Reminder : Computation on matrices

Copernicus master
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Why Focus on Matrices in Statistical Learning ?

- Principles of statistical learning :
  - We aim to estimate an output variable (e.g., detection of objects in an image).
  - For this purpose, we have a series $X \in P \times N$ of **P measurements in N dimensions** (color, contrast, etc.) with a corresponding reference to the target object (training data).

    $\Longrightarrow$ We need to analyze this matrix (find relations, simplification, transformations, ...)

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Why Focus on Matrices in Statistical Learning ?

- We've seen that correlation or covariance matrices allow us to establish connections between data.

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X =

    -2     3    -1
    -1     1     0
     2    -1    -1
     1    -3     2

mat_var_cov =

    3.3333   -4.0000    0.6667
   -4.0000    6.6667   -2.6667
    0.6667   -2.6667    2.0000

mat_corr =

    1.0000   -0.8485    0.2582
   -0.8485    1.0000   -0.7303
    0.2582   -0.7303    1.0000
```

- How to summarize the information ? How to express the fact that the 3rd column of X depends on the first 2 ?
- Matrix analysis.

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Eigenvalues / Eigenvectors

Definition of an Eigenvalue

Let M be an $N \times N$ square matrix. A value λ will be an **eigenvalue** of M if there exists a non-zero vector x of size $N \times 1$ such that :

$$Mx = \lambda x \tag{1}$$

Definition of an Eigenvector

The values of x that satisfy equation (1) are called the **eigenvectors associated with** λ.

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Calculation of Eigenvalues / Eigenvectors

- We have :

$$Mx = \lambda x \Leftrightarrow (M - \lambda I_d)x = 0$$

where I_d is the identity matrix.

- For this relation to have non-trivial solutions ($x \neq 0$), the matrix $(M - \lambda I_d)$ must be linearly dependent.

$\Longrightarrow$

$$\det(M - \lambda I_d) = 0$$

Principle

1. We find the eigenvalues as the solutions of the equation $\det(M - \lambda I_d) = 0$
2. For each eigenvalue, we find a corresponding eigenvector by solving the equation $Mx = \lambda x$ where $x \neq 0$.

Examples/exercises

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Properties of Eigenvalues / Eigenvectors

- Eigenvectors associated with different eigenvalues are **linearly independent** (see Proof)
- **Matrix Decomposition** : Let Λ be the diagonal matrix with the N eigenvalues and Q be the diagonal matrix with, in the i-th column, the eigenvector corresponding to the i-th eigenvalue of Λ. We have :

$$M = Q\Lambda Q^{-1}$$

Verification.

- Many applications (raising to the power n, diagonalization, ...)
- If M is symmetric, the matrix of unit vectors of Q forms an **orthonormal basis**

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