

5

Минимален автомат

Релация на Майхил-Нероу:

Нека $L \subseteq \Sigma^*$ $R_L \subseteq \Sigma^* \times \Sigma^*$

$$x R_L y \Leftrightarrow \forall z \in \Sigma^* \quad xz \in L \Leftrightarrow yz \in L$$

\hookrightarrow релация на еквивалентност

Класовете на екв. са състоянията на минималният автомат за L .

заг \downarrow $\Sigma = \{a, b\}$

$L = \{w \in \Sigma^* \mid w \text{ започва и завършва с различна буква}\}$

Класове на екв. на R_L ?

$a R_L b$? не!

$[a] = \{w \mid w \text{ започва с } a \text{ и завършва с } a\}$

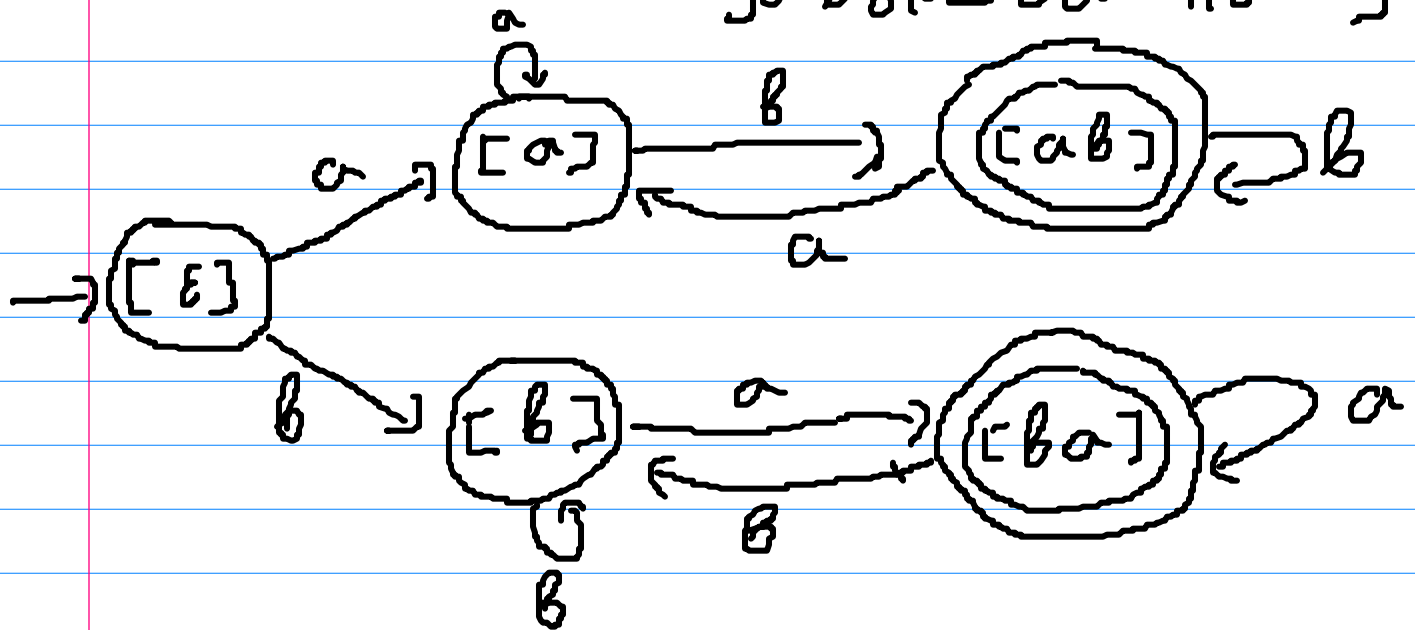
$a \text{ — } b$
 $a \text{ — } b$

$[b] = \{w \mid w \text{ започва с } b \text{ и завършва с } b\}$

$$[\varepsilon] = \{\varepsilon\}$$

$$[ab] = \left\{ w \mid \begin{array}{l} w \text{ започва с } a \\ \text{и завършва с } b \end{array} \right\} \text{sf}$$

$$[ba] = \left\{ w \mid \begin{array}{l} w \text{ започва с } b \\ \text{и завършва на } a \end{array} \right\} \text{sf}$$

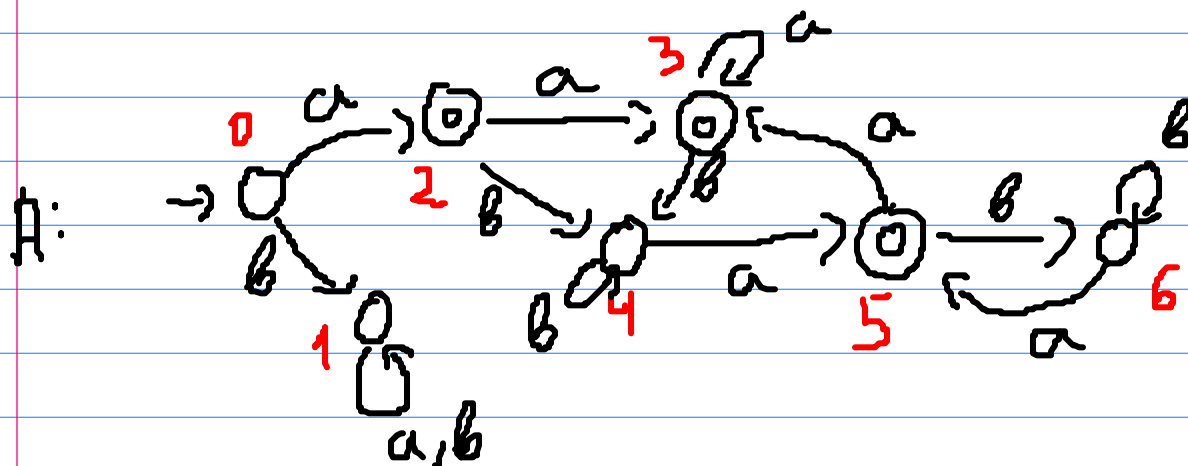


Минимизация

$$A \rightarrow K \rightarrow A \longrightarrow \text{Min}(A) \left\{ \begin{array}{l} \text{минимален} \\ \text{за езика} \end{array} \right.$$

$$L(A) = L(\text{Min}(A))$$

Пример за автомат,
който не е минимален:



$$\text{Min}(A) = ?$$

$$L_A(q_4) = \{a, aa, abaa \dots\}$$

$$L_A(q_1) = \emptyset$$

$$L_A^2(q_5) = \{\epsilon, a, aa\}$$

$$L_A^1(q_4) = \{a\}$$

$$L_A^3(q_5) = L_A^2 \cup \{aaaa, abaa, baab, bbaa\}$$

$$L(p) = \{w \in \Sigma^* \mid \delta^*(p, w) \in F\}$$

$$L(A) = L(q_0) \quad \text{Б.О.О.} \quad \text{ст. свст. на } q_0$$

$$p \equiv_A q \iff L(p) = L(q) \quad \text{реп. свт.}$$

Апроксимация

$$p \equiv_A^n q \iff L_A^n(p) = L_A^n(q) \quad (\text{реп. свт.})$$

$$L_A^n(q) = \{w \mid |w| \leq n \wedge \delta^*(q, w) \in F\}$$

\equiv_A^n е апроксимация на \equiv_A

\equiv_A^{n+1} е по-фина от \equiv_A^n

Тържи такова $n \in \mathbb{N} : \equiv_A^n = \equiv_A^{n+1}$

$$\Rightarrow \equiv_A^n = \equiv_A$$

Алгоритъм: $\equiv_A^0 \neq \equiv_A^1 \neq \dots \neq \equiv_A^k \neq \underbrace{\equiv_A^{k+1} = \equiv_A^{k+2} = \equiv_A^{k+3} \dots}_{\equiv_A}$

$$A_{k0} \equiv_A^n \neq \equiv_A^{n+1} \Rightarrow |\equiv_A^n| < |\equiv_A^{n+1}|$$

За произволен автомат

$$A = \langle Q, \Sigma, s, F, \delta \rangle$$

Класове на екв на \equiv_A^0 ?

$$F \cup Q \setminus F$$

решение на горната задача

$$\equiv_A^0 \rightarrow S_1 = \{ \underline{0, 1, 4, 6} \} \quad S_2 = \{ \underline{2, 3, 5} \}$$

$$TBRCA \equiv_A^1$$

	a	b	
{ 0, 1, 4, 6 }	s2	s1	→ {1}
	s1	s1	
	s2	s1	
	s2	s1	
{ 2, 3, 5 }	s2	s1	
	s2	s1	
	s2	s1	

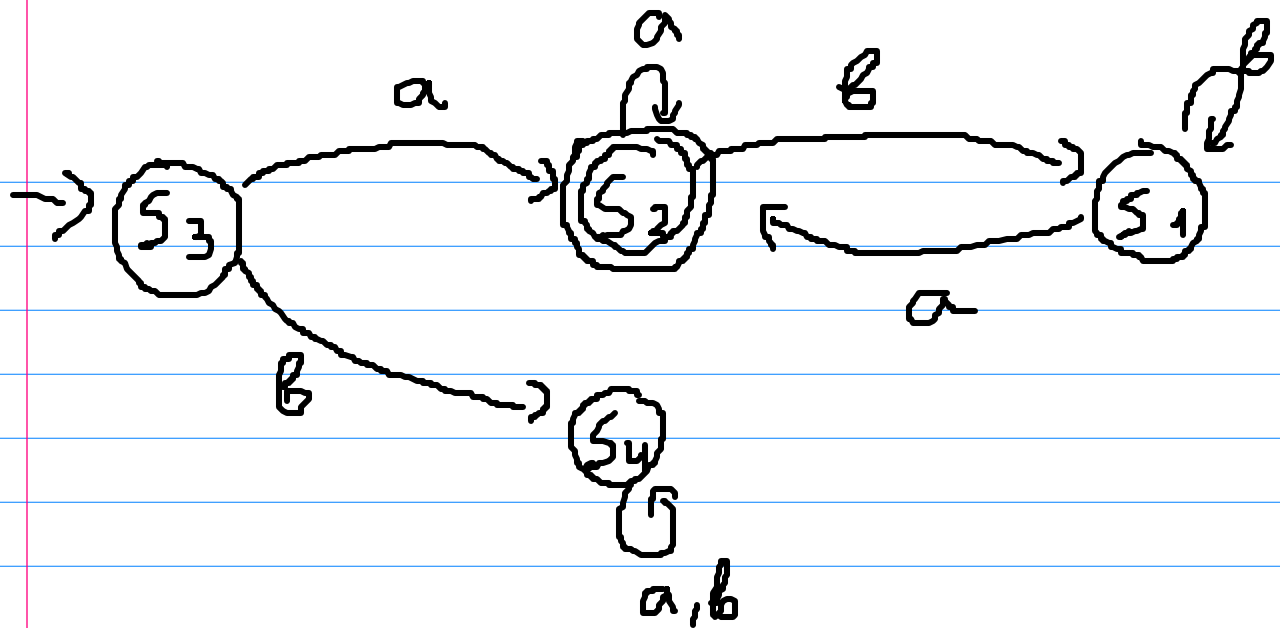
$$\equiv_A^1 \quad S_1 = \{0, 4, 6\}, \quad S_2 = \{2, 3, 5\}, \quad S_3 = \{1\}$$

$$\begin{array}{c} S_1 \\ S_2 \end{array} \left\{ \begin{array}{c|c|c} a & b \\ \hline 0 & S_2 & S_3 \\ 4 & S_2 & S_1 \\ 6 & S_2 & S_1 \\ 2 & S_2 & S_1 \\ 3 & S_2 & S_1 \\ 5 & S_2 & S_1 \end{array} \right\}$$

$$\equiv_A^2 \quad S_1 = \{4, 6\}, \quad S_2 = \{2, 3, 5\}, \quad S_3 = \{0\}, \quad S_4 = \{1\}$$

$$\begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \left\{ \begin{array}{c|c|c} a & b \\ \hline 4 & S_2 & S_1 \\ 6 & S_2 & S_1 \\ 2 & S_2 & S_1 \\ 3 & S_2 & S_1 \\ 5 & S_2 & S_1 \\ 0 & S_2 & S_4 \\ 1 & S_4 & S_4 \end{array} \right\}$$

$$\Rightarrow \equiv_A^3 = \equiv_A^2 \Rightarrow \equiv_A^2 = \equiv_A$$



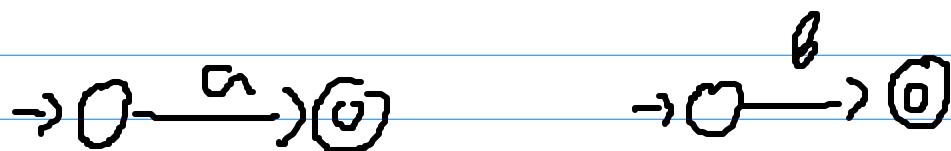
Зад Постройте минимальный детерминированный автомат для

$$L = \{ a b^n \mid n \in \mathbb{N} \wedge n \text{ нечетно} \}$$

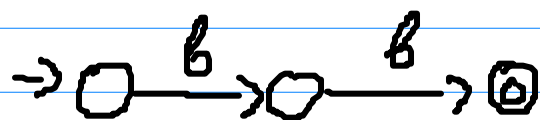
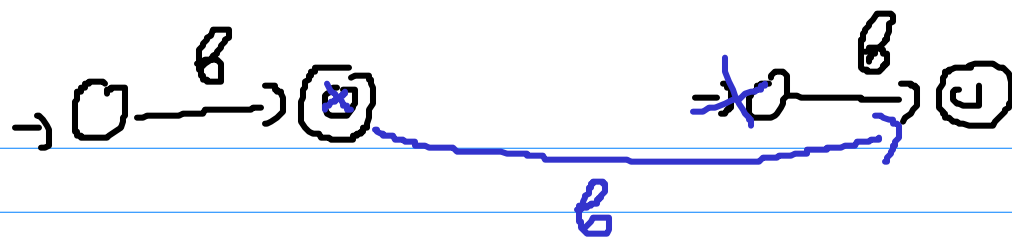
I рез. израз

$$a(bb)^*b$$

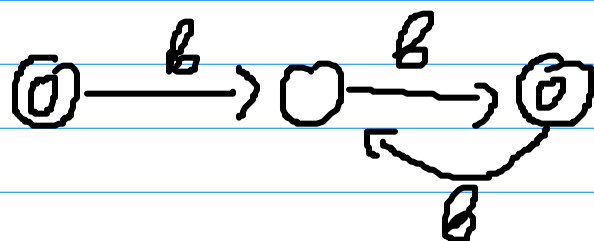
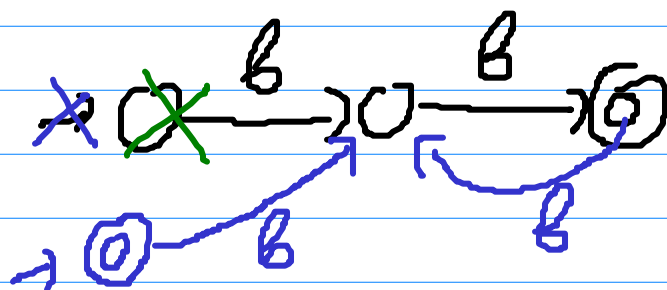
II строим автомата



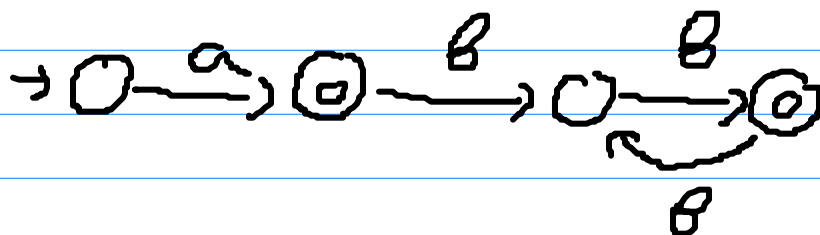
bb



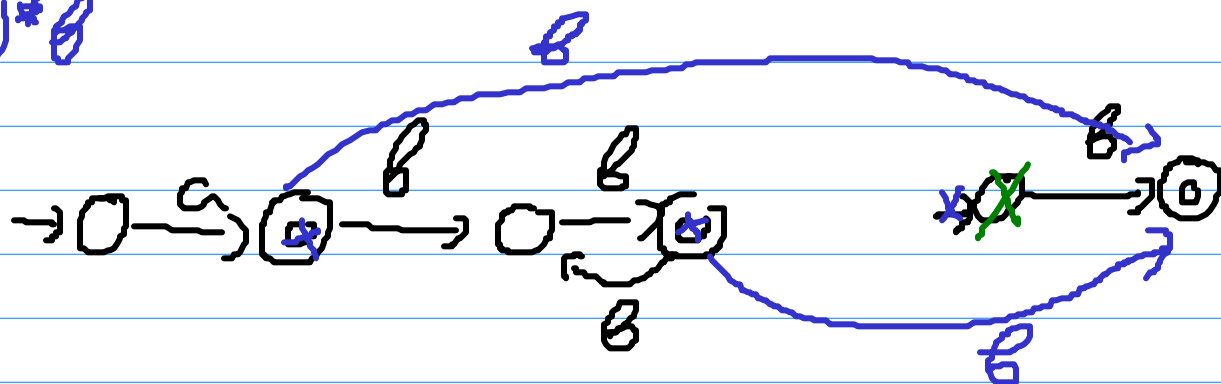
$(bb)^*$



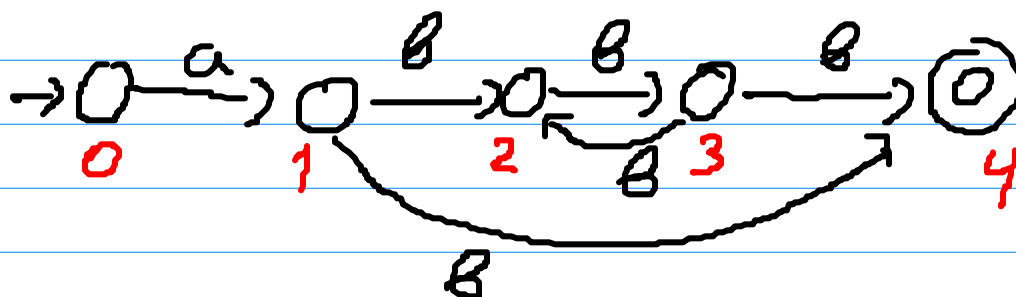
$a(bb)^*$



$a(bb)^*b$



A:

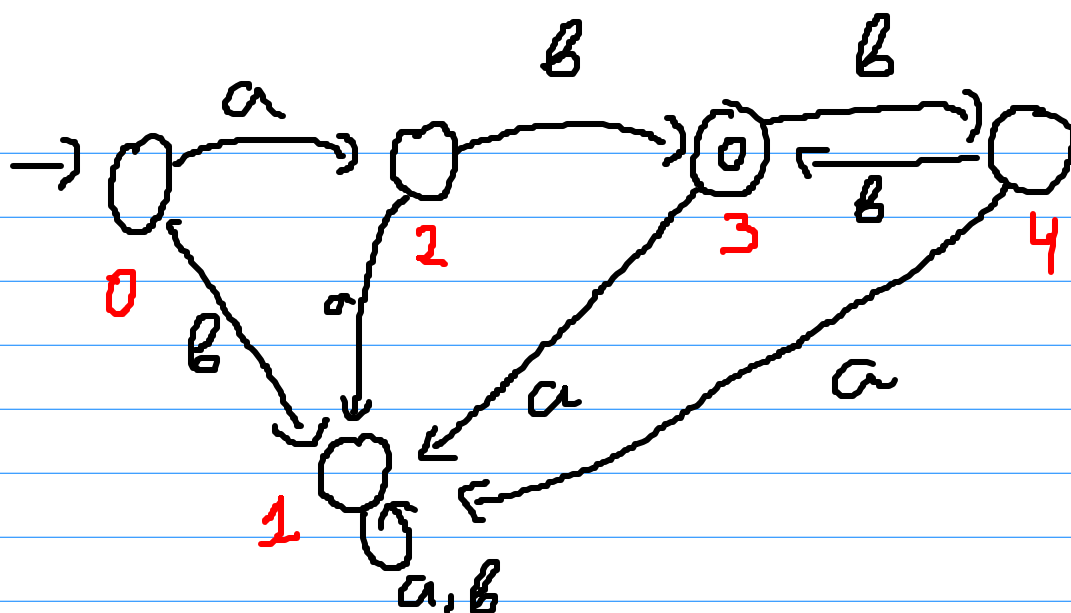


III

Тотален и детерминистичен

		a	b
0	{0}	{1}	\emptyset
1	\emptyset	\emptyset	\emptyset
2	{1}	\emptyset	{2,4}
3	{2,4}	\emptyset	{3}
4	{3}	\emptyset	{2,4}

$\det(A)$



IV минимален

$$\equiv_A^0 \quad S_1 = \underbrace{\{0, 1, 2, 4\}}_{Q \setminus F} \quad S_2 = \underbrace{\{3\}}_F$$

$$S_1 \left\{ \begin{array}{c|c|c} 0 & S_1 & S_1 \\ 1 & S_1 & S_1 \\ 2 & S_1 & S_2 \\ 4 & S_1 & S_2 \end{array} \right\}$$

$$\equiv_A^1 \quad S_1 = \{0, 1\}, \quad S_2 = \{2, 4\} \quad S_3 = \{3\}$$

		a	b
s_1	0	s_2	s_1
	1	s_1	s_1
s_2	2	s_1	s_3
	4	s_1	s_3

$$\equiv_A^2 \quad s_1 = \{0\}, s_2 = \{1\}, s_3 = \{2, 4\}, s_4 = \{3\}$$

		a	b
s_3	2	s_2	s_4
	4	s_2	s_4
s_1	0	s_3	s_2
s_2	1	s_2	s_2
s_4	3	s_2	s_3

$$\equiv_A^2 = \equiv_A^3$$

$$\Rightarrow \equiv_A^2 = \equiv_A$$

