

Assignment 1

Angela Zhai

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Document assignment

Consider the following situation:

A sloppy printer produces books with an average of 2 misprints per page. You want to know how many pages have more than k misprints in a book of n pages. Make an $n \times k$ table that shows the relationship between the total number of pages in a book and the number of pages with k misprints.

Show and explain your work. Include equations and calculations to teach the reader how to solve the problem. Include an image of a book.

Push your solution to a github repository and submit the url for repository on blackboard. Be sure your repo includes your document as a pdf file and as an RMD file. Include other files needed to recompile your document.

Result

Table 1: Probability of pages with k misprints

| | | Least Misprints | | | | | |
|-------------|-----|-----------------|--------|--------|--------|--------|--------|
| | | 2 | 3 | 4 | 5 | 6 | 7 |
| Total Pages | 100 | 0.5918 | 0.5918 | 0.5918 | 0.5918 | 0.5918 | 0.5918 |
| | 200 | 0.3166 | 0.3166 | 0.3166 | 0.3166 | 0.3166 | 0.3166 |
| | 300 | 0.1359 | 0.1359 | 0.1359 | 0.1359 | 0.1359 | 0.1359 |
| | 400 | 0.0491 | 0.0491 | 0.0491 | 0.0491 | 0.0491 | 0.0491 |
| | 500 | 0.0139 | 0.0139 | 0.0139 | 0.0139 | 0.0139 | 0.0139 |
| | 600 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 |

For this question, I think the probability of “more than k misprints” is invariant for a specified k . But if we would like to find out the probability of total misprints within n pages book, the situation follows poisson distribution.

The table above is generated by simulation. A vector contains 10000 random elements which follow poisson distribution, and lamda equal 2. The proportion of numbers greater than k in 10000 elements is nearly identical to the theoritical probability.

Another table

I also consider the situation of least errors appear within n pages, so the cumulative probability of poisson distribution is needed.

The equation to get CDF of poisson distribution is: $e^{-\lambda} \sum_{i=0}^k \frac{\lambda^i}{i!}$

Like if we get more than 100 misprints in a book of 100 pages, the equation should look like: $e^{-2} \sum_{i=100}^n \frac{2^i}{i!}$

Table 2: Probability of total misprints within n pages

| | | pages | | | | | |
|-----------------|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| | | 100 | 200 | 300 | 400 | 500 | 600 |
| Least Errors | 100 | 0.5939942 | 0.8646647 | 0.8646647 | 0.8646647 | 0.8646647 | 0.8646647 |
| | 200 | 0.3233236 | 0.5939942 | 0.8646647 | 0.8646647 | 0.8646647 | 0.8646647 |
| | 300 | 0.1428765 | 0.5939942 | 0.5939942 | 0.8646647 | 0.8646647 | 0.8646647 |
| | 400 | 0.0526530 | 0.3233236 | 0.5939942 | 0.5939942 | 0.8646647 | 0.8646647 |
| | 500 | 0.0165636 | 0.3233236 | 0.5939942 | 0.5939942 | 0.5939942 | 0.8646647 |
| | 600 | 0.0045338 | 0.1428765 | 0.3233236 | 0.5939942 | 0.5939942 | 0.5939942 |
| | 700 | 0.0010967 | 0.1428765 | 0.3233236 | 0.5939942 | 0.5939942 | 0.5939942 |
| | 800 | 0.0002374 | 0.0526530 | 0.3233236 | 0.3233236 | 0.5939942 | 0.5939942 |

The main part of the code is: `#ppois(q=freq, lamda=2, lower.tail=FALSE)`. Function `ppois` is used to calculate CDF for poisson distribution; `q` assigns the average errors for one page in a book of `n` pages; `lamda` assigns average misprints per page of this printer; `lower.tail=FALSE` means we try to get the cumulated value for errors more than `q`.

Image

