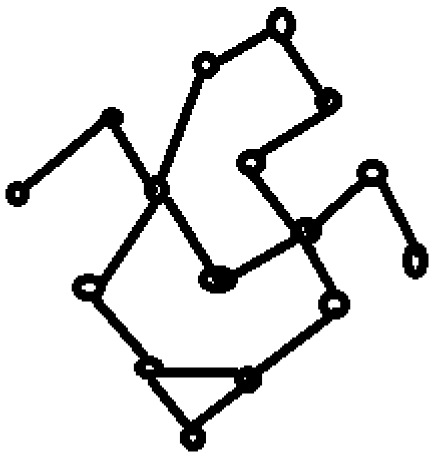
ECE227 HW1

1. 9!/2! + 12!/2! = 102

2.



For such a rhombus, the diameter is determined by the distance between the diagonal vertices. In each side of thombus we have k nodes. The path from one vertex to another can be understand as crossing 2 edges of the rhombus, which means passing k-1 + k-1 = 2k-2 nodes. And thus we get $2\sqrt{n}$-2, close to $2\sqrt{n}$, as diameter.

3. a.

	1	2	3	4	5
1	0	1	0	0	1
2	0	0	1	0	0
3	1	0	0	0	0
4	1	0	0	0	1
5	0	0	1	1	0

b. 1 → 2 → 5

2 → 3

3 → 1

4 → 1 → 5

5 → 3 → 4

4. Scenario 1: When n=1 and 2, we let N=3 where (N+1-n) out of N edges are self-connected edges. Therefore each node connects to (n-1) other nodes and (N+1-n) times to itself, (n-1) + (N+1-n) = N = 3 nodes in total, and they are all inter-connected. Thus clustering coefficient is 1.

Scenario 2: When n⩾3, that is when N>3, we let n = 3k+t where t=0, 1, or 2. We construct a graph that consists of k separated triangles and t nodes. If t=0, the graph is simply k independent triangles; otherwise, we treat that t nodes in a same way as in scenario 1, which is drawing 3 edges, including self-connected edges. In this way, the total edges of the graph is 3k+3 or 3k = O(N). In each triangle, all 3 nodes are interconnected, so each of them has a clustering coefficient as 1. For the t nodes aside, according to scenario 1, they can have clustering coefficients as 1 with respect to self-connected edges.

5. a. the graph has $\binom{n}{3}$ sets of triangle vertices (with or without edges). To form a triangle, all 3 edges should occur, with a probability of p^3. The number of expected triangles is $\binom{n}{3} * p\char`^3 = \binom{n}{3} * (\frac{c}{n-1})\char`^3 = (\frac{c\char`^3}{6}) * (\frac{n^2-2n}{n^2-2n+1})$. When n is very large, $\frac{n^2-2n}{n^2-2n+1} \approx 1$. Therefore we get $\frac{c\char`^3}{6}$.

b. To form a connected triple, 2 edges should occur. $3 * \binom{n}{3} * p\char`^2 = (\frac{c\char`^2}{2}) *$

($\frac{n(n-2)}{n-1}$). When n is large, $\frac{n(n-2)}{n-1}$ $\approx$ 1. Thus we get $\frac{1}{2}nc$^2.

c. CI(G) = 3*$\frac{c\text{^}3}{6}/\frac{1}{2}nc$^2 = $\frac{c}{n}$, when n is large n $\approx$ n-1, $CI(G) = \frac{c}{n-1}$ $\approx$ p