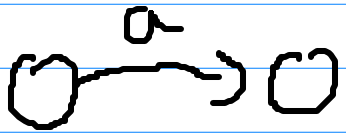
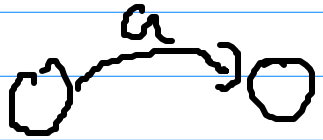
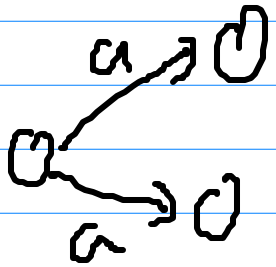
3

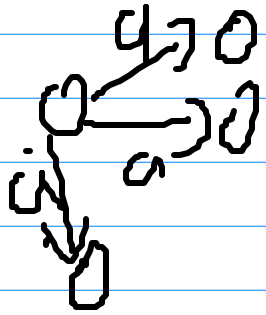
Ретроспекция

дет автомати ↓ VS недет. автомати ↓







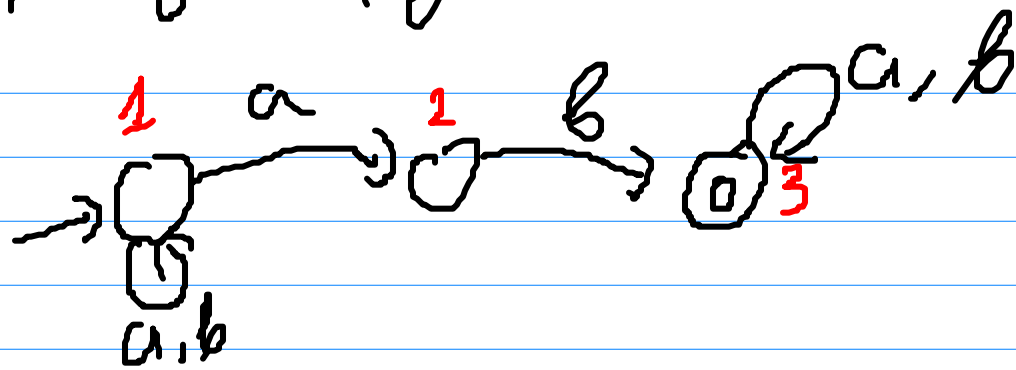


! За всеки недет. автомат A същ. дет автомат A' такъв че

$$L(A) = L(A')$$

Пример за Нед. автомат:

A:



$aab \overset{?}{\in} L(A)$

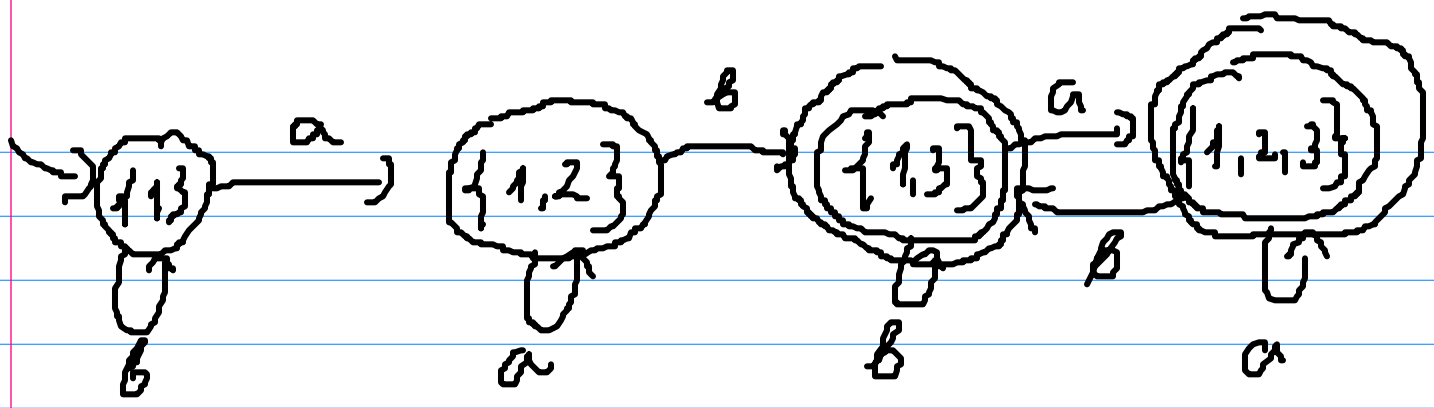
$1 \xrightarrow{a} 1 \xrightarrow{a} 1 \xrightarrow{b} 1$ ← не е финално

$1 \xrightarrow{a} 1 \xrightarrow{a} 2 \xrightarrow{b} 3$ ← финално

$\Rightarrow aab \in L(A)$

Зад. Постройте дет автомат, който има същия език като A (Th. Rabin - Scott)

	a	b
старт → {1}	{1,2}	{1}
{1,2}	{1,2}	{1,3}
{1,3}	{1,2,3}	{1,3}
фин. {1,2,3}	{1,2,3}	{1,3}



Твърдение ④ За всеки език $L \subseteq \Sigma^*$

L е автоматен $\Leftrightarrow$ L е регулярен

2) • L е автоматен $\Rightarrow$ L е регулярен
1) • L е регулярен $\Rightarrow$ L е автоматен

1) За да докажем 1) трябва да покажем, че за всеки регулярен език L, $\exists$ автомат A: $L(A) = L$

Структурна индукция:

L е рег:

def
- $\emptyset$ е рег
- $\{a\}$ е рек.

L_1 и L_2 са рег.
- $L_1 \cup L_2$ е рег
- $L_1 \cdot L_2$ е рег

• L_1^* и L_2^* са рег.

1.1) • $\emptyset$ A: →○

1.2) • $\{a\}$ A: →○ —a→ ◎

1.3) • Обединение на автомати

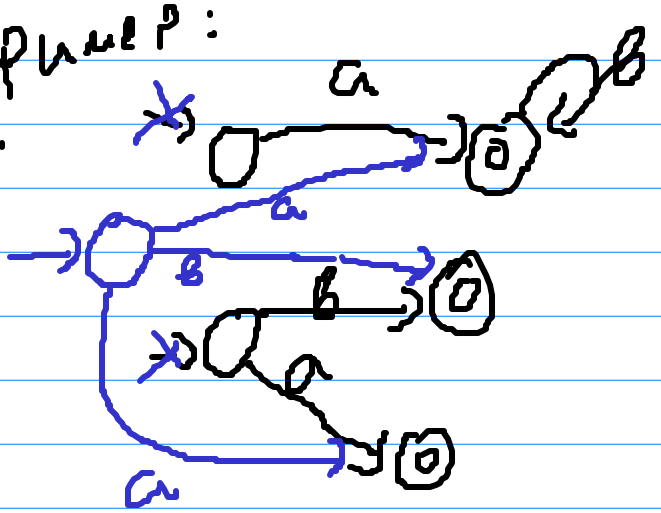
$A_1 = (Q_1, \Sigma, s_1, F_1, \delta_1)$

$A_2 = (Q_2, \Sigma, s_2, F_2, \delta_2)$

Твърдя A: $L(A) = L(A_1) \cup L(A_2)$

Пример:

A_1

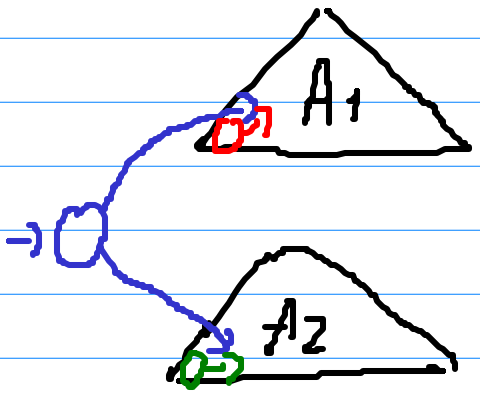


$L(A_1) = ab^*$

A_2

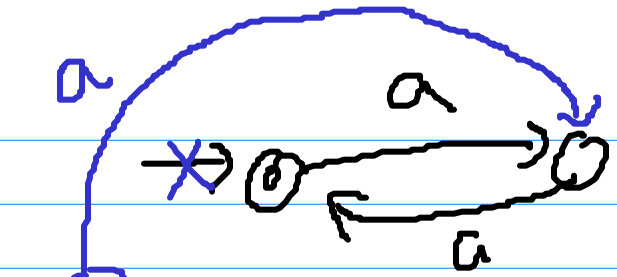
$L(A_2) = a + b$

$L(A) = ab^* + (a+b)$



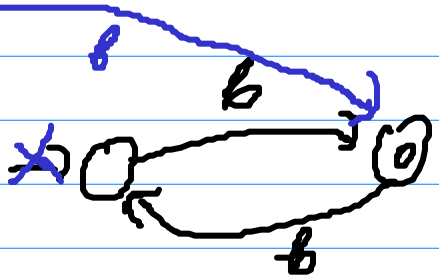
Пример 2

A_1:



$L(A_1) = (aa)^*$

A_2:



$L(A_2) = b(bb)^*$

$L(A) = (aa)^* + b(bb)^*$

$$A = \langle Q_1 \cup Q_2 \cup \{t\}, \Sigma, t, F', \delta' \rangle$$

$t \notin Q_1 \cup Q_2$

$$F' = \begin{cases} F_1 \cup F_2 & s_1 \notin F_1 \wedge s_1 \notin F_2 \\ F_1 \cup F_2 \cup \{t\} & s_1 \in F_1 \vee s_2 \in F_2 \end{cases}$$

$$\delta'(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \\ \delta_2(q,a) & q \in Q_2 \\ \delta_1(s_1,a) \cup \delta_2(s_2,a) & q = t \end{cases}$$

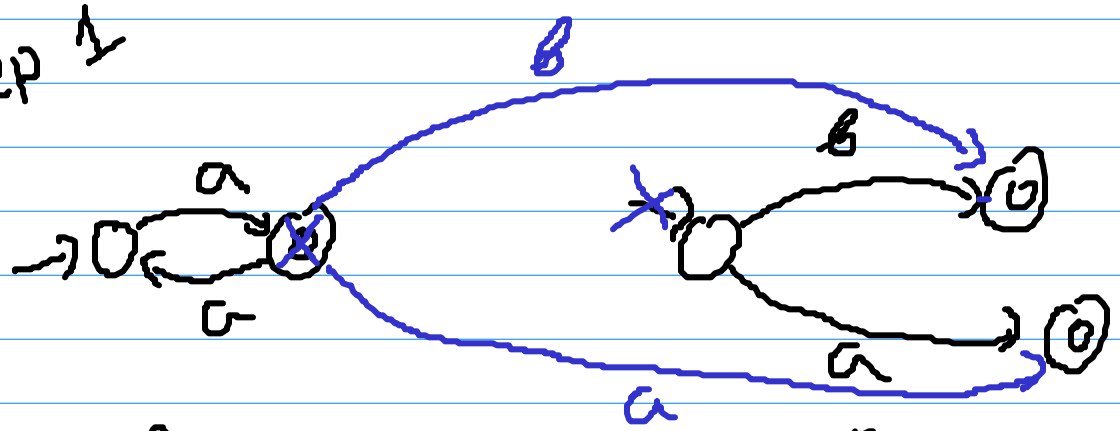
1.4) Конкатенация на автоматы

$A_1 = \langle Q_1, \Sigma, s_1, F_1, \delta_1 \rangle$

$A_2 = \langle Q_2, \Sigma, s_2, F_2, \delta_2 \rangle$

Тогда $A: L(A) = L(A_1) \cdot L(A_2)$

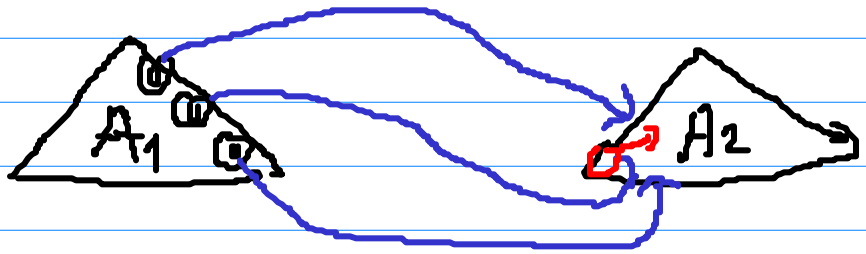
Пример 1



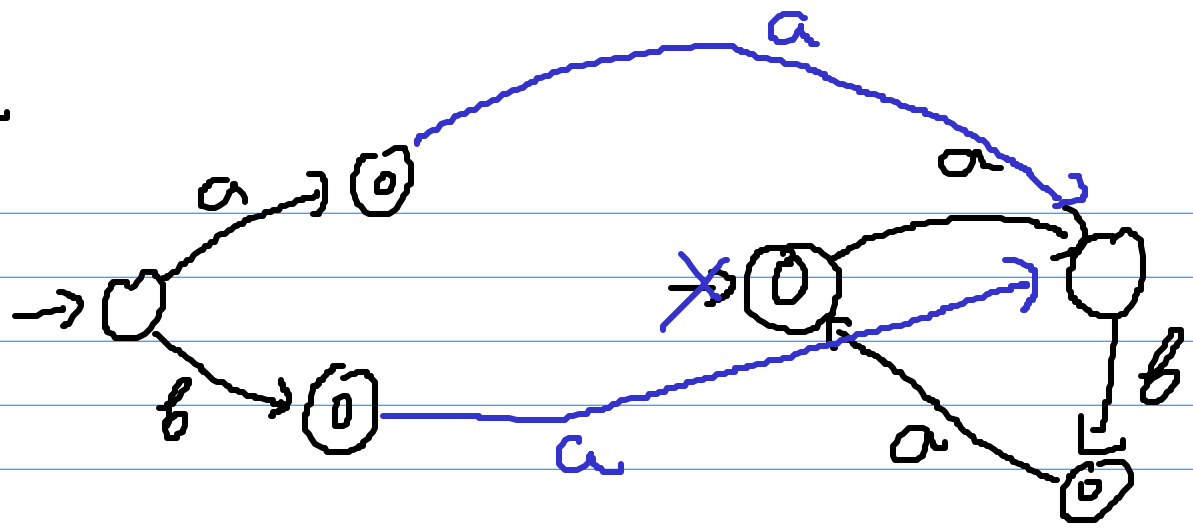
A_1 A_2

$L(A_1) = a(aa)^*$ $L(A_2) = a + b$

$L(A) = a(aa)^*(a + b)$



Пример 2



$L(A_1) = a + b$

$L(A_2) =$

$= (aba)^* + ab(aab)^*$

$L(A) = \{(a+b) \mid (aba)^* + ab(aab)^*\}$

$$A = \langle Q_1 \cup Q_2, \Sigma, s1, F', \delta' \rangle$$

$$F' = \begin{cases} F_1 \cup F_2 & s2 \in F_2 \\ F_2 & s2 \notin F_2 \end{cases}$$

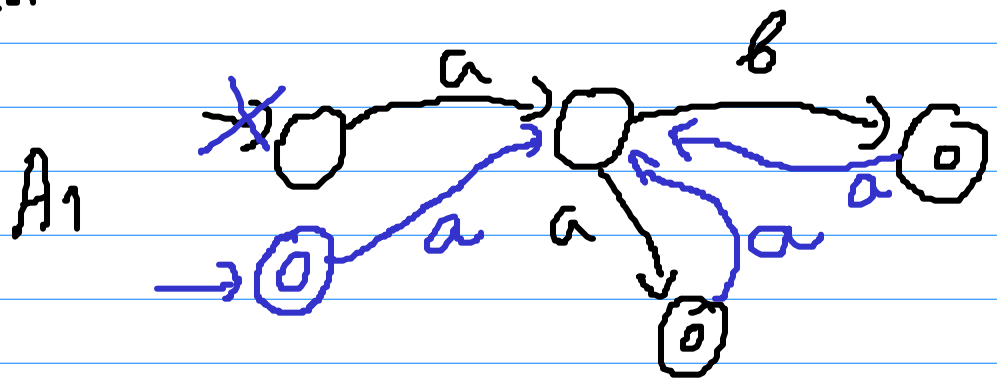
$$\delta'(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \setminus F_1 \\ \delta_1(q,a) \cup \delta_2(s_2,a) & q \in F_1 \\ \delta_2(q,a) & q \in Q_2 \end{cases}$$

1.5) Звезда на Клини на автомат

$$A_1 = \langle Q_1, \Sigma, S1, F1, \delta 1 \rangle$$

Търсим автомат $A : L(A) = L(A_1)^*$

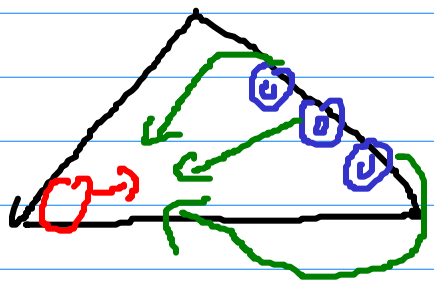
Пример 1



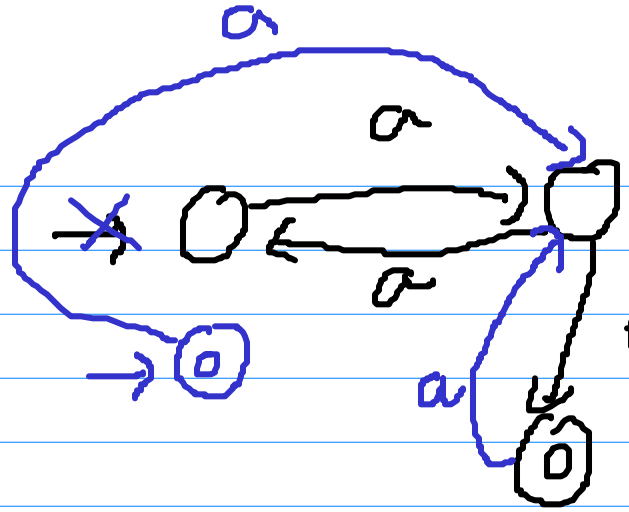
$$L(A_1) = aa + ab$$

$$L(A_1)^* = \{ \varepsilon, aa, ab, aaaa, abab, aaab, abaa \dots \}$$

"Рестартиране" автомата след прочитане на думата



Пример 2



A_1

$L[A_1] = a(aa)^*b$

$$L[A] = (a(aa)^*b)^*$$

$$A = \langle Q_1 \cup \{t\}, \Sigma, t, F_1 \cup \{t\}, \delta \rangle$$

$t \notin Q_1$

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \setminus F_1 \\ \delta_1(q,a) \cup \delta_1(s_1,a) & q \in F_1 \\ \delta_1(s_1,a) & q = t \end{cases}$$

зад. Построите автомат за $L \subseteq \{a,b\}^*$

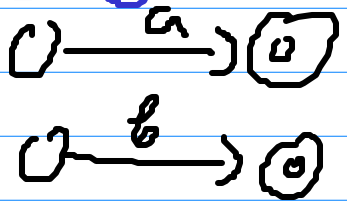
б) $L = \{w \mid w$ започва и завр. на $a\}$

а) $L = \{w \mid w$ започва с a и има 2 последователни a-та$\}$

а) Рег. израз $a(ba+b)^*$ ③

④ ① ②

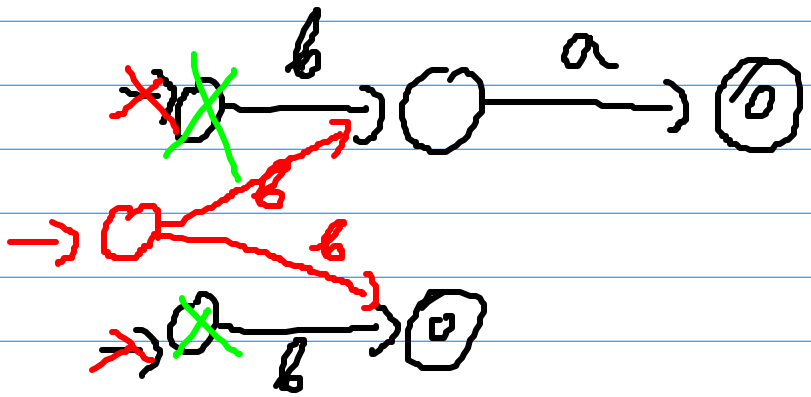
Имаме само:

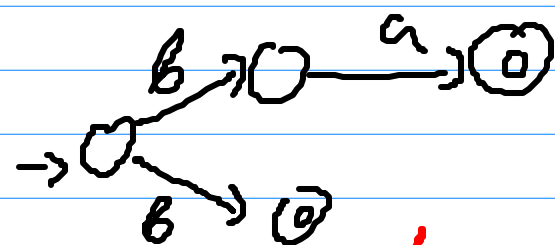


Ще премахваме недостижимите състояния

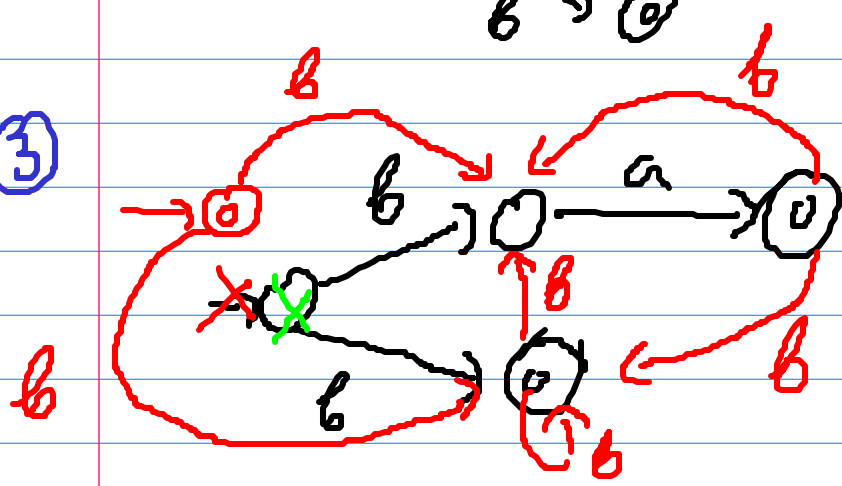
①

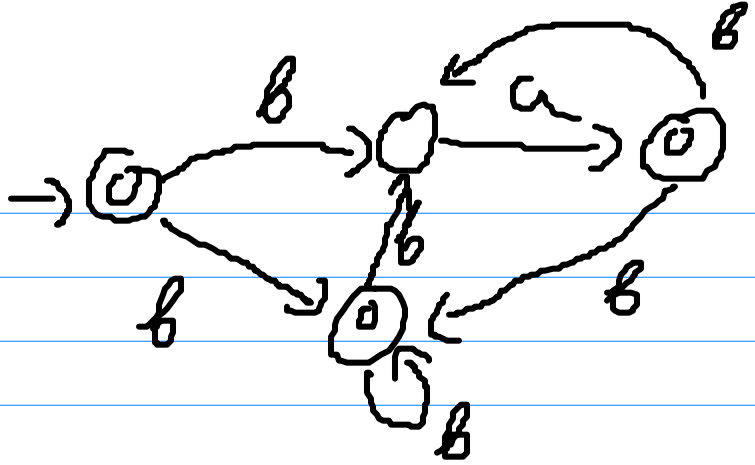
②



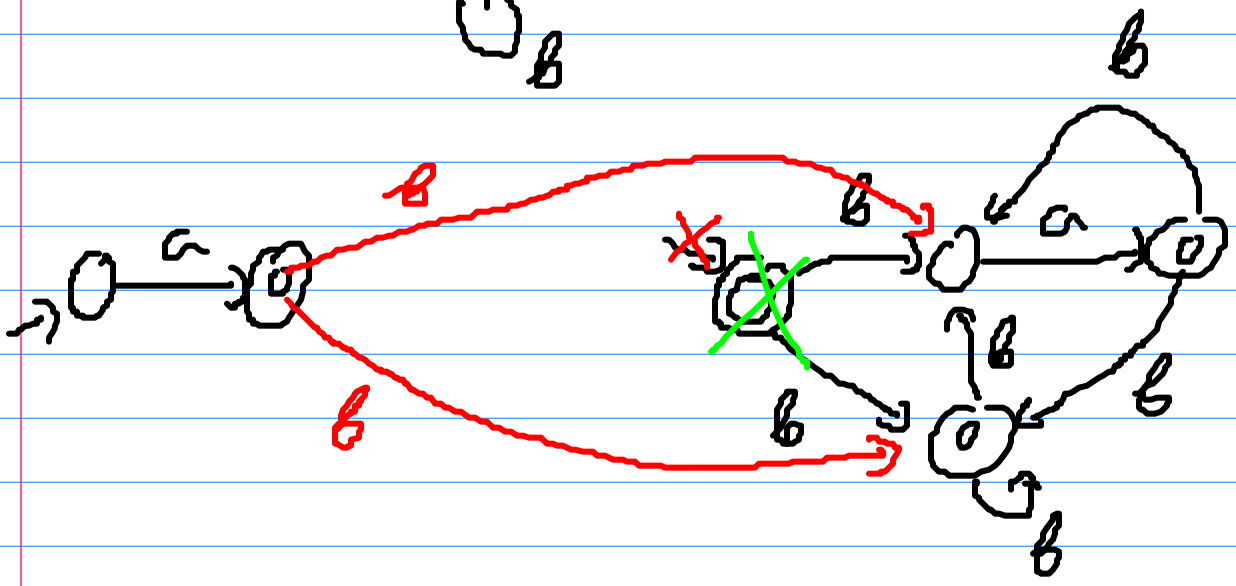


③

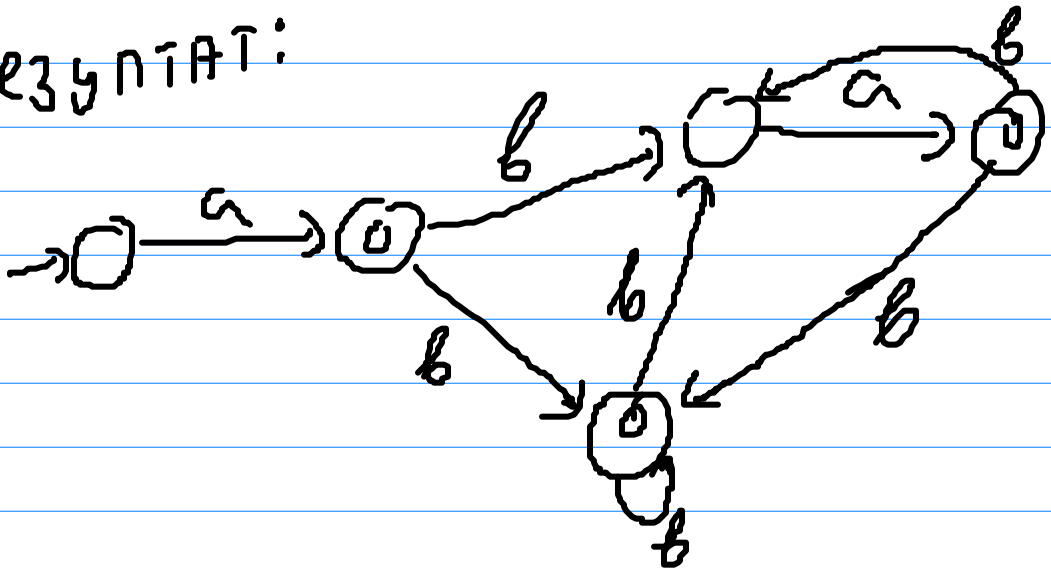




4)

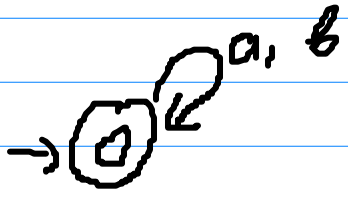


Результат:

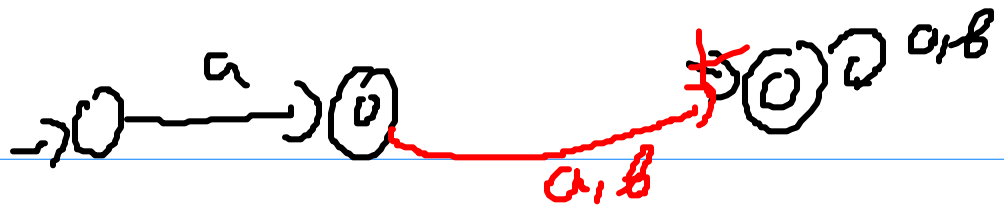


5) $a(a+b)^* a + a$

(1) $(a+b)^*$ (2) (3)



1



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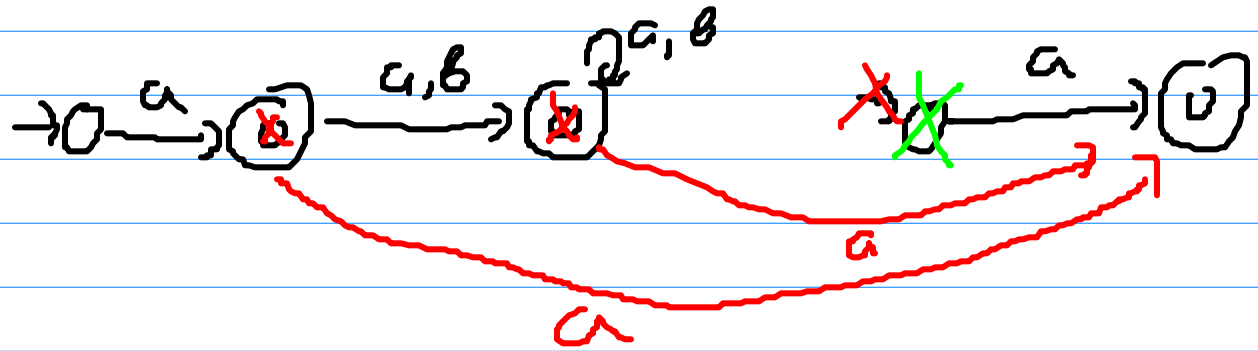
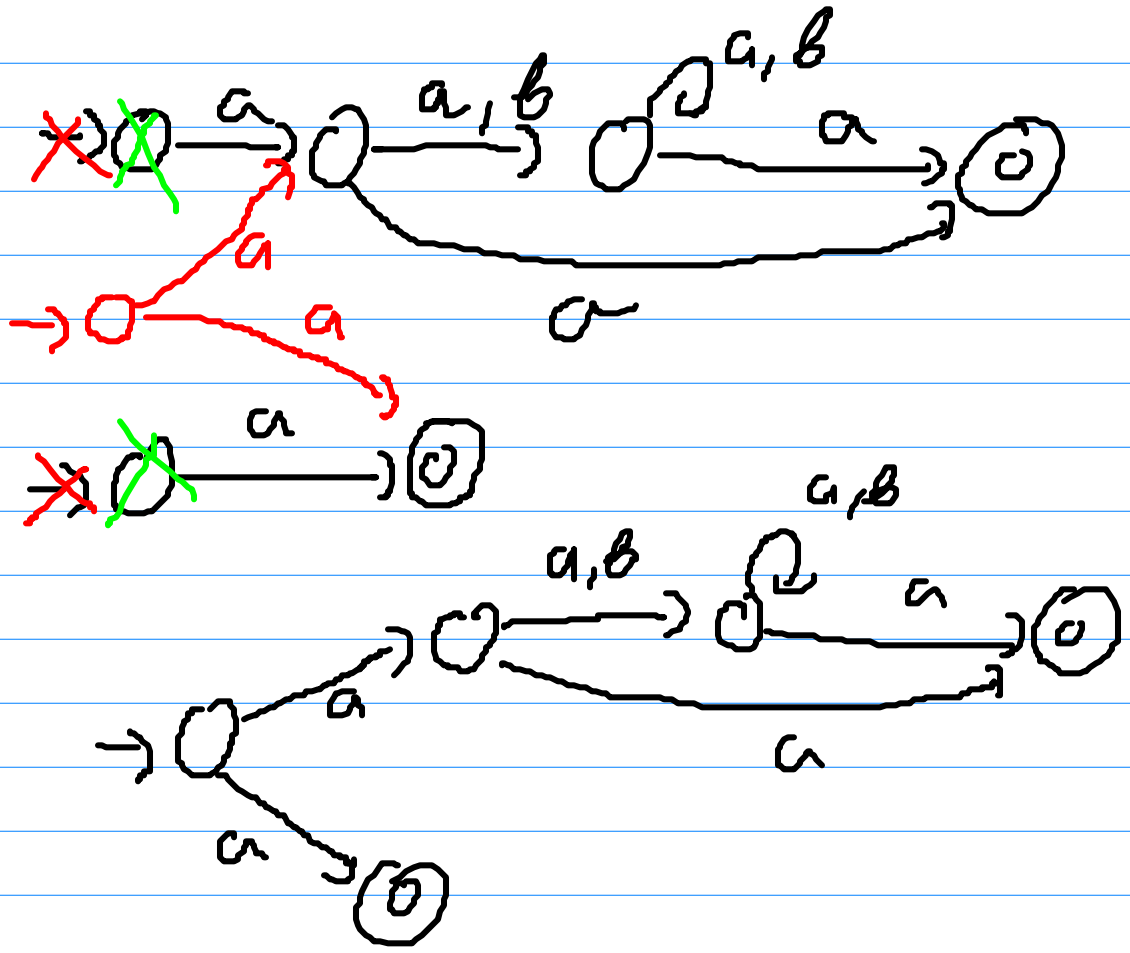
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