DEMOSTRACIONES

■ $x_1 < \bar{x} < x_k$

$$x_1 < x_i < x_k \iff n_i x_1 < n_i x_i < n_i x_k \iff$$

$$\iff \sum_{i=1}^{k} n_i x_1 < \sum_{i=1}^{k} n_i x_i < \sum_{i=1}^{k} n_i x_k \iff$$

$$\iff \frac{\sum_{i=1}^{k} x_1 n_i}{n} < \frac{\sum_{i=1}^{k} n_i x_i}{n} < \frac{\sum_{i=1}^{k} n_i x_k}{n} \iff$$

$$\iff x_1 \frac{\sum_{i=1}^{k} n_i}{n} < \bar{x} < x_k \frac{\sum_{i=1}^{k} n_i}{n} \iff x_1 < \bar{x} < x_k$$

■ $\sum_{i=1}^{k} f_i (x_i - \bar{x}) = 0$

$$\sum_{i=1}^{k} f_i (x_i - \bar{x}) = 0 \Rightarrow \sum_{i=1}^{k} (f_i x_i - f_i \bar{x}) = 0 \Rightarrow \sum_{i=1}^{k} f_i x_i - \sum_{i=1}^{k} f_i \bar{x} = 0 =$$

$$= \bar{x} - \bar{x} \sum_{i=1}^{k} f_i = \bar{x} - \bar{x} = 0$$

■ $\bar{y} = a\bar{x} + b$

$$\bar{y} = \sum_{i=1}^{k} f_i y_i = \sum_{i=1}^{k} f_i (a x_i + b) = \sum_{i=1}^{k} a f_i x_i + b \sum_{i=1}^{k} f_i =$$

$$= a \sum_{i=1}^{k} a x_i + b = a\bar{x} + b$$

$$\sigma_x^2 = \sum_{i=1}^{k} f_i (x_i - \bar{x})^2 < \sum_{i=1}^{k} f_i (x_i - a)^2 \quad , \quad \forall a \neq \bar{x}$$

Sea $\varphi(a) = \sum_{i=1}^{k} f_i (x_i - a)^2$

$$\varphi'(a) = -2 \sum_{i=1}^{k} f_i (x_i - a) = 0 \iff \sum_{i=1}^{k} f_i (x_i - a) = 0 \Rightarrow$$

$$\Rightarrow \sum_{i=1}^{k} f_i x_i - \sum_{i=1}^{k} f_i a = \bar{x} - \sum_{i=1}^{k} f_i a = 0 \iff \bar{x} = a$$

$$\varphi''(a) = 2 \cdot \sum_{i=1}^{k} f_i (x_i - a)^0 = 2 > 0 \Rightarrow$$ Tiene un mín en el punto a

La varianza está acotada

$$0 \leq \min (x_i - \bar{x})^2 < \sigma^2 = \sum_{i=1}^{k} f_i (x_i - \bar{x})^2 < \max (x_i - \bar{x})^2$$

= si $x_i = \bar{x}$
(los datos coinc.)

Tma König: $\sum_{i=1}^{k} f_i (x_i - a)^2 = \sum_{i=1}^{k} f_i (x_i - \bar{x}) + (a - \bar{x})^2$

$$\sum_{i=1}^{k} f_i (x_i - a)^2 = \sum_{i=1}^{k} f_i (x_i - a - \bar{x} + \bar{x})^2 = \sum_{i=1}^{k} f_i \left((x_i - \bar{x}) - (a - \bar{x}) \right)^2 =$$

$$= \sum_{i=1}^{k} f_i (x_i - \bar{x})^2 + (a - \bar{x})^2 \cdot \sum_{i=1}^{k} f_i - \cancel{2(a - \bar{x}) \cdot \sum_{i=1}^{k} (x_i - \bar{x}) f_i} \rightarrow 0 =$$

$$= \sum_{i=1}^{k} f_i (x_i - \bar{x})^2 + (a - \bar{x})^2 \sum_{i=1}^{k} f_i =$$

$$= \sum_{i=1}^{k} f_i (x_i - \bar{x}) + (a - \bar{x})^2$$

⋆ Si $a = 0$:

$$\sum_{i=1}^{k} f_i x_i^2 = \sigma_x^2 + \bar{x}^2 \Rightarrow \sigma_x^2 = \sum_{i=1}^{k} f_i x_i^2 - \bar{x}^2 \geq 0$$ Entonces:

$$\sum_{i=1}^{k} f_i x_i^2 \geq \bar{x}^2 \Rightarrow \sqrt{\sum_{i=1}^{k} f_i x_i^2} \geq \bar{x} \Rightarrow \underset{\text{Media cuadr.}}{Q_x} \geq \bar{x} \iff \sigma_x^2 > 0 \iff$$ al menos dos datos ≠

$\boxed{}$ $\sigma_y^2 = a^2 \sigma_x^2$

$y_i = ax_i + b \quad , \quad i=1,\dots,k \quad \Rightarrow \quad \bar{y} = a\bar{x} + b$

$$\sigma_y^2 = \sum_{i=1}^{k} f_i (y_i - \bar{y})^2 = \sum_{i=1}^{k} f_i (ax_i + b - (a\bar{x}+b))^2 = \sum_{i=1}^{k} f_i (ax_i - a\bar{x})^2 =$$

$$= \sum_{i=1}^{k} f_i a^2 (x_i - \bar{x})^2 = a^2 \sum_{i=1}^{k} f_i (x_i - \bar{x})^2 = a^2 \sigma_x^2 \quad \Rightarrow \sigma_y = |a| \sigma_x$$

$\boxed{}$ $\gamma_{1,y} = \pm \gamma_{1,x}$

$y_i = ax_i + b \quad i=1,\dots,k$

$$\gamma_{1,y} = \sum_{i=1}^{k} f_i \left(\frac{ax_i + b - a\bar{x} - b}{|a| \sigma_x}\right)^3 = \sum_{i=1}^{k} \frac{a^3}{|a|} \cdot f_i \left(\frac{x_i - \bar{x}}{\sigma_3}\right)^3 = \pm \gamma_{1,x}$$

$\boxed{}$ $\mu_r = \sum_{t=0}^{r} (-1)^t \binom{r}{t} \bar{x}^t \cdot \sum_{i=1}^{k} f_i x_i^{r-t}$

$$\mu_r = \sum_{i=1}^{k} f_i (x_i - \bar{x})^r = \sum_{i=1}^{k} f_i \sum_{t=0}^{r} (-1)^t \binom{r}{t} \bar{x}^t \cdot x_i^{r-t} =$$

$$= \sum_{t=0}^{r} (-1)^t \binom{r}{t} \underbrace{\bar{x}^t}_{\bar{x}=m_1} \cdot \underbrace{\sum_{i=1}^{k} f_i x_i^{r-t}}_{m_{r-t}}$$

Ej ¿μ_3?

$$\mu_3 = m_3 - 3m_1 m_2 + 3m_1^2 m_1 - m_1^3 = m_3 - 3m_1 m_2 + 2m_1^3$$