(5)

Минимален автомат

Релация на Майхил – Нероуд:

Нека $L \subseteq \Sigma^*$ $R_L \subseteq \Sigma^* \times \Sigma^*$

$x R_L y \leftrightarrow \forall z \in \Sigma^* \; xz \in L \leftrightarrow yz \in L$

↳ релация на еквивалентност

Класовете на екв. са състоянията на минималният автомат за L.

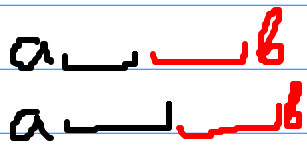
зад 1 $\Sigma = \{a, b\}$

- $L = \{w \in \Sigma^* \mid w$ започва и завършва с различна буква$\}$

Класове на екв. на R_L?

$a \not R_L b$? не!

$[a] = \{w \mid w$ започва с a и завършва с $a\}$

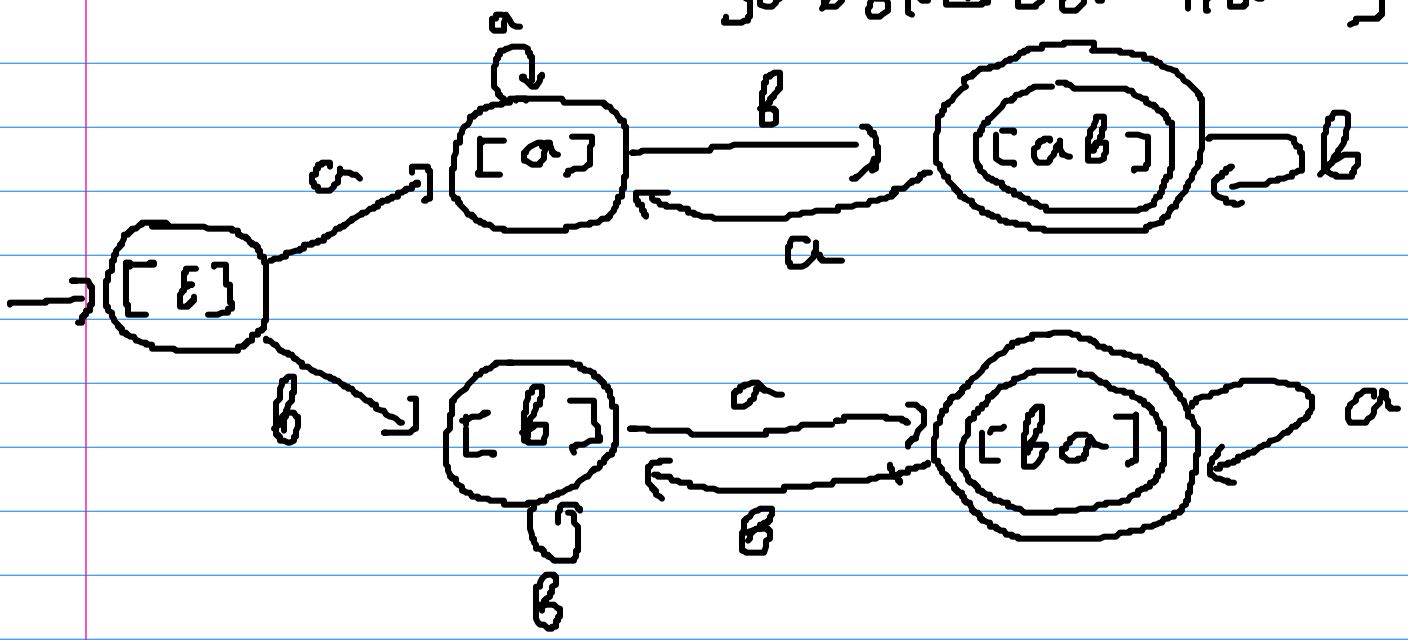


$[b] = \{w \mid w$ започва с b и завършва с $b\}$

$[\varepsilon] = \{\varepsilon\}$

$[ab] = \{w \mid w \text{ започва с } a \text{ и завършва с } b\} \subseteq F$

$[ba] = \{w \mid w \text{ започва с } b \text{ и завършва на } a\} \subseteq F$

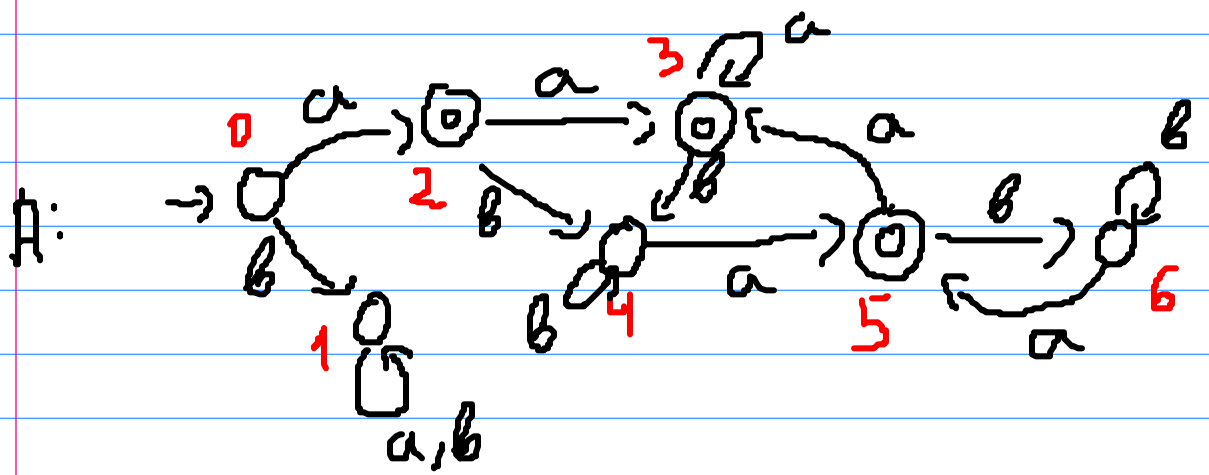


Минимизация

$A \to КА \to A \longrightarrow Min(A)$ { минимален за езика }

$L(A) = L(Min(A))$

Пример за автомат, който не е минимален:



$Min(A) = ?$

$L_A(q_4) = \{a, aa, aba \dots\}$

$L_A(q_1) = \emptyset$

$L_A^2(q_5) = \{\varepsilon, a, aa\}$

$L_A^1(q_4) = \{a\}$

$L_A^3(q_5) = L_A^2 \cup \{aaa, aba, baa, bba\}$

$L(P) = \{ w \in \Sigma^* \mid \delta^*(p, w) \in F \}$

$L(A) = L(q_0)$ Б.О.О. ст. свст $\overset{e}{\underset{c}{}}$ инд. 0

← Търся кл. на еквивалентност

$p \equiv_A q \leftrightarrow L(p) = L(q)$ (рел. екв)

↓ Апроксимираме

$p \equiv_A^n q \leftrightarrow L_A^n(p) = L_A^n(q)$ (рел. екв.)

$L_A^n(q) = \{ w \mid |w| \leq n \wedge \delta^*(q, w) \in F \}$

$\equiv_A^n$ е апроксимация на $\equiv_A$

$\equiv_A^{n+1}$ е по-фина от $\equiv_A^n$

Търсим такова $n \in \mathbb{N}$: $\equiv_A^n = \equiv_A^{n+1}$

Тогава: $\equiv_A^n = \equiv_A$

Алгоритъм: $\equiv_A^0 \neq \equiv_A^1 \neq \dots \neq \equiv_A^k \neq \underbrace{\equiv_A^{k+1} = \equiv_A^{k+2} = \equiv_A^{k+3} \dots}_{\equiv_A}$

(!) $p \equiv_A^{n+1} q \leftrightarrow p \equiv_A^n q \wedge (\forall a \in \Sigma)[\delta(p, a) \equiv_A^n \delta(q, a)]$

брой кл. на екв.

Ако $\equiv_A^n \neq \equiv_A^{n+1}$ $\Rightarrow |\equiv_A^n| < |\equiv_A^{n+1}|$

За произволен автомат

$A = \langle Q, \Sigma, s, F, \delta \rangle$

Класове на екв на $\equiv_A^0$?

F и $Q \setminus F$

Решение по горната задача

$\equiv_A^0 \rightarrow S_1 = \{0, 1, 4, 6\}$ $S_2 = \{2, 3, 5\}$

Тогава $\equiv_A^1$

	a	b
0	S_2	S_1
1	S_1	S_1
4	S_2	S_1
6	S_2	S_1
2	S_2	S_1
3	S_2	S_1
5	S_2	S_1

$\rightarrow \{1\}$

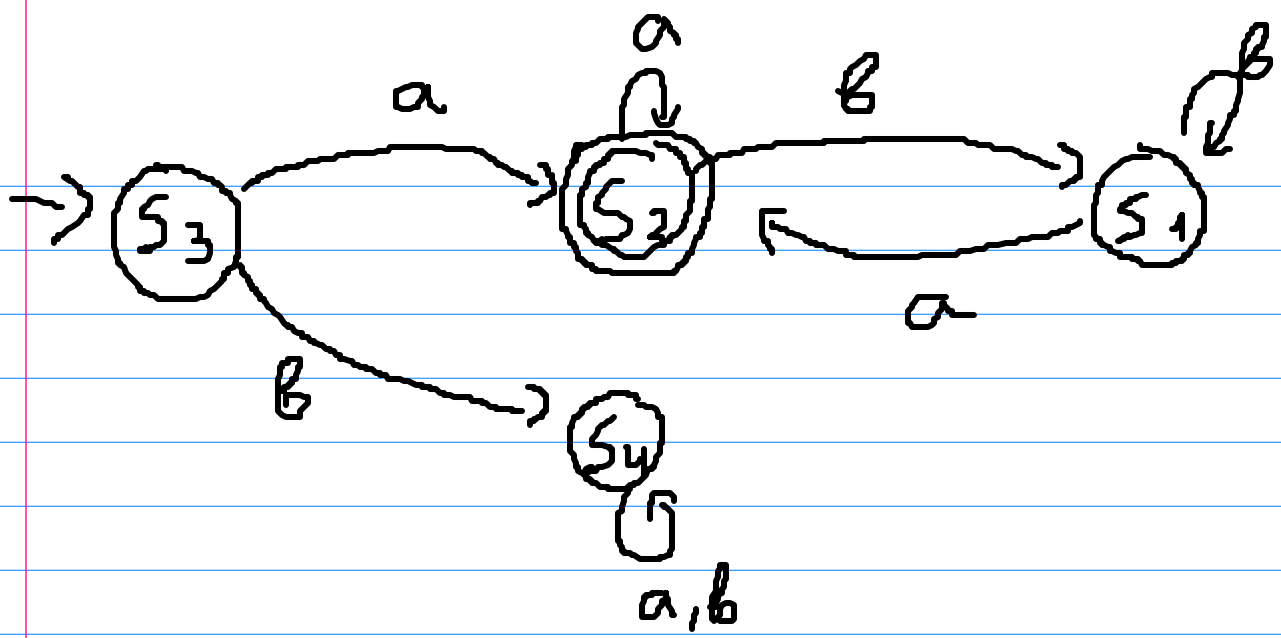
$\equiv_A^1$ $S_1 = \{0,4,6\}$, $S_2 = \{2,3,5\}$, $S_3 = \{1\}$

		a	b	
S_1	0	S_2	S_3	→
	4	S_2	S_1	
	6	S_2	S_1	
S_2	2	S_2	S_1	
	3	S_2	S_1	
	5	S_2	S_1	

$\equiv_A^2$ $S_1 = \{4,6\}$, $S_2 = \{2,3,5\}$, $S_3 = \{0\}$, $S_4 = \{1\}$

		a	b	
S_1	4	S_2	S_1	S_1
	6	S_2	S_1	
S_2	2	S_2	S_1	S_2
	3	S_2	S_1	
	5	S_2	S_1	
S_3	0	S_2	S_4	S_3
S_4	1	S_4	S_4	S_4

$\Rightarrow$ $\equiv_A^3 = \equiv_A^2$ $\Rightarrow$ $\equiv_A^2 = \equiv_A$



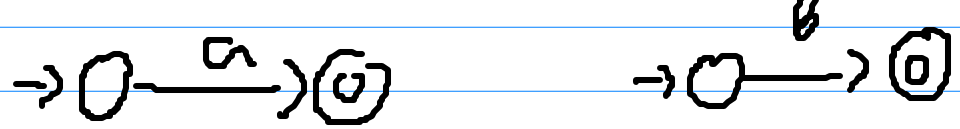
Зад Постройте минимален тотален детерминиран автомат за

$$L = \{a b^n \mid n \in \mathbb{N} \wedge n \text{ е нечетно}\}$$

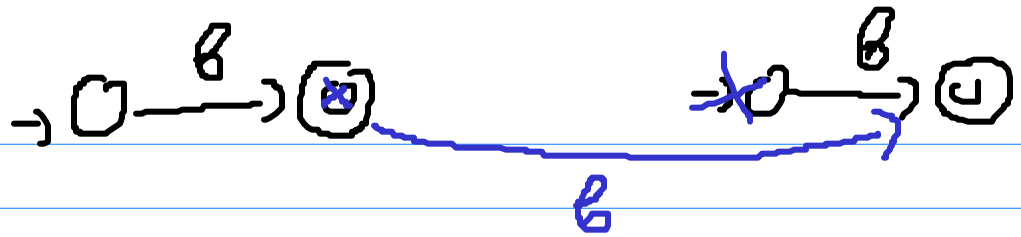
1. рег. израз

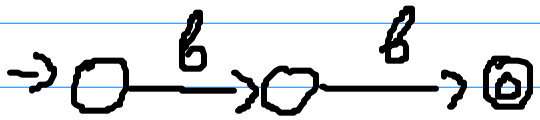
$$a(bb)^* b$$

2. Строим автомата

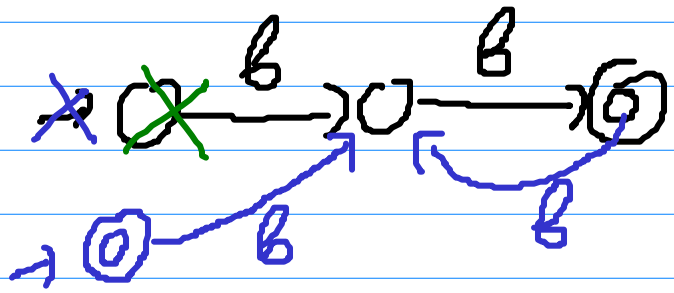


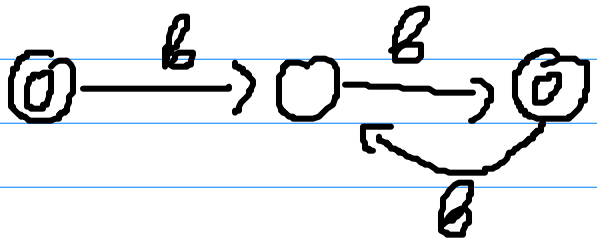
bb



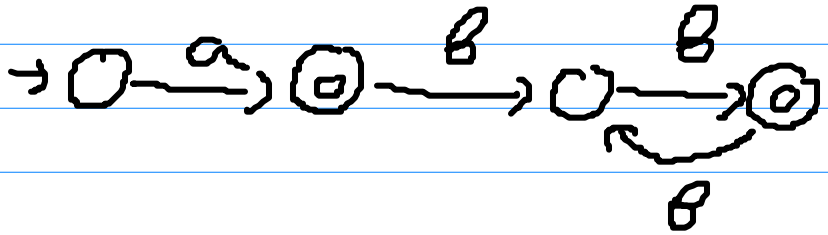


$(bb)^*$

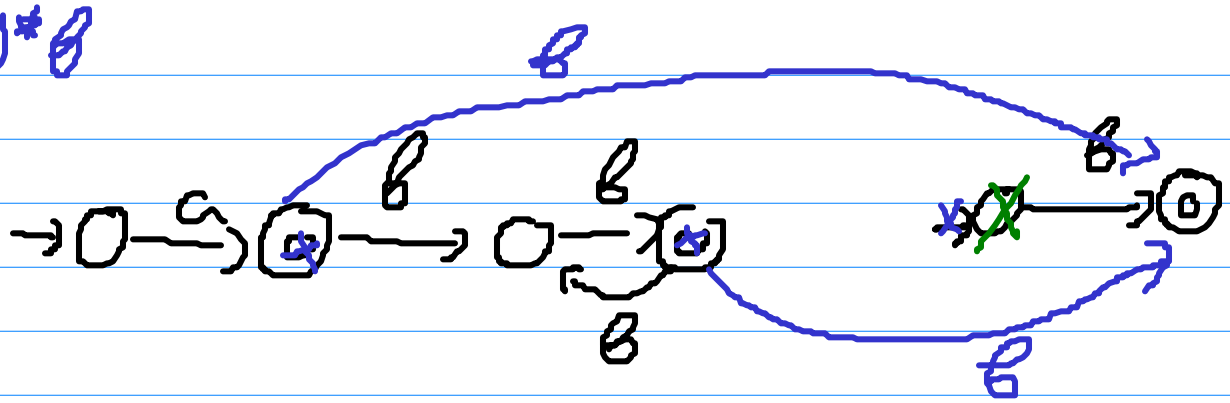




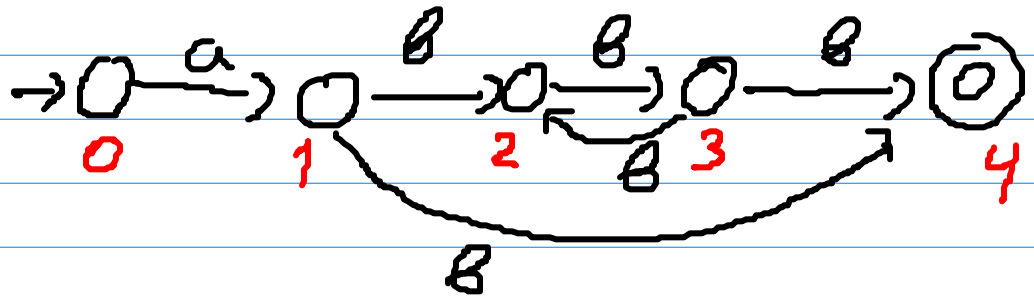
$a(bb)^*$



$a(bb)^*b$

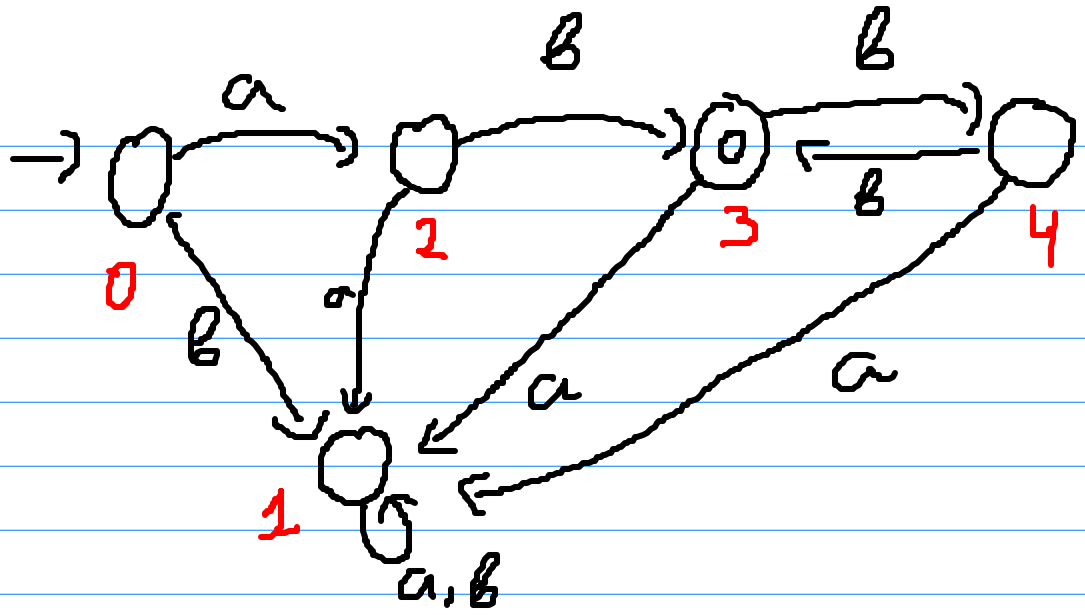


A:



III Тотален и детерминистичен

		a	b
0	{0}	{1}	∅
1	∅	∅	∅
2	{1}	∅	{2,4}
3	{2,4}	∅	{3}
4	{3}	∅	{2,4}



det(A)

IV минимален

$\equiv_A^0$ $S_1 = \{0, 1, 2, 4\}$ $S_2 = \{3\}$

$Q \setminus F$ F

		a	b
S_1	0	S1	S1
	1	S1	S1
	1	S1	S2
	4	S1	S2

$\equiv_A^1$ $S_1 = \{0, 1\}$, $S_2 = \{2, 4\}$ $S_3 = \{3\}$

		a	b
S_1	0	S_1	S_1
	1	S_1	S_1
S_2	2	S_1	S_3
	4	S_1	S_3

$\equiv_A^2$ $S_1 = \{0\}$, $S_2 = \{1\}$, $S_3 = \{2,4\}$, $S_4 = \{3\}$

		a	b
S_3	2	S_2	S_4
	4	S_2	S_4
S_1	0	S_3	S_2
S_2	1	S_2	S_2
S_4	3	S_2	S_3

$\equiv_A^2 = \equiv_A^3$

$\Rightarrow \equiv_A^2 = \equiv_A$