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Заг. Докажете, че L е к.-с. $\Sigma = \{a, b\}$

$$L = \{w \mid w \in \Sigma^* \wedge w = w^{rev}\}$$

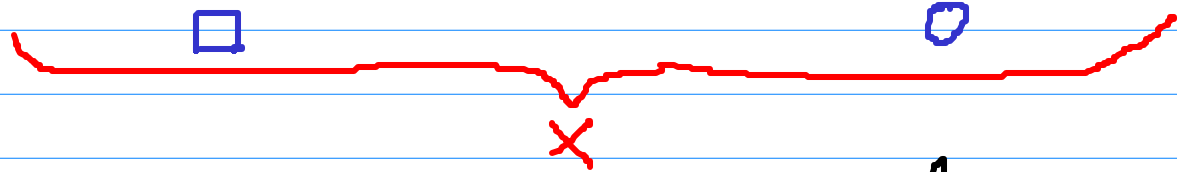
$$G: S \rightarrow aSa \mid bSb \mid a \mid b \mid \varepsilon$$

$$L(G) = L$$

$$1) L(G) \subseteq L$$

$$(\forall w \in \Sigma^*) (w \in L(G) \Rightarrow w \in L)$$

$$S^* \rightarrow \{uSu^{rev} \mid u \in \Sigma^*\} \cup \{u \mid u \in \Sigma^* \wedge u = u^{rev}\}$$



Унг. по дължината на извода

$$\text{База: } 0 \text{ стъпки} \quad S \xrightarrow{0} S = \varepsilon S \varepsilon^{rev} \quad \square$$

У.П. Нека за k стъпки:

$$S \xrightarrow{k} uSu^{rev} \quad \text{1 ст.}$$

$$S \xrightarrow{k} u \quad (u = u^{rev}) \quad \text{2 ст.}$$

И.С: разделим на $k+1$ столбца

1 лн. $S \xrightarrow{k} u \leq u^{rev}$

$$\begin{array}{lcl}
 & \nearrow & \overbrace{ua}^w \leq \overbrace{a u^{rev}}^{w^{rev}} \quad \square \quad (*) \\
 & \rightarrow & u \leq u^{rev} \quad \square \\
 & \searrow & u \leq u^{rev} \quad \square \\
 & \downarrow & u \leq u^{rev} \quad \square \quad (***) \\
 & \rightarrow & u \leq u^{rev} \quad \square
 \end{array}$$

(*) $(wa)^{rev} = aw^{rev} \quad (a \in \Sigma, w \in \Sigma^*)$

(**) $(u \leq u^{rev}) = u^{rev^{rev}} \leq u^{rev} = u \leq u^{rev}$
 $(b \in \Sigma, u \in \Sigma^*)$

2 лн. $S \xrightarrow{k} u \rightarrow X \quad (k+1 \text{ столбца не ма})$

$\Rightarrow S \xrightarrow{*} X$

$S \xrightarrow{*} X \quad X \cap \Sigma^* = \{u \in \Sigma^* \mid u = u^{rev}\}$

$\Rightarrow L(G) \subseteq L$

$$L \subseteq L(G)$$

$$\forall w \in L \quad (S \xrightarrow{*} w)$$

индукция по дължината на думата

база:

$$|w| = 0$$

$$w = \varepsilon$$

$$S \rightarrow \varepsilon$$

$$|w| = 1$$

$$w = a$$

$$S \rightarrow a$$

$$w = b$$

$$S \rightarrow b$$

И.П. Допускаме, че $\forall u \in L \quad |u| \leq k$
 $k \geq 2$
 $S \xrightarrow{*} u$

И.С. Разглеждаме произволна дума

$$w \in L \quad |w| = k$$

$$(w = w^{\text{rev}})$$

$$\text{1 сл.} \quad w = aua \quad u = u^{\text{rev}} \wedge |u| = k-2 < k$$

$$S \rightarrow aSa \xrightarrow{\text{И.П.}} aua = w \quad \checkmark$$

2 сл. $w = v u v$ $u = u^{rev} \wedge |u| = k-2 \leq k$

$S \rightarrow v S v \xrightarrow{u.p.} v u v = w \quad \checkmark$

$\Rightarrow L \subseteq L(G)$

$\Rightarrow L = L(G)$

Тверждение:

Всеки рег. язык е к.-с.:

Структурна индукция:

БАЗА	\emptyset	$S \rightarrow S$
	$\{a\}$	$S \rightarrow a \quad \checkmark$
СТЕПЕН	\cup	1)
	\cdot	2)
	$*$	3)

$$1) G_1 = \langle V_1, \Sigma, S_1, R_1 \rangle$$

$$V_1 \cap V_2 = \emptyset$$

$$G_2 = \langle V_2, \Sigma, S_2, R_2 \rangle$$

ТЪРСЕЛИМ $G: L(G) = L(G_1) \cup L(G_2)$

$$G = \langle V_1 \cup V_2 \cup \{S\}, \Sigma, S, R_1 \cup R_2 \cup \{S \rightarrow s_1 | s_2\} \rangle$$

$$S \notin V_1 \cup V_2$$

Зад. Докажете, че L е к.-с.

$$L = \{a^n b^k \mid n \neq k\}$$

$$L = \{a^n b^k \mid n < k\} \cup \{a^n b^k \mid n > k\}$$

$$S_1 \rightarrow aS_1b \mid S_1b \mid b$$

$$S_2 \rightarrow aS_2b \mid aS_2 \mid a$$

Доказателството
е различно
с групата

Доказана е.

$G:$	$S \rightarrow S_1 \mid S_2$ $S_1 \rightarrow aS_1b \mid S_1b \mid b$ $S_2 \rightarrow aS_2b \mid aS_2 \mid a$
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$$2) \quad G_1 = \langle V_1, \Sigma, S_1, R_1 \rangle$$

$$V_1 \cap V_2 = \emptyset$$

$$G_2 = \langle V_2, \Sigma, S_2, R_2 \rangle$$

$$\text{Требуем } G : L(G) = L(G_1) \cdot L(G_2)$$

$$G = \langle V_1 \cup V_2 \cup \{S\}, \Sigma, S, R_1 \cup R_2 \cup \{S \rightarrow S_1 S_2\} \rangle$$

$$S \notin V_1 \cup V_2$$

зад. Докажете, че L е к.-с.

$$L = \{a^n b^m a^k \mid m = n + k\}$$

$$L = \{a^n b^n \mid n \in \mathbb{N}\} \cdot \{b^n a^n \mid n \in \mathbb{N}\}$$

$$\downarrow$$

$$S_1 \rightarrow a S_1 b \mid \varepsilon$$

доказана е!

$$\downarrow$$

$$S_2 \rightarrow b S_2 a \mid \varepsilon$$

аналогично
доказателс; во

$$G: \begin{array}{|l} S \rightarrow S_1 S_2 \\ S_1 \rightarrow a S_1 b \mid \varepsilon \\ S_2 \rightarrow b S_2 a \mid \varepsilon \end{array}$$

$$3) G_1 = \langle V_1, \Sigma, S_1, R_1 \rangle$$

$$\text{Требуем } G : L(G) = L(G_1)^*$$

$$G = \langle V_1 \cup \{S\}, \Sigma, S, R_1 \cup \{S \rightarrow S_1 S \mid \varepsilon\} \rangle$$

$$S \notin V_1$$

309 \Rightarrow покажите, что L — к.-с.

$$L = \{ a^{n_1} b^{n_1} a^{n_2} b^{n_2} \dots a^{n_k} b^{n_k} \mid n_i \in \mathbb{N} \}$$

$$L = \{ a^n b^n \mid n \in \mathbb{N} \}^*$$

\downarrow

$$S_1 \rightarrow a S_1 b \mid \varepsilon \quad (\text{Доказана е})$$

$$G: \begin{array}{l} S \rightarrow S_1 S \mid \varepsilon \\ S_1 \rightarrow a S_1 b \mid \varepsilon \end{array}$$

Покажем, че всеки рег. език е к.-с.

зад Постройте безконтекстна граматика

за $L = \{w \mid w \text{ започва с } a\}$

$a(a+b)^*$

$\underline{S_1} \rightarrow a$
 $\underline{S_2} \rightarrow b$

1) $+$ $\left| \begin{array}{l} \underline{S_3} \rightarrow S_1 S_2 \\ S_1 \rightarrow a \\ S_2 \rightarrow b \end{array} \right.$

2) $*$ $\left| \begin{array}{l} \underline{S_4} \rightarrow S_3 S_4 \mid \epsilon \\ S_3 \rightarrow S_1 S_2 \\ S_1 \rightarrow a \\ S_2 \rightarrow b \end{array} \right.$

3) | S_5 $\rightarrow a$

① преименуване

S_6 $\rightarrow S_5 S_4$

$S_5 \rightarrow a$

$S_4 \rightarrow S_3 S_4 \mid \epsilon$

$S_3 \rightarrow S_1 \mid S_2$

$S_1 \rightarrow a$

$S_2 \rightarrow b$