EMPHASIS-GO is supported by the European Union (Grant Agreement: 101079772).

Completely Randomized Designs
&
Completely Randomized Block Designs
&
Fractional Factorial Experiments

Angélina El Ghaziri

angelina.elghaziri@institut-agro.fr

Experimental design principles
EMPHASIS training programme

1. Introduction
2. Completely Randomized Designs (CRD)
3. Completely Randomized Block Designs (CRBD)
4. Factorial and non-factorial designs

Objectives of the experimental methodology

Prepare data collection to improve their **quality**

- planned experiments in which variables are controlled

$\Rightarrow$ clear interpretation avoiding confusion

$\Rightarrow$ maximize the accuracy of the experiment.

Weighing example, Hotelling 1944

We consider three objects A, B and C with an unbiased balance which gives each result with an independent centered error and variance σ^2. It is assumed that the scale is also capable of weighing **two objects together**. The aim is to find the weights of these three objects.

1. We weight A, B, and C separately : the **cost** of the experiment is then that of three weighings, and the **precision** (variance of the error) obtained on the weights of A, B and, C is σ^2

Weighing example, Hotelling 1944

We consider three objects A, B and C with an unbiased balance which gives each result with an independent centered error and variance σ^2. It is assumed that the scale is also capable of weighing two objects together.

2. If we repeat each weighing twice : the **cost** of the experiment is then that of six weighings and the **accuracy** (variance of the error) obtained on the weighings of A, B and C is $\sigma^2/2$ (because average of the two weighings)

Weighing example, Hotelling 1944

We consider three objects A, B and C with an unbiased balance which gives each result with an independent centered error and variance σ^2. It is assumed that the scale is also capable of weighing two objects together.

3. We weigh two objects at the same time $A + B$, $A + C$ and $B + C$: the **cost** of the experiment is three weighings and its **accuracy** is $3\sigma^2/4$ (because the weights are obtained as the average of two weighings for three objects)

Conclusion

When a certain latitude is given in the preparation of the experimental design, gains can be easily obtained (in cost, accuracy...)

Some terminology

Experimental Units

The *experimental unit* is the basic element of the experiment.

- Each of the units is subjected to a particular object or treatment and leads to one or more *observations* at the end of the experiment.

Example

Agronomic field, particularly in crop production :
Base unit = *Parcel*.

- A parcel (in fields, forests, greenhouses) containing a certain number of plants.
- It can also be a single plant, part of a plant (a branch, a leaf, a fruit...), a group of trees.

Some terminology

Shape of an experimental unit

- When the plot (or available material) is homogeneous :
  - $\Rightarrow$ shape as **square** as possible (to reduce interference between neighboring plots and that the relative importance of possible borders is the weakest).
- When the terrain (or available material) shows heterogeneity in a given direction.
  - $\Rightarrow$ shape **rectangular**, elongated parallel to the general direction of this heterogeneity.

Appelation : ***Fertility gradient*** (in case of heterogeneity)

- A gradient of light or temperature in a greenhouse.
- Water supply condition along sloping plot.

Completely Randomized Designs (CRD)

A. Completely Randomized Designs for one factor

B. Completely Randomized Designs for two factors

Completely Randomized Designs : for one factor

Objective of the experiment : to study the effects of three Technical Itineraries (TI) (a,b,c) on the yield of wheat.

a 93.5	c 83.4	b 98	c 83.7	a 87.9	b 83.6	c 82.8	b 95.9	c 73.8
b 89.6	b 94.6	a 88.1	c 95.5	a 84.3	b 95.9	c 76.7	c 88.6	a 95.3
a 92.2	c 89.9	a 101	b 86.3	a 98	c 82.3	a 90.9	b 86.7	a 79.4
c 75.8	b 78.5	a 89.9	b 87.1	c 77.7	b 100	b 91.3	a 100	c 92.2

Experimental setup : **random partition** of 12 repetitions of each TI inside the parcel, supposedly **homogeneous**. The random partition of the factor levels ensure that each level has an equal chance of being affected by any other factors and thus to minimize any bias or confounding factors.

Two Statistical Problems :

1. Is there a significant effect of TI on the yield ?
2. Which TI is more efficient ?

The parameters of the design and the statistical model

- The experimental unit is a parcel
- The technical itinerary is the only factor of the experiment. It has 3 levels (qualitative).
- The dependent variable is the yield (quantitative).
- We have 12 repetitions for each TI ; the measures are distributed randomly.
- The analysis model is a one-way analysis of variance :

$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij} \tag{1}$$

  - Y_{ij} the dependent variable (*i.e.* the yield)
  - μ is the general average effect (*i.e.* reference yield)
  - α_i is the effect of the i^{th} level of the factor (*i.e.* the differential effect of level i)
  - ϵ_{ij} the residual random variable

Fertilizer experiment on tomatoes

Research question

Does the application of a certain fertilizer affect the growth and yield of tomato plants ?

Experimental design :

A completely randomized design where tomato plants are randomly assigned to one of two treatment groups : a control group that receives no fertilizer and a treatment group that receives the new fertilizer. Each treatment group consists of 20 tomato plants.

- In view of the dataset obtained ("fertilizer.xlsx", https ://github.com/AngelinaEG), propose the adequate statistical analysis for the researcher question.
- Perform the analysis with R following the three steps :

Step1 Statistical description of the data.

Step2 Adjustment of the model and verification of model requirements.

Step3 Hypothesis testing and conclusions.

How can we define the number of observations ?

To define the number of observations, we need to choose two elements :

- Test power
- Effect size

Test power

Definition

- We call test power, the probability “p” to reject the null hypothesis when it’s wrong (taking the right decision).
- We write the test power as : $p = 1 - \beta$ with β the type II error.

Reality / Decision test	H0 is true	H0 is false
H0 not rejected	Right decision Confidence level $1 - \alpha$	Bad decision (wrong acceptance) Error β of type II
H0 rejected	Bad decision (wrong rejection) Error α of type I	Right decision Power of the test $1 - \beta$

The test power should be at least equal to 0.8.

Effect size

The effect size is usually determined by the practitioner. He is most capable to say starting which size an effect constitute a significant difference.

In analysis of variance, effect size calculation

- For two levels of a factor (equivalent to t-test), the effect size is given by :

$$f = \frac{m_1 - m_2}{\sigma} \quad (2)$$

With m_1 and m_2 the mean values of the two modalities and σ the standard deviation on all observations.

- for more than two levels for the factor ($I > 2$), the effect size is given by :

$$f = \frac{\sqrt{\sum_{i=1}^{I} \frac{n_i}{n} (\mu_j - \mu)^2}}{\sigma} \quad (3)$$

$\Rightarrow$ If the effect size is small, we are close to the null hypothesis (the means are equal), we don't see the influence of the factor effect.

Effect size

How to determine effect size ?

Several methods exists to define the effect size :

- using past experiences or literature, to make assumptions on the values of the means and standard deviation.
- using conventional values suggested by Cohen :$f = 0.10$ (low), $f = 0.25$ (moderate) et $f = 0.4$ (high).

J.Cohen, 1988, Statistical power analysis for the behavioral sciences, Scd Edition, Lawrence Erlbaum Associates (pages : 274-284).

- In analysis of variance example, when the effect size is small, the null hypothesis is verified.

$\Rightarrow$ more we suspect that the factor effect is significant (Null hypothesis rejected), more the effect size is big and therefore we need less observations to prove it.

Example with R : libraries pwr, pwr2, ANOVA one factor

Find the test power in function of the number of observations and the effect size

For a one-way analysis of variance. $I = 3$ levels for the factor and $n_i = 5$ observations per level ($i = 1, .., 3$).

```
> library(pwr2) ; library(pwr)
> effectsize <- sapply(c("small", "medium", "large"),
+                     function(size) cohen.ES("anov", size = size)$effect.size)
> effectsize
small medium  large
0.10   0.25   0.40
> pwr.1way(k = 3, n=5, alpha = 0.05, f = effectsize)
Balanced one-way analysis of variance power calculation

k = 3
n = 5
effect.size = 0.10, 0.25, 0.40
sig.level = 0.05
power = 0.05896537, 0.10952969, 0.21374351

NOTE: n is number in each group, total sample = 15 power = 0.0589653654563358
n is number in each group, total sample = 15 power = 0.109529687261554
n is number in each group, total sample = 15 power = 0.213743511235321
```

Example with R : libraries pwr, pwr2, ANOVA one factor

Calculate the number of observation for a fixed power test and effect size

For a one-way analysis of variance, $I = 3$ levels for the factor and a power test equal to 0.8.

```
> ss.1way(k = 3, alpha = 0.05, beta = 0.20, f = effectsize["small"], B = 1000)
Balanced one-way analysis of variance sample size adjustment
k = 3
sig.level = 0.05
power = 0.8
n = 323
NOTE: n is number in each group, total sample = 969
> ss.1way(k = 3, alpha = 0.05, beta = 0.20, f = effectsize["medium"], B = 1000)
Balanced one-way analysis of variance sample size adjustment
k = 3
sig.level = 0.05
power = 0.8
n = 53
NOTE: n is number in each group, total sample = 159
> ss.1way(k = 3, alpha = 0.05, beta = 0.20, f = effectsize["large"], B = 1000)
Balanced one-way analysis of variance sample size adjustment
k = 3
sig.level = 0.05
power = 0.8
n = 22
NOTE: n is number in each group, total sample = 66
```

Example with R : library(pwr), one-way ANOVA

Calculate the number of observations using past experiment results

For a one-way ANOVA with three levels. If we have a past experiment results such as the mean squares of the factor and the residuals : $CM_{FA} = 237$ et $CM_{res} = 43$.

```
> library(pwr)
> power.anova.test(groups = 3, between.var = 237, within.var = 43, power=0.80)

Balanced one-way analysis of variance power calculation

groups = 3
n = 2.221835
between.var = 237
within.var = 43
sig.level = 0.05
power = 0.8

NOTE: n is number in each group
```

Completely Randomized Designs : for two factors

Objective of the experiment : compare four devices (Device factor) intended for measuring the humidity level of organic matter ; we chose three samples of this organic matter (Sample factor).

Mesures		Sample e1	Sample e2	Sample e3
Device	A1	19.4	18.9	16.8
		19.4	19.0	17.1
		18.8	18.2	17.1
	A2	18.6	18.9	17.1
		18.7	18.7	17.1
		18.2	18.5	16.8
	A3	18.9	19.5	17.8
		19.0	19.7	18.1
		20.0	19.0	17.5
	A4	19.0	19.3	16.9
		19.5	19.5	17.9
		19.1	19.4	18.0

It's a balanced completely randomized design for two factors : 36 observations (3 replication per Device x Sample).

A1e1	A1e2	A1e3	A2e1	A2e2	A2e3	A3e1	A3e2	A3e3	A4e1	A4e2	A4e3
A4e3	A3e1	A1e3	A2e3	A3e2	A1e1	A2e2	A1e2	A3e3	A2e1	A4e1	A4e2
A2e1	A1e1	A2e2	A3e1	A1e3	A2e3	A1e2	A3e3	A3e2	A4e1	A4e3	A4e2

Two statistical problems :

1. Is there interaction between device and sample ?
2. Is there a significant effect of the device or of the sample ?

The experimental design parameters and the model

- Number of observations $n = 36 := 3 \times 4 \times 3$; Nombre de répétitions par croisement $R = 3$:
- The experimental design is
  balanced : Same number of observations per crossing device x sample.

Device	A1	A2	A3	A4
sample e1	3	3	3	3
sample e2	3	3	3	3
sample e3	3	3	3	3

- The model is a two-way analysis of variance with interaction :

$$\boxed{\text{Humidity level} \sim \text{device} + \text{sample} + \text{device:sample}}$$

la notation : *device*:*sample* refers to the interaction between the two factors.

An experiment to analyze object detection by radar

An engineer is studying methods for improving the ability to detect targets on a radar scope. Two factors she considers to be important are the amount of background noise, or "ground clutter," on the scope and the type of filter placed over the screen. An experiment is designed using three levels of ground clutter and two filter types. We will consider these as fixed type factors. The experiment is performed by randomly selecting a treatment combination (ground clutter level and filter type) and then introducing a signal representing the target into the scope. The intensity of this target is increased until the operator observes it. The intensity level at detection is then measured as the response variable.

Source : Montgomery (1997) Design and analysis of experiments, page. 273.

- What is the experimental design used by this engineer ?
- In view of the data obtained ("telediction.xlsx", https ://github.com/AngelinaEG), what is the statistical analysis model of the data ?
- Perform the analysis with R following the three steps :

Step1 Statistical description of the data.

Step2 Adjustment of the model and verification of model requirements.

Step3 Hypothesis testing and conclusions.

An experiment to analyze object detection by radar

- What is the test power ?

```
>library(pwr2)
>pwr.2way(a=2,b=3,alpha=0.05,size.A=6,size.B=6,f.A=0.4,f.B=0.4)
     Balanced two-way analysis of variance power calculation

a = 2
b = 3
n.A = 6
n.B = 6
sig.level = 0.05
power.A = 0.6434088
power.B = 0.5239121
power = 0.5239121

NOTE: power is the minimum power among two factors
```

Completely Randomized Block Designs (CRBD)

A. Completely Randomized Block Designs

B. Latin Square

About "Blocks"

What is a block in an experimental design ?

A block refers to a group of subjects or experimental units that are 'similar' to each other. The blocks are used to decrease the residual variability while taking into consideration a part of the heterogeneity.

Examples

- A gradient of illumination or temperature in a greenhouse
- Variable conditions of drainage or irrigation of a slope

Remark

The block effect is considered as random. Considering a block as fixed effect does not change the test on the other effects, unless there's an interaction of the bloc factor with the other effect.

The Complete Randomized Block Design for one factor

Objectif : we compare the yield of tomatoes for 5 treatments a,b,c,d,t)

Bloc 1	a	d	t	b	c
	50.7	52.1	39.8	49.2	51.4

Bloc 2	b	d	c	a	t
	49.8	54.0	50.9	49.5	38.8

Bloc 3	b	c	a	t	d
	48.7	54.3	48.6	37.5	49.8

Bloc 3	a	t	d	c	b
	50.8	35.9	50.1	53.8	51.5

Bloc 5	c	a	t	b	d
	52.4	50.0	38.5	49.8	51.5

Each treatment appears only once in each of the blocks ; the bloc is supposedly “homogenous” on an agronomic level.

⇒ It’s a Randomized Complete Block Design with one factor.

The parameters of the design and the statistical model

- Number of observations $n = 5 \times 5 = 25$; Number of repetitions per treatment $R = 5$; this is the number of blocks
- The design is balanced ; therefore, it is orthogonal : each treatment appears once in each bloc.

Traitement	*A*	*B*	*C*	*D*	*T*
*block*1	1	1	1	1	1
*block*2	1	1	1	1	1
*block*3	1	1	1	1	1
*block*4	1	1	1	1	1
*block*5	1	1	1	1	1

- There are two explanatory sources for the variability : treatment and block. The analysis model is :

$$\boxed{\text{Rendement} \sim \text{Treatment} + \text{Block}}$$

The Complete Randomized Block Design for two factors

We compare the yield of the two varieties of tomatoes (V1,V2) for three treatments (T1,T2,T3) :

Bloc 1					
T1 V2	T3 V2	T1 V1	T2 V1	T2 V2	T3 V1

Bloc 2					
T2 V1	T3 V2	T3 V1	T1 V2	T2 V2	T1 V1

Bloc 3					
T2 V1	T3 V1	T1 V2	T1 V1	T2 V2	T3 V2

Bloc 4					
T1 V2	T1 V1	T3 V2	T3 V1	T2 V2	T2 V1

Bloc 5					
T3 V1	T1 V2	T1 V1	T2 V1	T2 V2	T3 V2

The explanatory sources of variability are : the two factors (treatment, variety) $= (F1, F2)(\{T1, T2, T3\}, \{V1, V2\})$ their interaction and the block. The analysis model is :

$$\boxed{y \sim F1 + F2 + F1{:}F2 + Bloc}$$

Block : are blocks fixed or random effects ?

What's a random effect ?

Fixed effect	Random effect
When the practitioner **choose** the treatments he wants to study	The practitioner **select randomly** the treatments to study among all treatments available.

- Effect of three seeds producers on the germination time. The aim is to study if there is a producer effect among these three on the seeds germination rate.

$\Rightarrow$ Fixed effect (the result is only true for these three tested producers)

- Effect of the source of seeds on the germination rate. Here, the practitioner select at random some producers among all that are available (a catalog).

$\Rightarrow$ Random effect (the results can be generalized to all producers)

In practice

- In most of the examples using analysis of variance, at least one factor is random (e.g. block effect, animal effect, parcel effect etc.) .
- If we consider block factor as fixed, it will not influence the results of the other factors, only if there is an interaction between the block and another factor.

$\Rightarrow$ If there is only **a block factor without interaction**, considering block factor as fixed or random does not change the tests results of other factors and the conclusions made.

Why it influence when there is interaction ?

Mixed models : a model with fixed and random factors.

Two-way analysis of variance with interaction

$$Y_{ijr} = \mu + \alpha_i + B_j + (C)_{ij} + \epsilon_{ijr}$$

α_i Fixed effect. B_j, $C_{ij} = (\alpha B)_{ij}$ random variables normally distributed independent, mean zero and constant variance (σ_B^2, σ_C^2), ϵ_{ijr} random variable $N(0, \sigma^2)$

$$H_0 : \alpha_1 = ... = \alpha_I = 0 \text{ et } H_0' : \sigma_B^2 = 0 \text{ et } H_0'' : \sigma_C^2 = 0$$

Interaction test : $F_{ab} = \frac{CM_{ab}}{CM_{residuel}}$
Main effects tests : $F_a = \frac{CM_a}{CM_{ab}}$ et $F_b = \frac{CM_b}{CM_{residuel}}$

The difference of means for the fixed factor are dependent of the chosen levels of the random factor. The random factor influence by means of the interaction term in the results of the fixed factor.

With R

With R

package lmerTest ; function *lmer* instead of *lm* function and factor in parenthesis : $(1|factor)$.

Latin Square

Question. How to define blocks when the field present two gradients ?

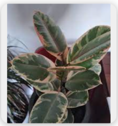
Idea. we define a block for each type of gradient.

Bloc ligne	Bloc colonnes					
	c1	c2	c3	c4	c5	c6
l1	2 28.7	5 28.4	4 25.4	3 30.7	1 30.6	6 30.9
l2	6 31.4	3 30.1	2 27.4	5 26.8	4 29.8	1 29.8
l3	4 29.4	6 29.7	1 30.4	2 22	5 24.1	3 32.9
l4	1 29.6	2 21.8	5 22.5	6 30	3 30.6	4 28.5
l5	3 25.8	4 21.9	6 23.1	1 24.3	2 20.7	5 17.7
l6	5 18.1	1 23.6	3 22.5	4 20.2	6 23.7	2 18.9

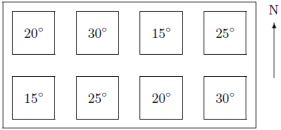
- The design is balanced for each type of block : each levels appear one time in each block line and column.
- Constraint. The number of levels of the factor is the same for each block. The number of levels p is the latin square order.

Example on latin square

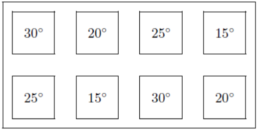
Experience of ground heating in greenhouse, in Belgium during winter on a decorative variety of *Ficus elastica*.



Ficus elastica Roxb.



Serre A



Serre B

Temp	Line	Column	Height
20	1	1	185
30	1	2	242
15	1	3	177
25	1	4	214
15	2	1	117
25	2	2	229
20	2	3	209
30	2	4	238
30	3	1	200
20	3	2	200
25	3	3	222
15	3	4	154
25	4	1	218
15	4	2	174
30	4	3	247
20	4	4	205

Mean height evolution by 'parcel', in mm, in function of the ground temperature, in degree celsius, per line and per column.

Model and choice of effects type

The model of analysis of a Latin square is

$$\boxed{Y \sim FA + Line + Column}$$

$$Y_{ijk} = \mu + a_i + b_j + c_r + \epsilon_{ijr} \tag{4}$$

with a_i, b_j et c_r the effects of the three factors and ϵ_{ijr} the residuals (i, j, and $r = 1, ..., p$).
$ddl_{FA} = ddl_{Ligne} = ddl_{Colonne} = p - 1$.
$ddl_{res} = (p^2 - 1) - 3 \times (p - 1) = (p - 2)(p - 1)$
This model implies the additivity of the three factors.

- When the factors line and column are **random**, the additivity hypothesis is not restrictive since the factor FA could be tested with the residuals variability.
- However, wen factors line and column (or one of them) are **fixed**, additivity hypothesis is more restrictive. This hypothesis should be verified by **an additivity test**.

Test of additivity with one degree of freedom (Tukey's One DoF Test, 1955)

Two-way analysis of variance for two factors A and B with $n = I \times J$ observations :

$$y_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ij} \tag{5}$$

Without replications, not enough observations to estimate the interaction $(\alpha\beta)_{ij}$. One way to test the non-additivity between the two factors is with Tukey additivity test.

- Consider the model :

$$y_{ij} = \mu + \alpha_i + \beta_j + \lambda\alpha_i\beta_j + \epsilon_{ij} \tag{6}$$

  and test the hypothesis $H_0 : \lambda = 0$ vs $H_1 : \lambda \neq 0$

Since α_i and β_j are already estimated, we need one degree of freedom to test λ.

With R : Tukey's One DoF Test

With R

Package **dae** : *Functions useful in the* ***D****esign and* ***A****NOVA of* ***E****xperiments* ;

$\Rightarrow$ function : *tukey.1df*

With R

- Import the dataset ‘FicusElastica.xlsx’ (https ://github.com/AngelinaEG)
- Install the libraries : *lmerTest* and *dae*
- Transform the variables to factors.
- Apply both scenarios.
  - First case scenario : line and column random effects
  - Second case scenario : line and column fixed effects
    - ⇒ With the function *tukey*.1*df* test the additivity of the three factors.
    - ⇒ Apply the analysis of variance.

First case scenario (line and column random)

Test factor temperature

```
>summary(aov(Height~Temp+Error(Line+Column),data=FicusElastica))
Error: Line
Df Sum Sq Mean Sq F value Pr(>F)
Residuals  3  661.2   220.4

Error: Column
Df Sum Sq Mean Sq F value Pr(>F)
Residuals  3   2833   944.2

Error: Within
Df Sum Sq Mean Sq F value   Pr(>F)
Temp        3  13616    4539   42.99 0.000189 ***
Residuals  6     633     106
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

⇒ Factor temperature is significant

First case scenario (line and column random)

Test of the random variables

```
> ranova(mod1)
Model:
Height ~ Temp + (1 |Line) + (1 |Column)
npar  logLik     AIC     LRT Df Pr(>Chisq)
<none>          7 -52.146 118.29
(1 | Line)      6 -52.435 116.87 0.5762  1    0.44779
(1 | Column)    6 -54.843 121.69 5.3923  1    0.02023 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

⇒ Factor column is significant (influence of direction East-West).

Second case scenario (all factors fixed)

Additivity test

```
>mod2<- lm(Height~Temp + Line + Column, data=FicusElastica)
>tukey.1df(mod2, data = FicusElastica)
$Tukey.F
[1] 3.425082
$Tukey.p
[1] 0.09123067
```

- For $\alpha = 0.05$, $F = 2.788 < qF(1, 5, 1 - \alpha) = 6.61$ (or $p - value = 0.09 \geq 0.05$)

$\Rightarrow$ We don't reject the null hypothesis of additivity.

Factors line and column fixed

```
>resF <- aov(Height~Temp + Line + Column, data=FicusElastica)
>(tabF <- summary(resF))
            Df Sum Sq Mean Sq F value   Pr(>F)
Temp         3  13616    4539  42.994 0.000189 ***
Line         3    661     220   2.088 0.203293
Column       3   2833     944   8.945 0.012397 *
Residuals    6    633     106
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$\Rightarrow$ There is influence of temperature and column factor (direction East-West).

How to construct a latin square of fourth order ?

- Etape 1. On construit un Carré Latin "régulier" en glissant d'une case chaque ligne
- Etape 2a. On randomise les colonnes
- Etape 2b. On randomise les lignes

Etape 1	Colonne 1	2	3	4
Ligne 1	a	b	c	d
2	d	a	b	c
3	c	d	a	b
4	b	c	d	a

Etape 2a	Colonne 2	4	1	3
Ligne 1	b	d	a	c
2	a	c	d	b
3	d	b	c	a
4	c	a	b	d

Etape 2b	Colonne 2	4	1	3
Ligne 4	c	a	b	d
1	b	d	a	c
2	a	c	d	b
3	d	b	c	a

Factorial and Non-factorial designs

A. Split-plot design

B. Hierarchical design

Factorial and Non-factorial designs

Factorial designs

The experiments when the levels of a factor are fully crossed with the other factors. All levels combinations are investigated simultaneously in a single experiment.

If there are two factors with three levels each, there will be $3 \times 3 = 32$ combinations to test in the experiment.

A special case of Factorial design is the split-plot.

Non-factorial designs

Non-factorial designs are experimental designs where the levels of a factor are not fully crossed with the levels of a second factor. A factor level will not have a value for all the levels of the second factor.

$\Rightarrow$ Hierarchical designs

Split-plot

Bloc 1		
G	P	Y
Victory	0	111
	0.2	130
	0.4	157
	0.6	174
Golden Rain	0	117
	0.2	114
	0.4	161
	0.6	141
Marvellous	0	105
	0.2	140
	0.4	118
	0.6	156

Bloc 2		
G	P	Y
Victory	0	61
	0.2	91
	0.4	97
	0.6	100
Golden Rain	0	70
	0.2	108
	0.4	126
	0.6	149
Marvellous	0	96
	0.2	124
	0.4	121
	0.6	144

Bloc 3		
G	P	Y
Victory	0	68
	0.2	64
	0.4	112
	0.6	86
Golden Rain	0	60
	0.2	102
	0.4	89
	0.6	96
Marvellous	0	89
	0.2	129
	0.4	132
	0.6	124

Bloc 4		
G	P	Y
Victory	0	74
	0.2	89
	0.4	81
	0.6	122
Golden Rain	0	64
	0.2	103
	0.4	132
	0.6	133
Marvellous	0	70
	0.2	89
	0.4	104
	0.6	117

Bloc 5		
G	P	Y
Victory	0	62
	0.2	90
	0.4	100
	0.6	116
Golden Rain	0	80
	0.2	82
	0.4	94
	0.6	126
Marvellous	0	63
	0.2	70
	0.4	109
	0.6	99

Bloc 6		
G	P	Y
Victory	0	53
	0.2	74
	0.4	118
	0.6	113
Golden Rain	0	89
	0.2	82
	0.4	86
	0.6	104
Marvellous	0	97
	0.2	99
	0.4	119
	0.6	121

- A split-plot is a block design where the factors are applied to two experimental units (e.u) : a big e.u (called also plot) divided into several smaller e.u. (called subplot).

Variety-Fertiliser, yield of oats

- Y= plant "yield"
- V = "Variety" of oats with $g = 3$ levels applied in each plot.
- N = "nutrient" with $p = 4$ levels applied in each subplot.

The split-plot model

- A Split-plot model is written as(see Sahai and Ageel, 2000 p. 513) :

$$Y_{ijk} = m + Bloc_k + V_i + E_{ik} + N_j + (V:N)_{ij} + e_{ijk}$$

  where E_{ik} and e_{ijk} are respectively the errors associated to the big e.u and small e.u.

- The analysis of variance sum of squares and degrees of freedom are given by :

$$\underset{Rgp-1}{SCE_T} \underset{=}{=} \underset{R-1}{SCE_{Block}} + \underset{g-1}{SCE_V} + \underset{(R-1)(g-1)}{SCE_E} + \underset{p-1}{SCE_N} + \underset{(g-1)(p-1)}{SCE_{V:N}} + \underset{g(R-1)(p-1)}{SCE_e}$$

- Analysis principal : we study the effect of each factor compared to the residual variability associated with its experimental unit.

Split-plot with R

The Oats dataset

```
>library(MASS)
>data(oats)
>oats
>model <- lmer(Y ~ V + N + V:N + (1|B) + (1|B:V),
data = oats)
anova(model)

Type III Analysis of Variance Table with Satterthwaite's method
Sum Sq Mean Sq NumDF DenDF F value    Pr(>F)
V      526.1   263.0     2    10  1.4853    0.2724
N    20020.5  6673.5     3    45 37.6857 2.458e-12 ***
V:N    321.7    53.6     6    45  0.3028    0.9322
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$\Rightarrow$ Factor Nutrient is significant

Hierarchical design

Insecticide : each company propose different products

Société	Produit										
	1	2	3	4	5	6	7	8	9	10	11
A	151	118	131								
	135	132	137								
	137	135	121								
B				140	151						
				152	132						
				133	139						
C						96	84				
						108	87				
						94	82				
D								79	67	90	83
								74	78	81	89
								73	63	96	94

- Company A propose products 1, 2, 3.
- Company B propose products 4, 5.
- Company C propose products 6, 7.
- Company D propose products 8, 9, 10, 11.

$Y \sim Company + Product(Company)$

Factor product is Le facteur produit is dependent of the company : the levels of products are not the same according to the company.

The hierarchical design model

The linear model for the hierarchical design is given by :

$$y_{ijk} = \mu + \tau_i + \beta_{j(i)} + \epsilon_{(ij)k}$$

for $i = 1, \ldots, a$, $j = 1, \ldots, b$, $k = 1, \ldots, n_{ij}$.
where :

- a is the number of levels of factor A (the company here) ;
- b is the number of levels of factor B (products here) nested in the factor A ;
- n_{ij} number of repetitions.

The index $j(i)$ indicates that the j^{th} level of factor B is nested in the i^{th} level of factor A.

With R : if both factors are fixed

```
>model= lm(InsectsCount~(Company/Product), data = InsectsProducts)
>anova(model)

Analysis of Variance Table

Response: InsectsCount
                Df  Sum Sq Mean Sq F value    Pr(>F)
Company          3 22813.3  7604.4 132.776 3.048e-14 ***
Company:Product  7  1500.6   214.4   3.743  0.008098 **
Residuals       22  1260.0    57.3
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With R : if the company is fixed and product is random

```
>library(lmerTest)
>mixmodel = lmer(InsectsCount~Company + (1|Company:Product),
 data = InsectsProducts)

>anova(mixmodel) #test fixed effect

Type III Analysis of Variance Table with Satterthwaite's method
        Sum Sq Mean Sq NumDF DenDF F value    Pr(>F)
Company   6095  2031.7     3     7  35.474 0.0001327 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>ranova(mixmodel)#test random effect

ANOVA-like table for random-effects: Single term deletions

Model:
InsectsCount ~ Company + (1 | Company:Product)
                       npar  logLik    AIC    LRT Df Pr(>Chisq)
<none>                    6 -108.59 229.19
(1 | Company:Product)     5 -111.34 232.69 5.4951  1    0.01907 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With R : if both factors are random

```
randmodel= lmer(InsectsCount~(1|Company/Product), data = InsectsProducts)
ranova(randmodel)

ANOVA-like table for random-effects: Single term deletions

Model:
InsectsCount ~ (1 | Product:Company) + (1 | Company)
                        npar  logLik    AIC     LRT Df Pr(>Chisq)
<none>                    4 -123.74 255.48
(1 | Product:Company)     3 -126.49 258.98  5.4931  1  0.0190913 *
(1 | Company)             3 -130.66 267.32 13.8390  1  0.0001992 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Take away message

It's essential to think about the experimental design beforehand :

A. to gain time and accuracy.

B. to not miss studying any effects.

C. to not confuse main factors with other factors (*e.g.*, environmental factors).

D. to associate the correct model for the analysis.

References

- Hardeo Sahai, Mohammed I. Ageel, The Analysis of Variance : Fixed, Random and Mixed Models, Springer Science & Business Media, 2000.
- Philip M. Dixon, 2016, “Should blocks be fixed or random ?”, Conference on Applied Statistics in Agriculture.
- Pierre Dagnelie, analyse de la variance, statistique théorique et appliquée. Vol2, inférence statistique à une et à deux dimensions, 3e édition, 2013.
- Pierre Dagnelie, principe d’expérimentation : planification des experiences et analyse de leurs résulats, Les presses agronomiques de Gembloux, 2003