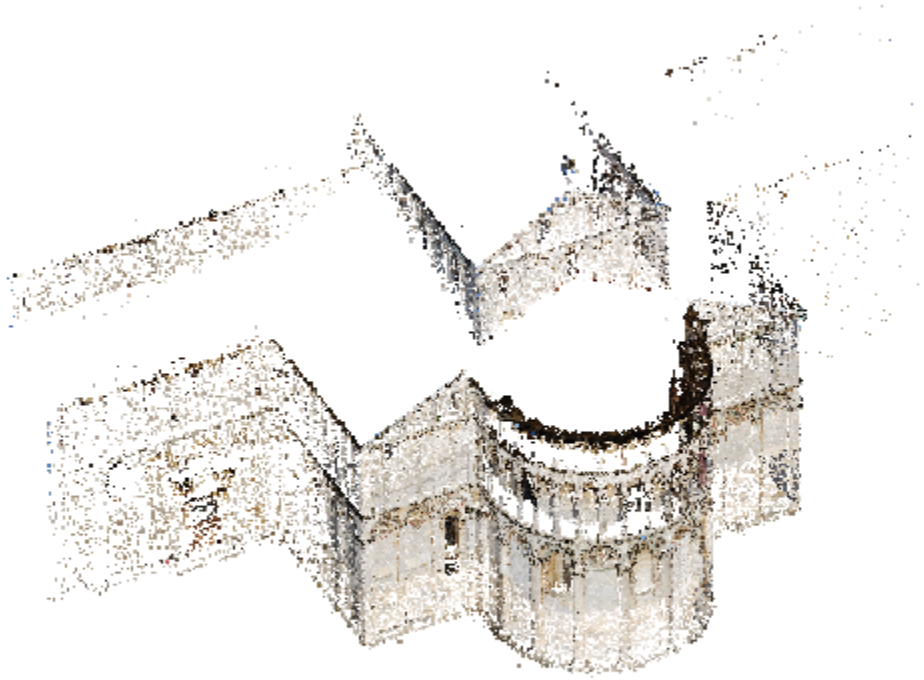
$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \sum_{\boldsymbol{x_i}} \rho(\operatorname{dist}(\boldsymbol{x_i}, \mathcal{M}_\theta))$$



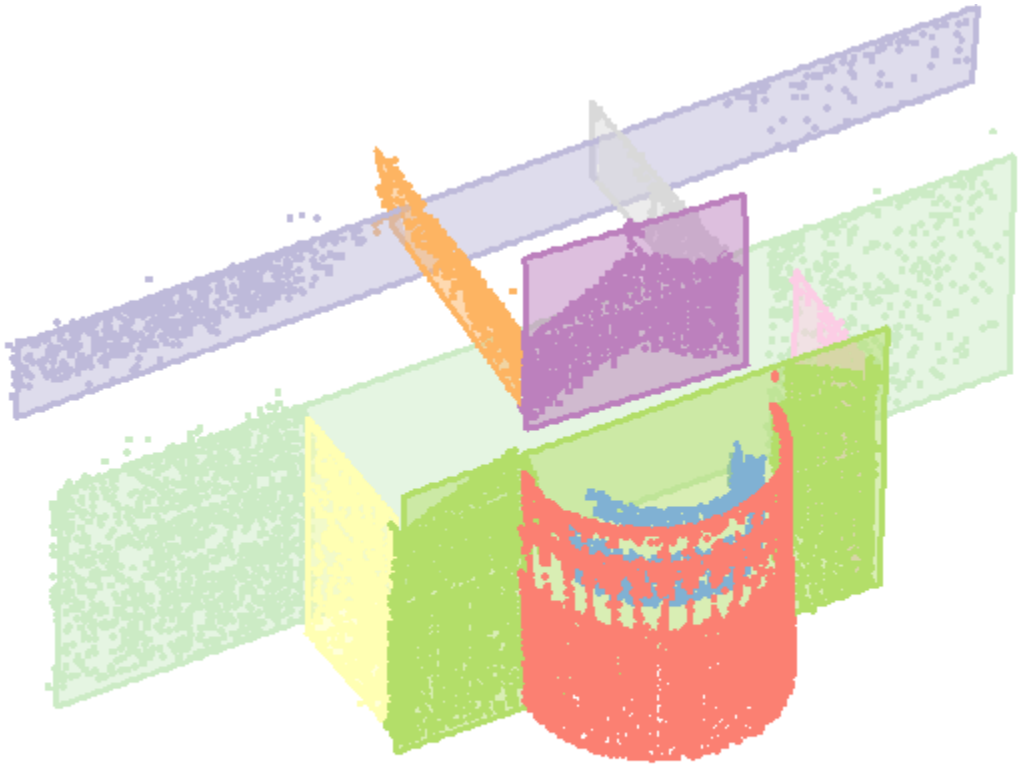




Who cares about images? *visual recognition systems*



(a) Input point cloud



(b) Recovered structures

This is a (robust) fitting problem

Magri, Leveni, Boracchi “MultiLink: Multi-class Structure Recovery via Agglomerative Clustering and Model Selection”, CVPR 2021

Who cares about images? *visual recognition systems*

Who cares about images? *visual recognition systems*

This is a (robust) fitting problem

Is this interesting for a (perspective) Mathematical Engineer?

Is this interesting? Sure!

All the algorithms build upon:

- a **clear problem formulation**
- a **simple mathematical model (…often linear combinations!)**
- Sound mathematical solutions (linear algebra, least squares, convex optimization)

…and the result is not just a number... it's an image!

Ok, to recap

Mathematical Models and Methods for Image Processing (5 CFU)

The primary **goal** *of this laboratory course is to* **let the students design, implement and practice algorithms** *based* **on**

simple mathematical models *from* **linear algebra and convex optimization**,

and **solve** *challenging inverse* **problems in image processing** *(denoising, deblurring, inpainting, anomaly detection)*

Mathematical Models and Methods for Image Processing (5 CFU)

The course **topics include**:

- **Image models based on orthonormal bases** (Fourier, wavelets), **data-driven basis** (PCA, Gram-Schmidt) and **local polynomial approximation**.

- **Sparsity and redundancy**.
  - Away from Orthonormal Basis, redundant set of generators
  - Sparse coding with ℓ^0 (OMP) or ℓ^1 norm (convex optimization ISTA, IRLS, LASSO)
  - Dictionaries yielding sparse representations and dictionary learning (KSVD)
- **Applications of sparse models** to image denoising, inpainting, anomaly detection and classification.

- **Robust fitting** methods (RANSAC, LMEDS, HOUGH) and their sequential counterparts for object detection in images.

Course Organization

Lectures: 20 hours

Laboratory: 30 hours

There will be short theory recap and then you will be invited to develop and practice presented algorithms. Some demo code to fill in will be provided.

Simple assignment provided during lectures, oral exam.

Frequently Asked Questions

Q: Any specific background?

A: linear algebra, statistics and calculus

Q: Any programming skill required?

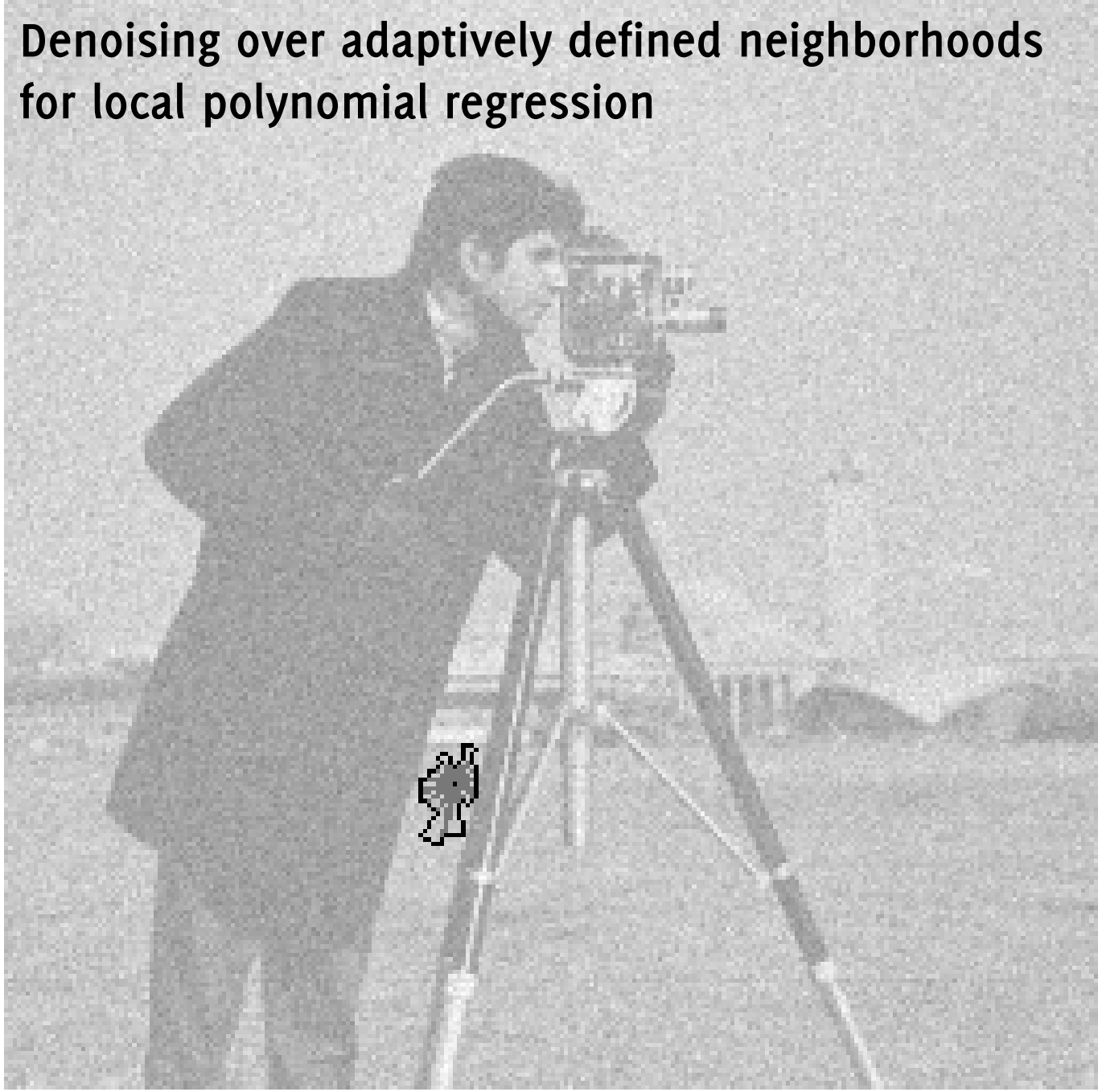
A: Proficiency in Matlab or Python

Q: Plenty of neural networks then?

A: No way. No neural networks allowed here ☺*
Only expert-driven algorithms designed upon a clear mathematical modeling that admits closed-form solutions / sound optimization schemes.

*** Interested in neural networks? Refer to «Artificial Neural Network and Deep Learning» in the first semester**

Questions?


Denoising over adaptively defined neighborhoods
for local polynomial regression