DEMOSTRACIONES

$$\bar{x} = \sum_{j=1}^{p} f_{\cdot j}\,\bar{x}_j$$

$$\bar{x} = \sum_{i=1}^{k} f_{i\cdot}\,x_i = \sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}\,x_i = \sum_{i=1}^{k}\sum_{j=1}^{p} f_{\cdot j}\,f_i^j\,x_i = \sum_{j=1}^{p} f_{\cdot j}\sum_{i=1}^{k} f_i^j\,x_i = \sum_{j=1}^{p} f_{\cdot j}\,\bar{x}_j$$

$$\bar{y} = \sum_{i=1}^{k} f_{i\cdot}\,\bar{y}_i$$

$$\bar{y} = \sum_{j=1}^{p} f_{\cdot j}\,y_j = \sum_{j=1}^{p}\sum_{i=1}^{k} f_{ij}\cdot y_j = \sum_{j=1}^{p}\sum_{i=1}^{k} f_{i\cdot}\,f_j^i\cdot y_j = \sum_{i=1}^{k} f_{i\cdot}\sum_{j=1}^{p} f_j^i\,y_j = \sum_{i=1}^{k} f_{i\cdot}\,\bar{y}_i$$

$$\sigma_x^2 = \sum_{j=1}^{p} f_{\cdot j}\,\sigma_{x,j}^2 + \sum_{j=1}^{p} f_{\cdot j}\,(\bar{x}_j - \bar{x})^2$$

$$\sigma_x^2 = \sum_{j=1}^{p} f_{i\cdot}\,(x_i - \bar{x})^2 = \sum_{i=1}^{k}\sum_{j=1}^{p} f_i^j\,f_{\cdot j}\,(x_i - \bar{x})^2 = \sum_{j=1}^{p}\sum_{i=1}^{k} f_{\cdot j}\,f_i^j\,(x_i - \bar{x})^2 =$$

$$= \sum_{j=1}^{p} f_{\cdot j}\left[\sum_{i=1}^{k} f_i^j\,(x_i - \bar{x})^2\right] = \sum_{j=1}^{p} f_{\cdot j}\left[\sum_{i=1}^{k} f_i^j\,(x_i - \bar{x}_j + \bar{x}_j - \bar{x})^2\right] =$$

$$= \sum_{j=1}^{p} f_{\cdot j}\left[\sum_{i=1}^{k} f_i^j\,(x_i - \bar{x}_j)^2 + \sum_{i=1}^{k} f_i^j\,(\bar{x}_j - \bar{x})^2 + \cancelto{0}{2\sum_{i=1}^{k} f_i^j\,(x_i - \bar{x}_j)(\bar{x}_j - \bar{x})}\right] =$$

$$= \sum_{j=1}^{p} f_{\cdot j}\left[\sigma_{x,j}^2 + (\bar{x}_j - \bar{x})^2\right]$$

X indep. Y $\Longleftrightarrow$ Y indep X

$\Rightarrow$

¿Si $f_i^j = f_{i.} \Rightarrow f_j^i = f_{.j}$?

$f_{ij} = f_{i.} f_j^i = f_{.j} f_i^j$

$\cancel{f_{i.}} f_j^i = f_{.j} \cancel{f_{i.}}$

$f_j^i = f_{.j} \Rightarrow$ Y indep X

$\Leftarrow$

¿Si $f_j^i = f_{.j} \Rightarrow f_i^j = f_{i.}$?

$f_{ij} = f_{.j} f_i^j = f_{i.} f_j^i$

$\cancel{f_{.j}} f_i^j = f_{i.} \cancel{f_{.j}}$

$f_i^j = f_{i.} \Longrightarrow$ X indep Y

$f_{i/j} = f_{i.}$

$f_{i.} = \sum_{j=1}^{p} f_{ij} = \sum_{j=1}^{p} f_{i/j} f_{.j} = f_{i/j} \sum_{j=1}^{p} f_{.j} = f_{i/j}$

X e Y indep. $\Leftrightarrow f_{ij} = f_{i.} f_{.j} \Leftrightarrow n_{ij} = \frac{n_{i.} n_{.j}}{n}$

$f_{i/j} = f_{i.} \Leftrightarrow \frac{f_{i/j} f_{.j}}{f_{ij}} = f_{i.} f_{.j} \Leftrightarrow n f_{ij} = n f_{i.} f_{.j} \Leftrightarrow n_{ij} = n_{i.} \cdot \frac{n_{.j}}{n}$

$m_{r0} = \sum_{i=1}^{k} f_{i.} x_i^2$

$m_{r0} = \sum_{i=1}^{k} \left(\sum_{j=1}^{p} f_{ij} x_i^r \right) = \sum_{i=1}^{k} x_i^2 \left(\sum_{j=1}^{p} f_{ij} \right) = \sum_{i=1}^{k} f_{i.} x_i^r$

$m_{rs} = m_{r0} m_{0s}$ (X e Y indep)

$m_{rs} = \sum_{i=1}^{k} \sum_{j=1}^{p} f_{ij} x_i^r y_j^s = \sum_{i=1}^{k} \sum_{j=1}^{p} f_{i.} f_{.j} x_i^r y_j^s = \sum_{i=1}^{k} f_{i.} x_i^r \left(\sum_{j=1}^{p} f_{.j} y_j^s \right) = m_{r0} m_{0s}$

$$\sigma_{xy} = \mu_{11} = m_{11} - m_{10}m_{01}$$

$$\sigma_{xy} = \sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}(x_i - \bar{x})(y_j - \bar{y}) = \sum_i^k \sum_j^p f_{ij}(x_i y_j - \bar{x} y_j - \bar{y} x_i + \bar{x}\bar{y}) =$$

$$= \sum_i^k \sum_j^p f_{ij} x_i y_j - \sum_i^k \sum_j^p f_{ij} \bar{x} y_j - \sum_i^k \sum_j^p f_{ij} \bar{y} x_i + \sum_i^k \sum_j^p f_{ij} \bar{x}\bar{y} =$$

$$= m_{11} - \bar{x}\bar{y} - \bar{x}\bar{y} + \bar{x}\bar{y} = m_{11} - \bar{x}\bar{y} = m_{11} - m_{10}m_{01}$$

$$\left.\begin{array}{l} \frac{\partial \varphi(a,b)}{\partial a} = 0 \\ \frac{\partial \varphi(a,b)}{\partial b} = 0 \end{array}\right\} \Rightarrow \begin{array}{l} m_{11} = a m_{20} + b m_{10} \\ m_{01} = a m_{10} + b \end{array} \Rightarrow a = \frac{\sigma_{xy}}{\sigma_x^2}, \quad b = \bar{y} - \frac{\sigma_{xy}}{\sigma_x^2} \cdot \bar{x}$$

$$\left.\begin{array}{l} \frac{\partial \varphi(a,b)}{\partial a} = 0 \\ \frac{\partial \varphi(a,b)}{\partial b} = 0 \end{array}\right\} \begin{array}{l} \sum_i \sum_j f_{ij}(y_j - a x_i - b)x_i = 0 = \sum_i \sum_j f_{ij} x_i y_j - a \sum_i \sum_j f_{ij} x_i^2 - b \sum_i \sum_j f_{ij} x_i \\ \sum_i \sum_j f_{ij}(y_j - a x_i - b) = 0 = \sum\sum f_{ij} y_j - a \sum\sum f_{ij} x_i - b \sum\sum f_{ij} \end{array} \Bigg\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \sum\sum f_{ij} x_i y_j = a \sum\sum f_{ij} x_i^2 + b \sum\sum f_{ij} x_i \\ \sum\sum f_{ij} y_j = a \sum\sum f_{ij} x_i + b \sum\sum f_{ij} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} m_{11} = a m_{20} + b m_{10} \\ m_{01} = a m_{10} + b \end{array}\right.$$

$\sigma_x^2 = m_{20} - m_{10}^2 \neq 0 \Rightarrow$ Tiene sol y es única

$$a = \frac{m_{11} - m_{10}m_{01}}{m_{20} - m_{10}^2} = \frac{\sigma_{xy}}{\sigma_x^2}$$

$$b = m_{01} - \frac{\sigma_{xy}}{\sigma_x^2} m_{10} = \bar{y} - \frac{\sigma_{xy}}{\sigma_x^2}\bar{x}$$

Buscar $f(x_1), \dots, f(x_k)$: $\sum_i^k \sum_j^p f_{ij} (y_j - f(x_i))^2$ sea mínima

$$\sum_i^k \sum_j^p f_{ij} (y_j - f(x_i))^2 = \sum \sum f_{j/i} f_{i\cdot} (y_j - f(x_i))^2 = \sum_{i=1}^k f_{i\cdot} \left[\sum_{j=1}^p f_{j/i} (y_j - f(x_i))^2 \right]$$

Minimizar $\sum_j^p f_{j/i} (y_j - f(x_i))^2$, $\forall i = 1, \dots, k$

↳ Media de desviaciones cuadr. de Y respecto de $f(x_i)$ en las desv. cuadr. $Y / X = x_i$

$$f(x_i) = \bar{y}_i = \sum_{j=1}^p f_{j/i} y_j$$

$\sum_{i=1}^k \bar{y}_i f_{i\cdot} = \bar{y}$

$$0 = \sum_i^k \sum_j^p f_{ij} (y_j - \bar{y}_i) = \sum_i^k \sum_j^p f_{ij} y_j - \sum_i^k \sum_j^p f_{ij} \bar{y}_i = m_{01} - \sum_i^k \bar{y}_i \left(\sum_j^p f_{ij}\right) = \bar{y} - \sum_{i=1}^k \bar{y}_i f_{i\cdot}$$

$\sigma_{r_y}^2 = \sum_i^k \sum_j^p f_{ij} (r_{ij}^{Ly})^2$ | $\sum^k f_{i\cdot} \left(\bar{y} + \frac{\sigma_{xy}}{\sigma_x^2}(x_i - \bar{x})\right) = \sum^k f_{i\cdot} \bar{y} + \sum^k f_{i\cdot} \frac{\sigma_{xy}}{\sigma_x^2} (x_i - \bar{x}) = \bar{y}$

$(r_{ij}^y = y_i - \bar{y}_i$, $\forall i = 1, \dots, k$, $\forall j = 1, \dots, p)$

$$\sigma_{r_y}^2 = \sum_i^k \sum_j^p f_{ij} (r_{ij}^y)^2 \overset{\text{error cuadr. medio}}{=} \sum_i^k \sum_j^p f_{ij} (y_j - \bar{y}_i)^2 = \sum_i^k \sum_j^p f_{j/i} f_{i\cdot} (y_j - \bar{y}_i)^2 =$$

$$= \sum_i^k f_{i\cdot} \underbrace{\sum_j^p f_{j/i} (y_j - \bar{y}_i)^2}_{\text{Varianza condic}} = \underbrace{\sum_{i=1}^k f_{i\cdot} \sigma_{y_i}^2}_{\text{Media de las varianzas condic}}$$

Los resid. lin. mín. cuad. tienen media 0 y su varianza es el error cuad. medio

$$\sum_i^k \sum_j^p f_{ij} r_{ij}^{Ly} = \sum_i^k \sum_j^p f_{ij} \left(y_j - \bar{y} + \frac{\sigma_{xy}}{\sigma_x^2}(x_i - \bar{x})\right) = \sum^k \sum^p f_{ij} y_j - \sum \sum f_{ij} \frac{\sigma_{xy}}{\sigma_x^2}(x_i - \bar{x}) - \bar{y} \sum^k \sum^p f_{ij} =$$

$$= \bar{y} - \bar{y} - \frac{\sigma_{xy}}{\sigma_x^2} \underbrace{\sum^k \sum^p f_{ij} (x_i - \bar{x})}_{0 \text{ (media)}} = 0$$

$$\Rightarrow \underset{\text{varianza de resid.}}{\sigma_{r_y^L}^2} = \sum_i^k \sum_j^p f_{ij} (r_{ij}^{Ly} - 0)^2 = \sum^k \sum^p f_{ij} (r_{ij}^{Ly})^2 = ECM$$

xq la media de los resid. es 0

Varianza valores observ. = varianza val. ajustados + varianza resid.

$$\sigma_y^2 = \sigma_{ey^*}^2 + \sigma_{ry^*}^2$$

- $$\sigma_{ey^*}^2 = \sum_{i=1}^{k} f_{i\cdot} \left(\bar{y} + \frac{\sigma_{xy}}{\sigma_x^2}(x_i - \bar{x}) - \bar{y}\right)^2 = \sum_i f_{i\cdot} \frac{\sigma_{xy}^2}{(\sigma_x^2)^2}(x_i - \bar{x})^2 =$$

$$= \frac{\sigma_{xy}^2}{(\sigma_x^2)^2} \sum_i^k f_{i\cdot} (x_i - \bar{x})^2 = \frac{\sigma_{xy}^2}{(\sigma_x^2)^{\cancel{2}}} \cancel{\sigma_x^2} = \frac{\sigma_{xy}^2}{\sigma_x^2} \Rightarrow \sigma_{ey^*}^2 = \frac{\sigma_{xy}^2}{\sigma_x^2}$$

- $$\sigma_{ry^*}^2 = \sum_i^k \sum_j^p f_{ij} \left(y_j - \bar{y} - \frac{\sigma_{xy}}{\sigma_x^2}(x_i - \bar{x})\right)^2 =$$

$$= \sum^k \sum^p f_{ij} \left[(y_j - \bar{y})^2 + \left(\frac{\sigma_{xy}}{\sigma_x^2}(x_i - \bar{x})\right)^2 - 2(y_j - \bar{y})\left(\frac{\sigma_{xy}}{\sigma_x^2}(x_i - \bar{x})\right)\right] =$$

$$= \sum^k \sum^p f_{ij} (y_j - \bar{y})^2 + \sum_i^k \sum_j^p f_{ij} \left(\frac{\sigma_{xy}}{\sigma_x^2}\right)^2 (x_i - \bar{x})^2 - 2\sum^k \sum^p f_{ij} (y_j - \bar{y})\left(\frac{\sigma_{xy}}{\sigma_x^2}(x_i - \bar{x})\right) =$$

$$= \sigma_y^2 + \frac{\sigma_{xy}^2}{(\sigma_x^2)^2}\sigma_x^2 - 2\frac{\sigma_{xy}}{\sigma_x^2}\sigma_{xy} = \sigma_y^2 + \frac{\sigma_{xy}^2}{\sigma_x^2} - 2\frac{\sigma_{xy}^2}{\sigma_x^2} = \sigma_y^2 - \frac{\sigma_{xy}^2}{\sigma_x^2} \Rightarrow$$

$$\Rightarrow \sigma_{xy^*}^2 = \sigma_y^2 - \frac{\sigma_{xy}^2}{\sigma_x^2}$$

X, Y indep $\Rightarrow \sigma_{xy} = 0$

$$\sigma_{xy} = \sum\sum f_{i\cdot} f_{\cdot j} (x_i - \bar{x})(y_j - \bar{y}) = \sum f_{i\cdot} \underbrace{\cancel{(x_i - \bar{x})}}_{0} \left[\sum f_{\cdot j} \underbrace{\cancel{(y_j - \bar{y})}}_{0}\right] = 0$$

$\sigma_y^2 = \sigma_{res}^2 + \sigma_{expl}^2$ (Caso lineal)

$$\sigma_y^2 = \sum_j f_{\cdot j}(y_j - \bar{y})^2 = \sum_j f_{\cdot j} (\underbrace{y_j - ax_i - b}_{r_{ij}} + \underbrace{ax_i + b - \bar{y}}_{\hat{y}_j - \bar{y}})^2 =$$

$$= \sum f_{\cdot j} (y_j - \hat{y}_j)^2 + \sum f_{\cdot j} (\hat{y}_j - \bar{y})^2 + \underbrace{\cancel{2\sum f_{\cdot j} (y_j - \hat{y}_j)(\hat{y}_j - \bar{y})}}_{0} =$$

$$= \sum f_{\cdot j} (r_{ij} - 0)^2 + \sigma_{expl}^2 = \sigma_{res}^2 + \sigma_{expl}^2$$

$r^2 = \frac{\sigma_{xy}^2}{\sigma_x^2 \sigma_y^2}$

$$\sigma_{res}^2 = \frac{1}{n} \sum \sum n_{ij} (y_j - ax_i - b)^2 = \sum \sum f_{ij} \left(y_j - \frac{\sigma_{xy}}{\sigma_x^2} x_i - \bar{y} + \frac{\sigma_{xy}}{\sigma_x^2} \bar{x} \right)^2 =$$

$$= \sum \sum f_{ij} \left[(y_j - \bar{y}) - \frac{\sigma_{xy}}{\sigma_x^2} (x_i - \bar{x}) \right]^2 =$$

$$= \sum \sum f_{ij} (y_j - \bar{y})^2 + \sum \sum f_{ij} \frac{\sigma_{xy}^2}{(\sigma_x^2)^2} (x_i - \bar{x})^2 - 2 \sum \sum f_{ij} \frac{\sigma_{xy}}{\sigma_x^2} (x_i - \bar{x})(y_j - \bar{y}) =$$

$$= \sigma_y^2 + \frac{\sigma_{xy}^2}{(\sigma_x^2)^2} \cancel{\sigma_x^2} - 2 \frac{\sigma_{xy}^2}{\sigma_x^2} = \sigma_y^2 - \frac{\sigma_{xy}^2}{\sigma_x^2}$$

$$\eta^2_{y/x} = 1 - \frac{\sigma_{res}^2}{\sigma_y^2} = 1 + \frac{\sigma_y^2 + \frac{\sigma_{xy}^2}{\sigma_x^2}}{\sigma_y^2} = \frac{\sigma_{xy}^2}{\sigma_x^2 \sigma_y^2} = r^2$$

"

$\eta^2_{x/y}$