5

Минимален автомат

Релация на Майхил – Нероуд:

Нека $L \subseteq \Sigma^*$ $\quad R_L \subseteq \Sigma^* \times \Sigma^*$

$x R_L y \iff \forall z \in \Sigma^* \; xz \in L \iff yz \in L$

↳ релация на еквивалентност

Класовете на екв. са състоянията на минималният автомат за L.

зад 1 $\quad \Sigma = \{a, b\}$

- $L = \{ w \in \Sigma^* \mid w$ започва и завършва с различна буква $\}$

Класове на екв. на R_L?

$aa \, R_L \, ba$? не!

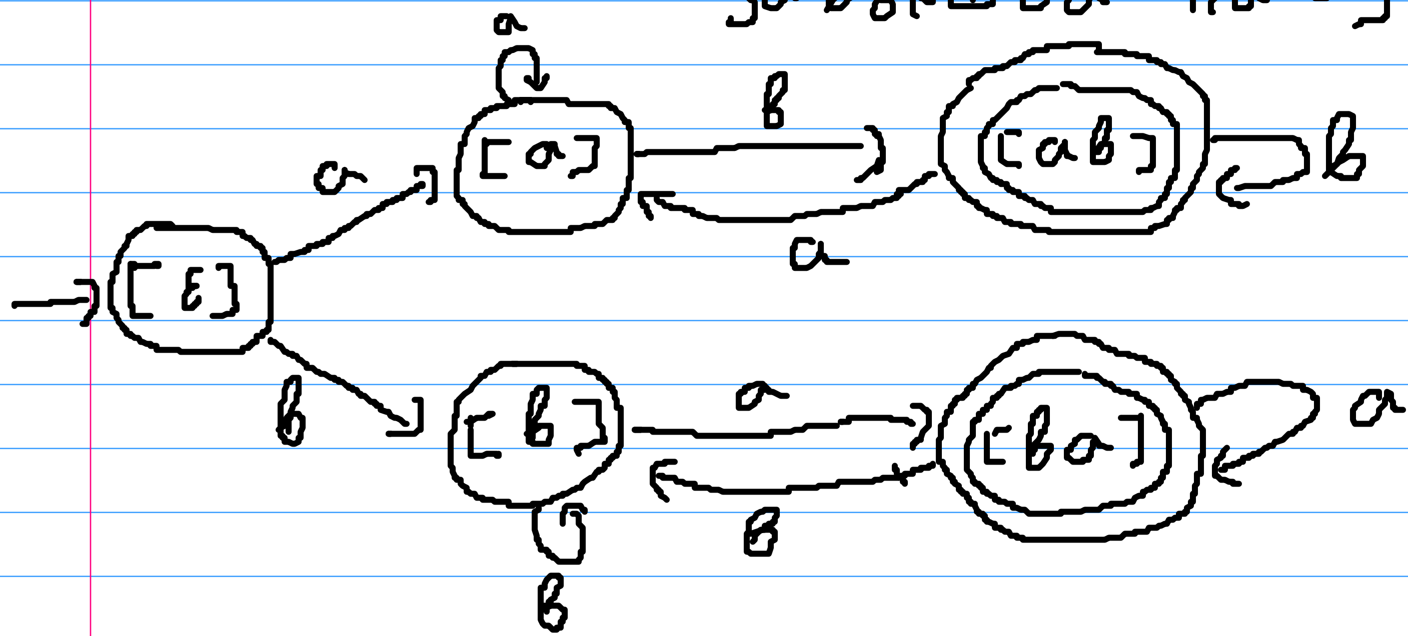
$[a] = \{ w \mid w$ започва с a и завършва с $a \}$ $\qquad$ a____b
a____b

$[b] = \{ w \mid w$ започва с b и завършва с $b \}$

$[\varepsilon] = \{\varepsilon\}$

$[ab] = \{w \mid w$ започва с a и завършва с $b\}$ $\subseteq F$

$[ba] = \{w \mid w$ започва с b и завършва на $a\}$ $\subseteq F$



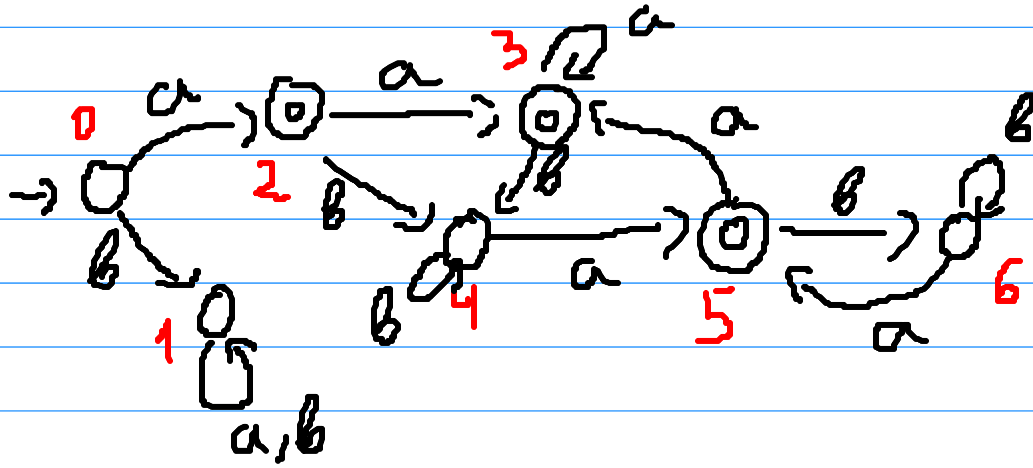
Минимизация

$A \to$ КДА $A \longrightarrow Min(A)$ (минимален за езика)

$L(A) = L(Min(A))$

Пример за автомат, който не е минимален:

A:



$Min(A) = ?$

$L_A(q_4) = \{a, aa, aba \dots\}$

$L_A(q_1) = \emptyset$

$L_A^2(q_5) = \{\varepsilon, a, aa\}$

$L_A^1(q_4) = \{a\}$

$L_A^3(q_5) = L_A^2 \cup \{aaa, aba, baa, bba\}$

$$L(p) = \{ w \in \Sigma^* \mid \delta^*(p,w) \in F \}$$

$L(A) = L(q_0)$ Б.О.О. ст. свст е с инд. 0

← търся кл. на еквивалентност

$p \equiv_A q \Leftrightarrow L(p) = L(q)$ (рел. екв.)

Апроксимираме

$p \equiv_A^n q \Leftrightarrow L_A^n(p) = L_A^n(q)$ (рел. екв.)

$$L_A^n(q) = \{ w \mid \underline{|w| \le n} \wedge \delta^*(q,w) \in F \}$$

$\equiv_A^n$ е апроксимация на $\equiv_A$

$\equiv_A^{n+1}$ е по-фина от $\equiv_A$

Търси такова $n \in \mathbb{N}$: $\equiv_A^n = \equiv_A^{n+1}$

$\Rightarrow \equiv_A^n = \equiv_A$

Алгоритъм: $\equiv_A^0 \ne \equiv_A^1 \ne \dots \ne \equiv_A^k \ne \underbrace{\equiv_A^{k+1} = \equiv_A^{k+2} = \equiv_A^{k+3} \dots}_{\equiv_A}$

Ако $\equiv_A^n \neq \equiv_A^{n+1} \Rightarrow |\equiv_A^n| < |\equiv_A^{n+1}|$

За произволен автомат

$A = \langle Q, \Sigma, s, F, \delta \rangle$

Класове на екв на $\equiv_A^0$?

F и $Q \setminus F$

решение на горната задача

$\equiv_A^0 \to S_1 = \{0, 1, 4, 6\}$ $S_2 = \{2, 3, 5\}$

търся $\equiv_A^1$

	a	b
0	S_2	S_1
1	S_1	S_1
4	S_2	S_1
6	S_2	S_1
2	S_2	S_1
3	S_2	S_1
5	S_2	S_1

$\to \{1\}$

$\equiv_A^1$ $\quad S_1=\{0,4,6\}$, $S_2=\{2,3,5\}$, $S_3=\{1\}$

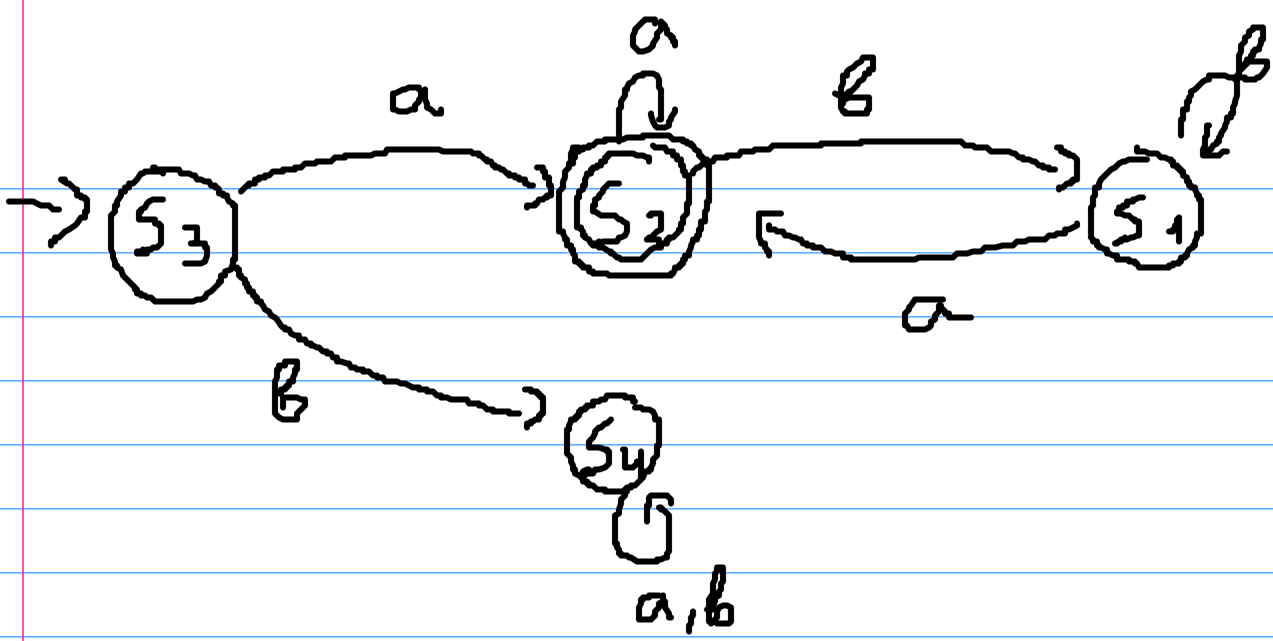
		a	b
S_1	0	S_2	S_3
	4	S_2	S_1
	6	S_2	S_1
S_2	2	S_2	S_1
	3	S_2	S_1
	5	S_2	S_1

$\equiv_A^2$ $\quad S_1=\{4,6\}$, $S_2=\{2,3,5\}$, $S_3=\{0\}$

$S_4=\{1\}$

		a	b	
S_1	4	S_2	S_1	S_1
	6	S_2	S_1	
S_2	2	S_2	S_1	S_2
	3	S_2	S_1	
	5	S_2	S_1	
S_3	0	S_2	S_4	S_3
S_4	1	S_4	S_4	S_4

$\Rightarrow \quad \equiv_A^3 = \equiv_A^2 \qquad \Rightarrow \quad \equiv_A^2 = \equiv_A$



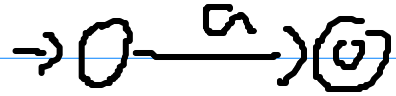
зад. Постройте минимален тотален детерминиран автомат за

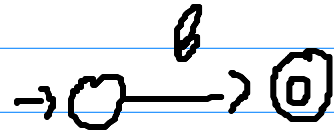
$$L = \{ a b^n \mid n \in \mathbb{N} \land n \text{ е нечетно} \}$$

I) рег. израз

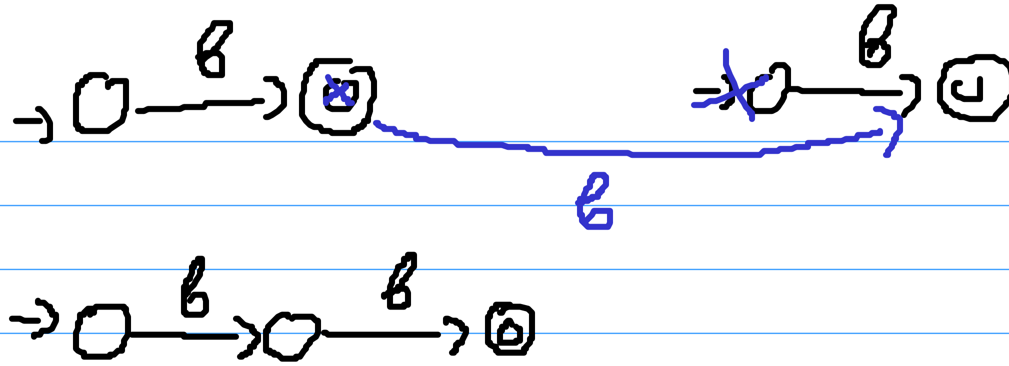
$$a(bb)^* b$$

II) Строим автомата

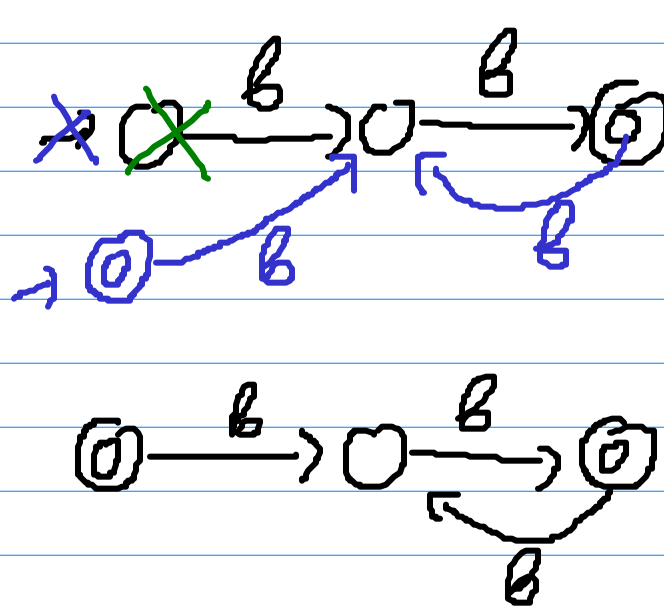




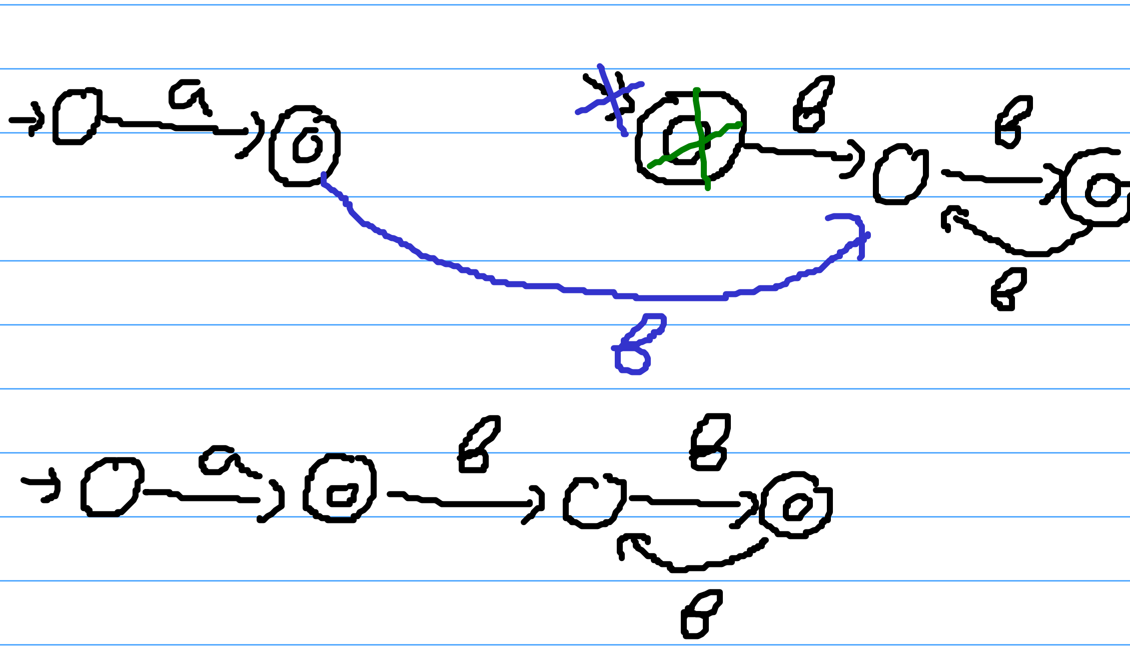
bb



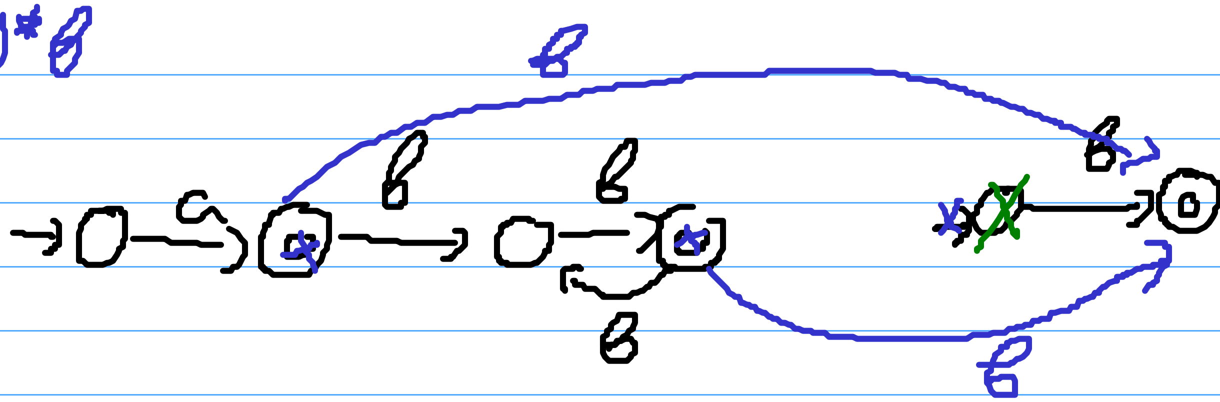
$(bb)^*$



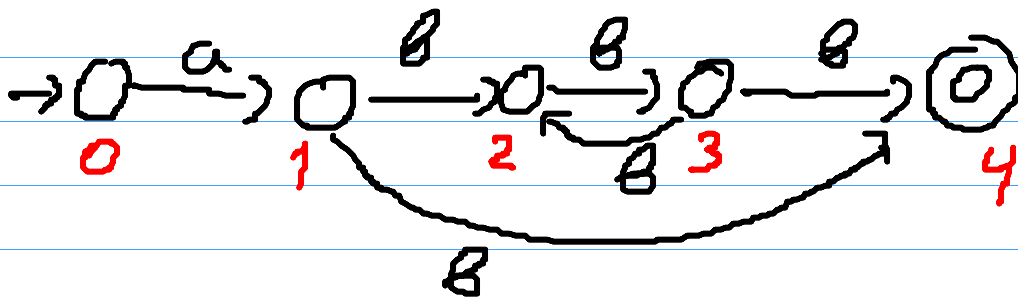
$a(bb)^*$



$a(bb)^*b$



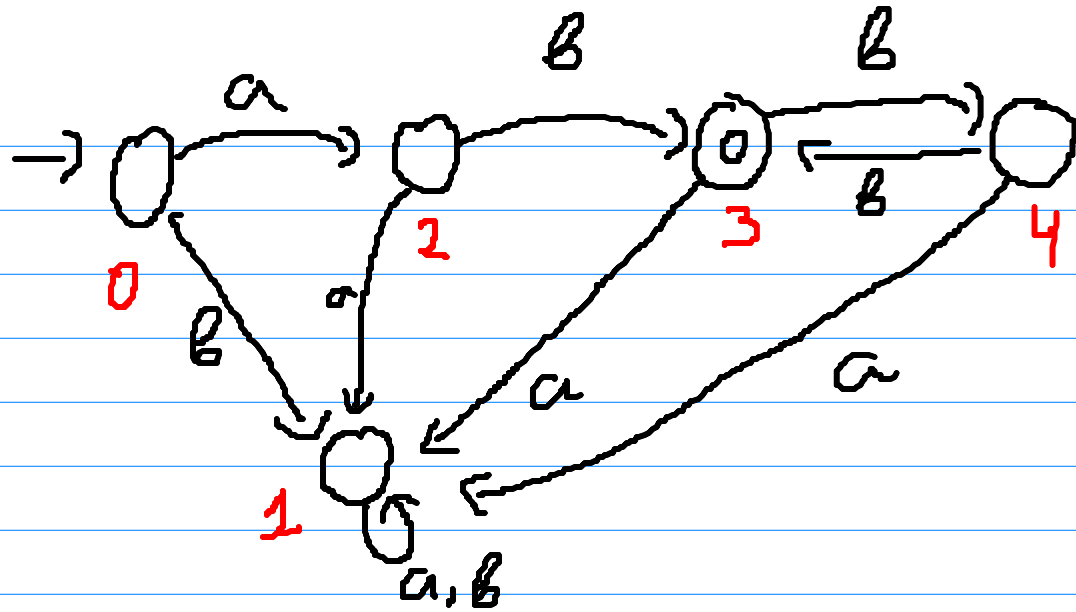
A:



III Тотален и детерминистичен

		a	b
0	{0}	{1}	∅
1	∅	∅	∅
2	{1}	∅	{2,4}
3	{2,4}	∅	{3}
4	{3}	∅	{2,4}

det(A)



$\overline{\underline{IV}}$ минимален

$\equiv^0_A$ $\qquad \underbrace{S_1 = \{0, 1, 2, 4\}}_{Q \setminus F} \qquad \underbrace{S_2 = \{3\}}_{F}$

		a	b
S_1	0	S_1	S_1
	1	S_1	S_1
	2	S_1	S_2
	4	S_1	S_2

$\equiv^1_A$ $\quad S_1 = \{0, 1\}, \quad S_2 = \{2, 4\} \qquad S_3 = \{3\}$

		a	b
S_1	0	S_1	S_1
	1	S_1	S_1
S_2	2	S_1	S_3
	4	S_1	S_3

$\equiv_A^2$ $\quad S_1 = \{0\}, \; S_2 = \{1\}, \; S_3 = \{2,4\}, \; S_4 = \{3\}$

		a	b
S_3	2	S_2	S_4
	4	S_2	S_4
S_1	0	S_3	S_2
S_2	1	S_2	S_2
S_4	3	S_2	S_3

$\equiv_A^2 = \equiv_A^3$

$\Rightarrow \; \equiv_A^2 = \equiv_A$