Barycentric Coordinates (and Some Texture Mapping)

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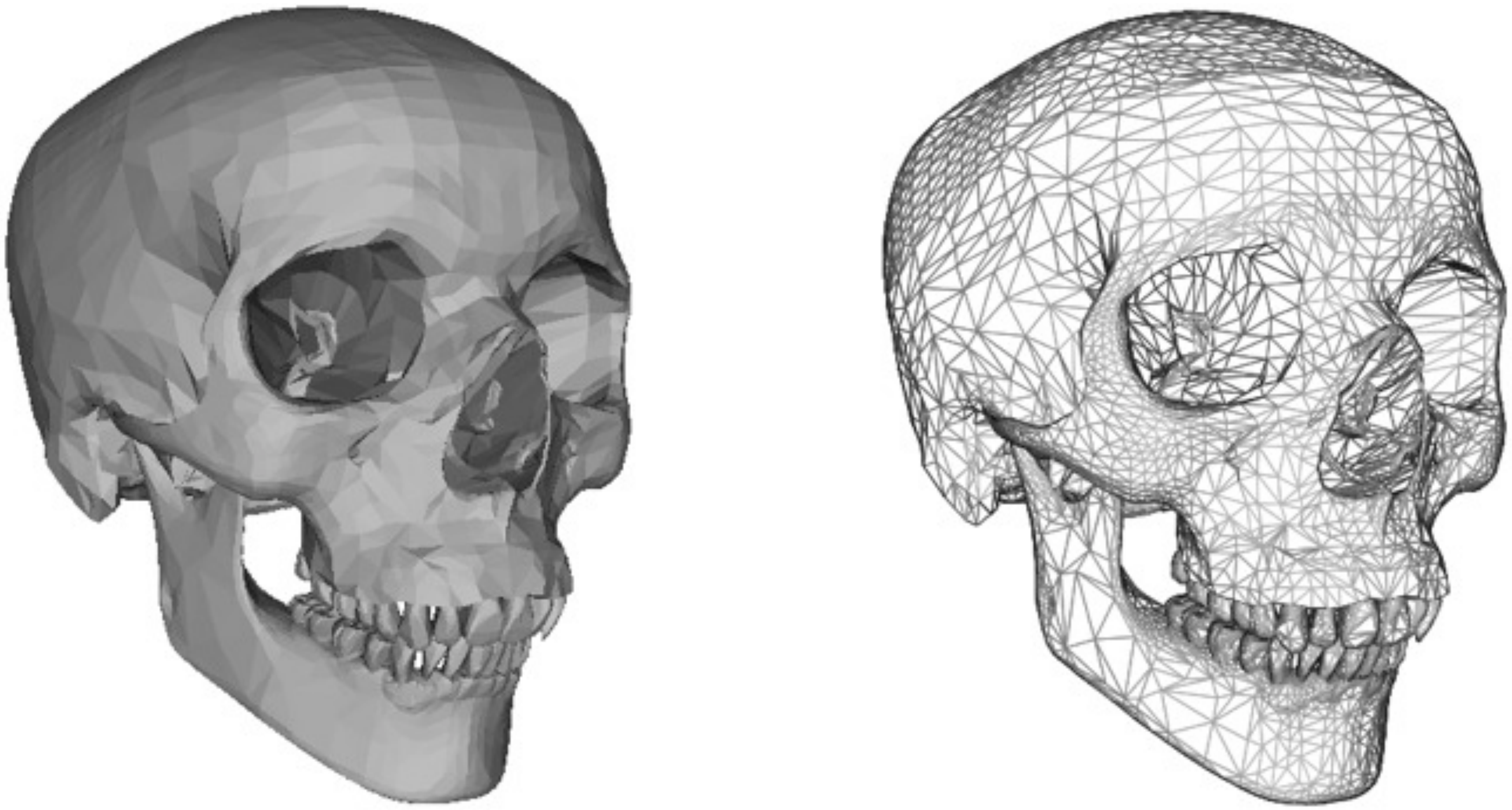
CS 4810: Graphics

Acknowledgment: slides by Misha Kazhdan, Allison Klein, Tom Funkhouser, Adam Finkelstein and David Dobkin

Triangles

These are the basic building blocks of 3D models.

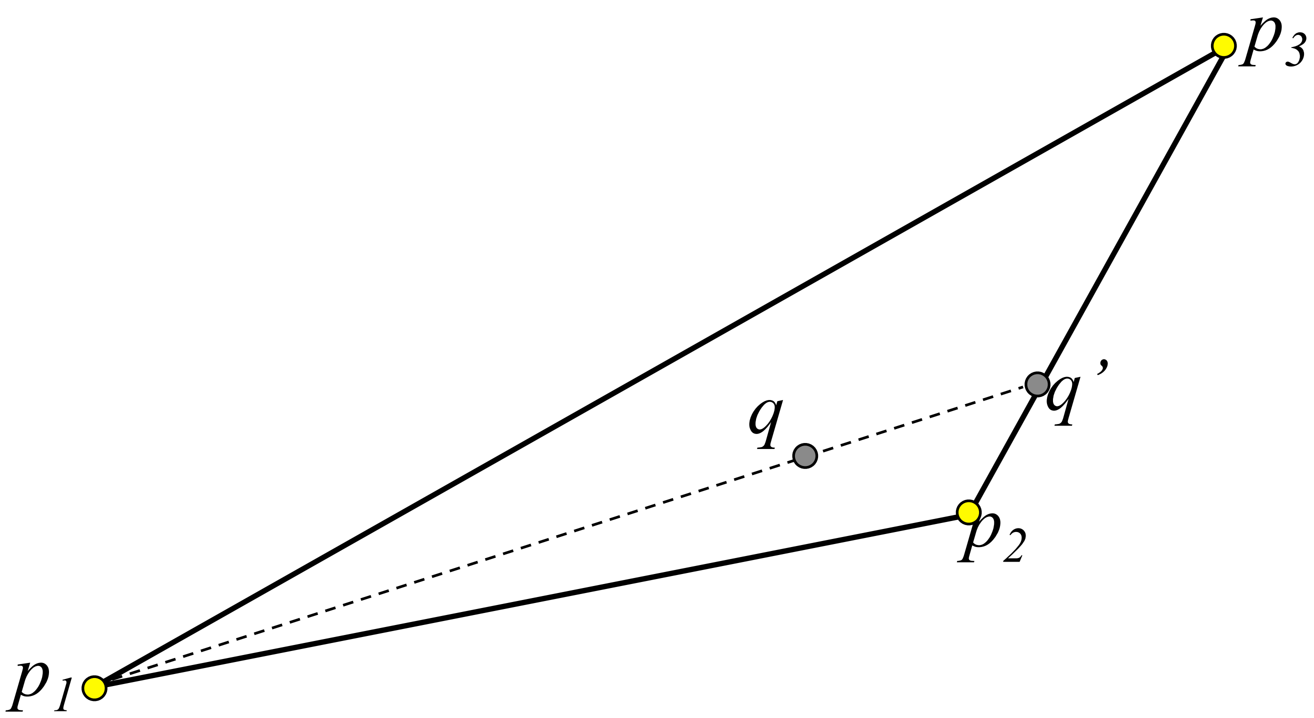
- Often 3D models are complex, and the surfaces are represented by a triangulated approximation.



Triangles

A triangle is defined by three non-collinear vertices:

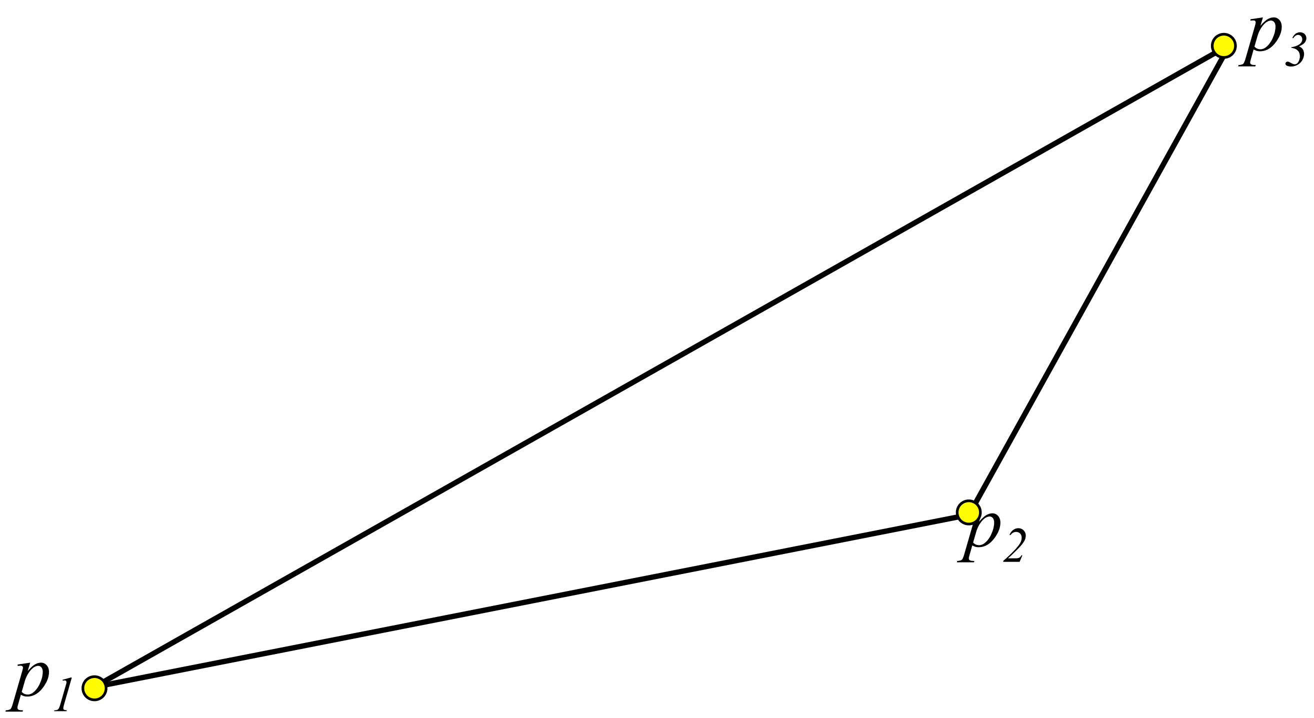
- Any point q in the triangle is on the line segment between one vertex and some other point q' on the opposite edge.



Barycentric Coordinates

A triangle is defined by three non-collinear vertices:

- Any point q in the triangle is on the line segment between one vertex and some other point q' on the opposite edge.
- Any point on the triangle can be expressed as:
  - $q = \{ \alpha p_1 + \beta p_2 + \gamma p_3 \mid \alpha + \beta + \gamma = 1,\ \alpha, \beta, \gamma \geq 0 \}$



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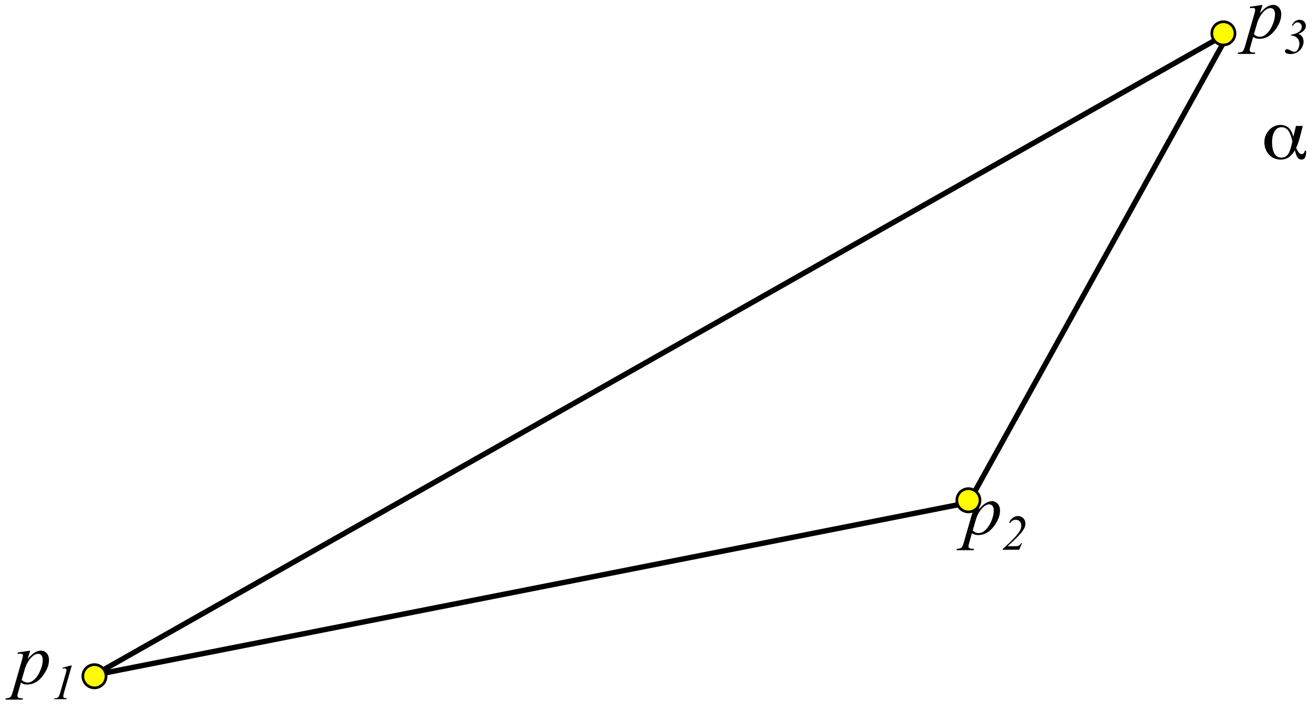

$$\alpha \mathrm{p}_1+\beta \mathrm{p}_2+\gamma \mathrm{p}_3=\alpha \mathrm{p}_1+(1-\alpha)\left(\frac{\beta \mathrm{p}_2+\gamma \mathrm{p}_3}{1-\alpha}\right)$$

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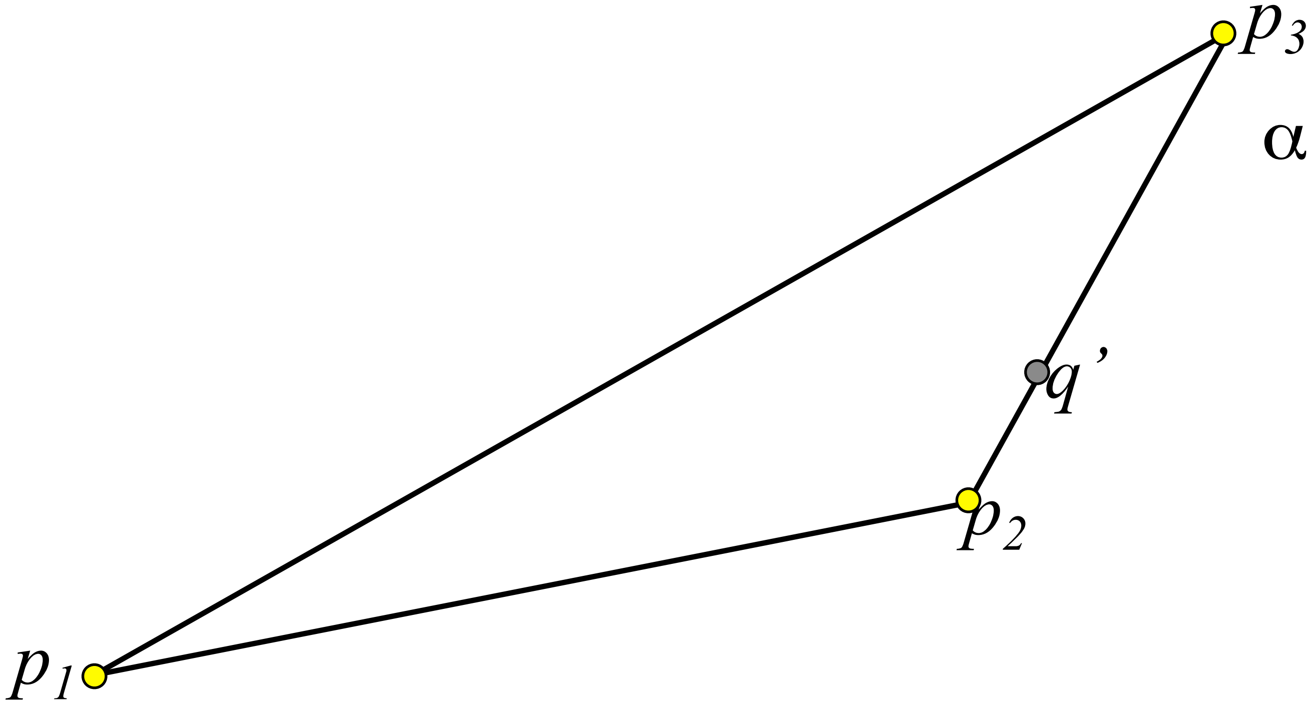

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$$= \alpha \mathrm{p}_1 + (1-\alpha)\boxed{\left(\frac{\beta \mathrm{p}_2 + \gamma \mathrm{p}_3}{\beta + \gamma}\right)}$$

A point q' on the segment between p_2 and p_3

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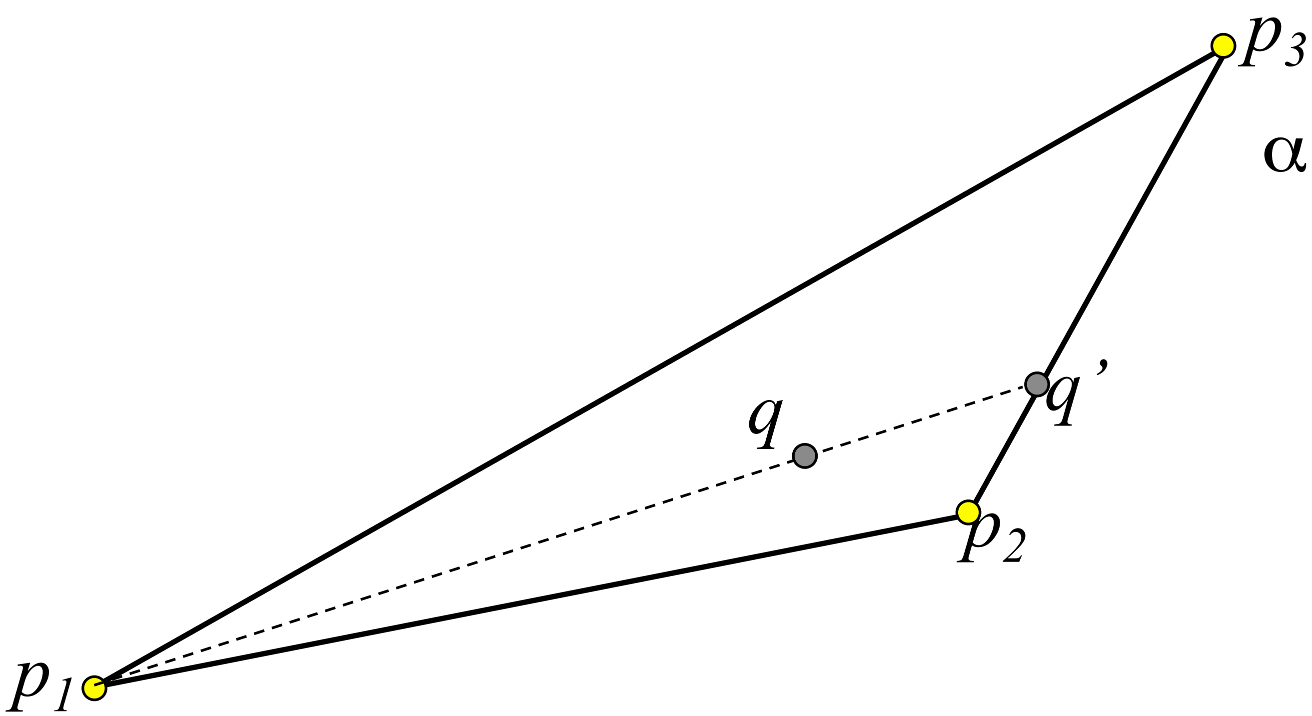

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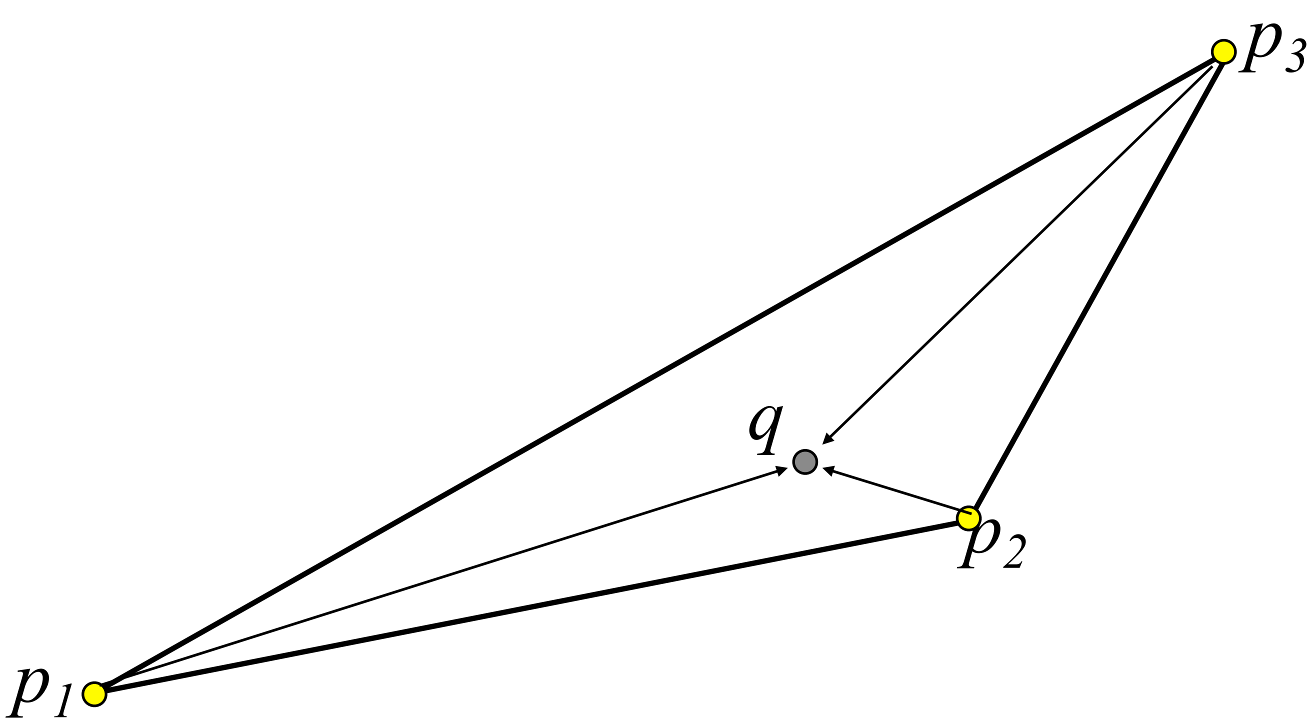
A point *q* on the segment between p_1 and *q*

Barycentric Coordinates

The barycentric coordinates of a point *q:*

$$q = \alpha p_1 + \beta p_2 + \gamma p_3$$

allow us to express *q* as a weighted average of the vertices of the triangles.



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Questions:

•What happens if α,β, or $\gamma < 0$?

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Questions:

- What happens if α,β, or $\gamma < 0$?
  - q is not inside the triangle but it is in the plane spanned by p_1, p_2, and p_3.

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Questions:

•What happens if α,β, or $\gamma < 0$?

•What happens if $\alpha+\beta+\gamma\neq 1$?

Note: If we force $\alpha=1-\beta-\gamma$, we always get $\alpha+\beta+\gamma=1$ so the point q is always in the plane containing the triangle

Barycentric Coordinates

Barycentric coordinates are needed in:

- Ray-Tracing, to test for intersection
- Rendering, to interpolate triangle information

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- Ray-Tracing, to test for intersection
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```
Float TriangleIntersect(Ray r, Triangle tgl) {
	Plane p=PlaneContaining( tgl );
	Float t = IntersectionDistance( r, p );
	if (t < 0 ) { return -1;}
	else {
		(α, β, γ) = Barycentric( r(t), tgl);
		if (α < 0  or β < 0 or γ < 0 ) { return -1;}
		else { return t; }
	}
}
```

Barycentric Coordinates

Barycentric coordinates are needed in:

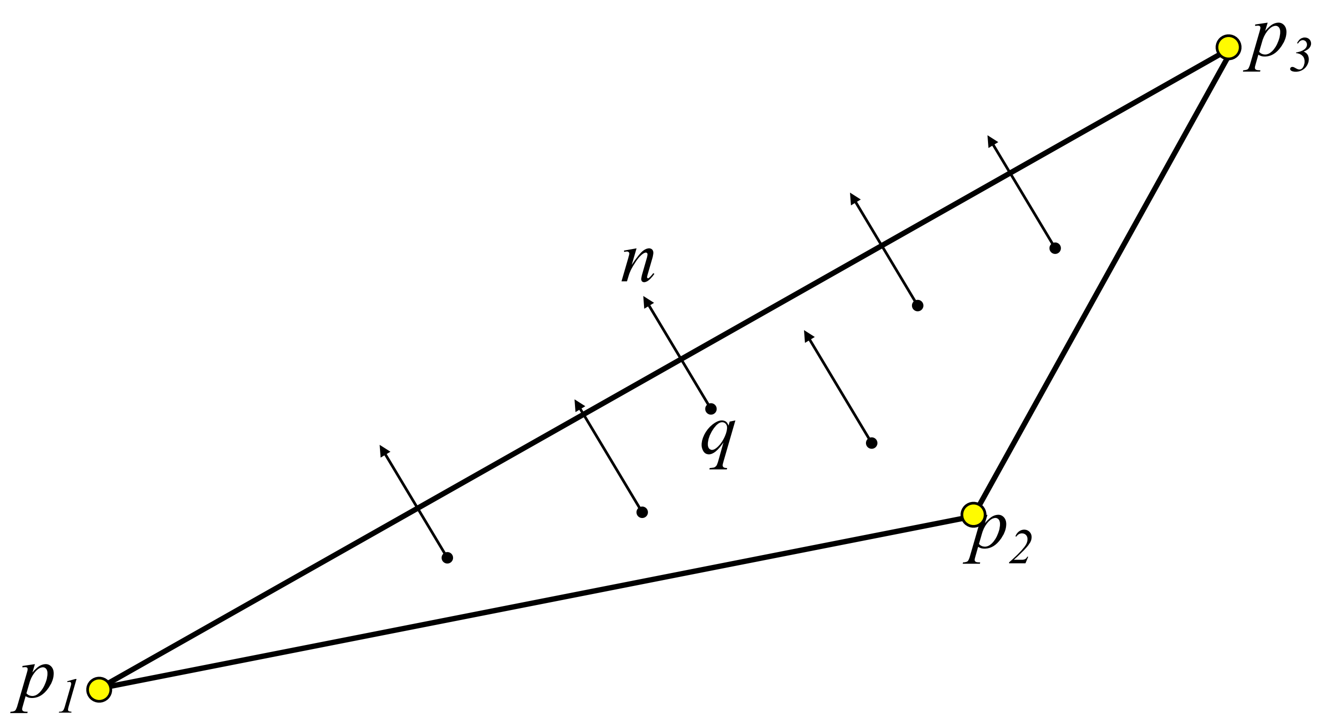
- Ray-Tracing, to test for intersection
- Rendering, to interpolate triangle information
  - In 3D models, information is often associated with vertices rather than triangles (e.g. color, normals, etc.)

Barycentric Coordinates

For example:

- We could associate the same normal/color to every point on the face of a triangle by computing:

$$\mathrm{n} = \frac{(\mathrm{p}_2 - \mathrm{p}_1)\times(\mathrm{p}_3 - \mathrm{p}_1)}{\left\|(\mathrm{p}_2 - \mathrm{p}_1)\times(\mathrm{p}_3 - \mathrm{p}_1)\right\|}$$



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- We could associate the same normal/color to every point on the face of a triangle by computing:

$$n = \frac{(p_2 - p_1)\times(p_3 - p_1)}{\left\|(p_2 - p_1)\times(p_3 - p_1)\right\|}$$

Triangle Normals

This gives rise to flat shading/ coloring across the faces

Barycentric Coordinates

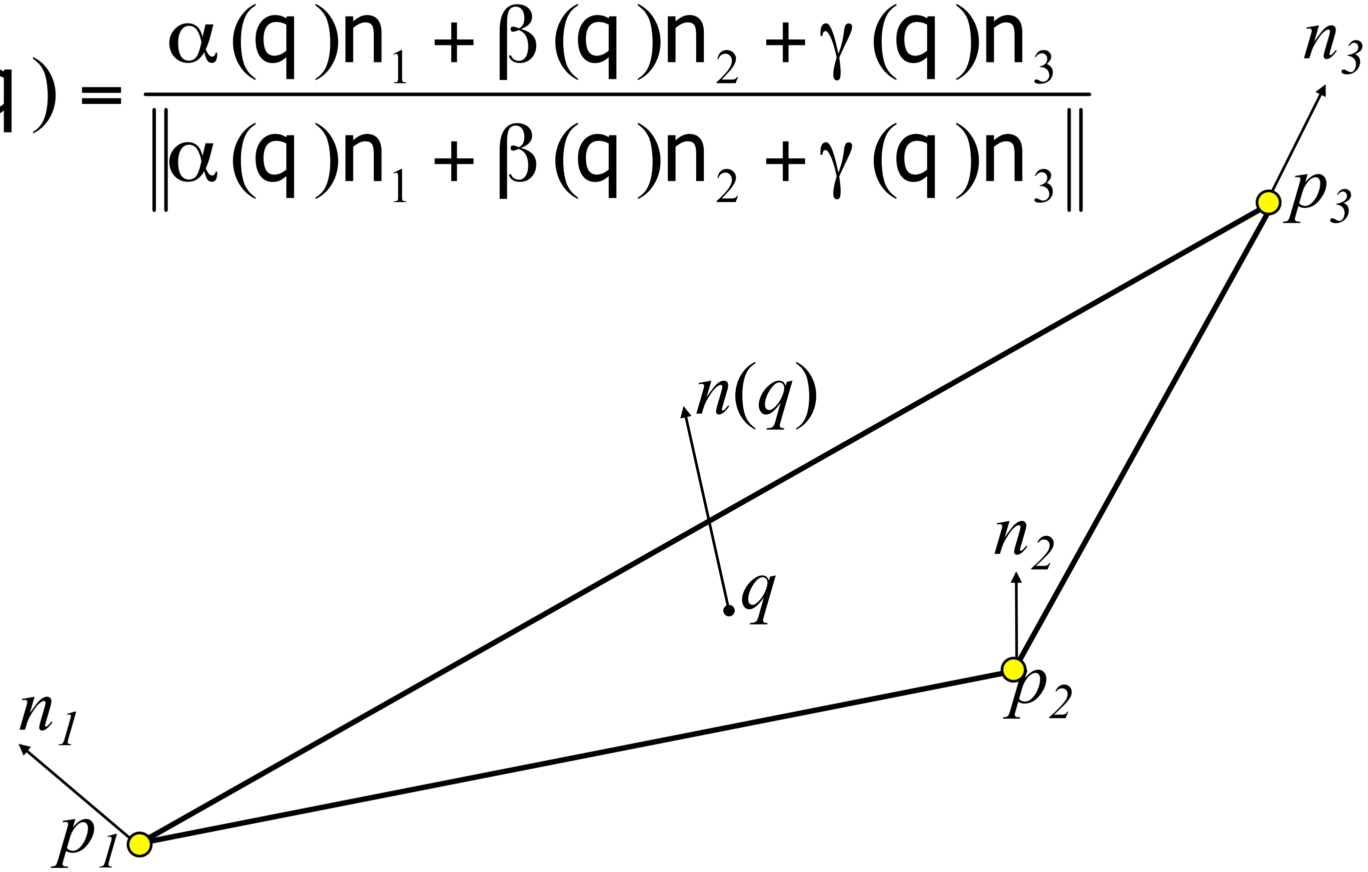
Instead:

- We could associate normals to every vertex:

$$T = \big((p_1, n_1), (p_2, n_2), (p_3, n_3)\big)$$

so that the normal at some point *q* in the triangle is the interpolation of the normals at the vertices:

$$n(q) = \frac{\alpha(q)n_1 + \beta(q)n_2 + \gamma(q)n_3}{\left\|\alpha(q)n_1 + \beta(q)n_2 + \gamma(q)n_3\right\|}$$



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Triangle Normals

Interpolated Point Normals

Barycentric Coordinates

So given the points p_1, p_2, and p_3, how do we compute the barycentric coordinates of a point q in the plane spanned by p_1, p_2, and p_3?

<u>Matrix Inversion</u>:

We can approach this is as a linear system with three equations and two unknowns:

$$q_x = (1-\beta-\gamma)p_{1x} + \beta p_{2x} + \gamma p_{2x}$$
$$q_y = (1-\beta-\gamma)p_{1y} + \beta p_{2y} + \gamma p_{2y}$$
$$q_z = (1-\beta-\gamma)p_{1z} + \beta p_{2z} + \gamma p_{2z}$$

Barycentric Coordinates

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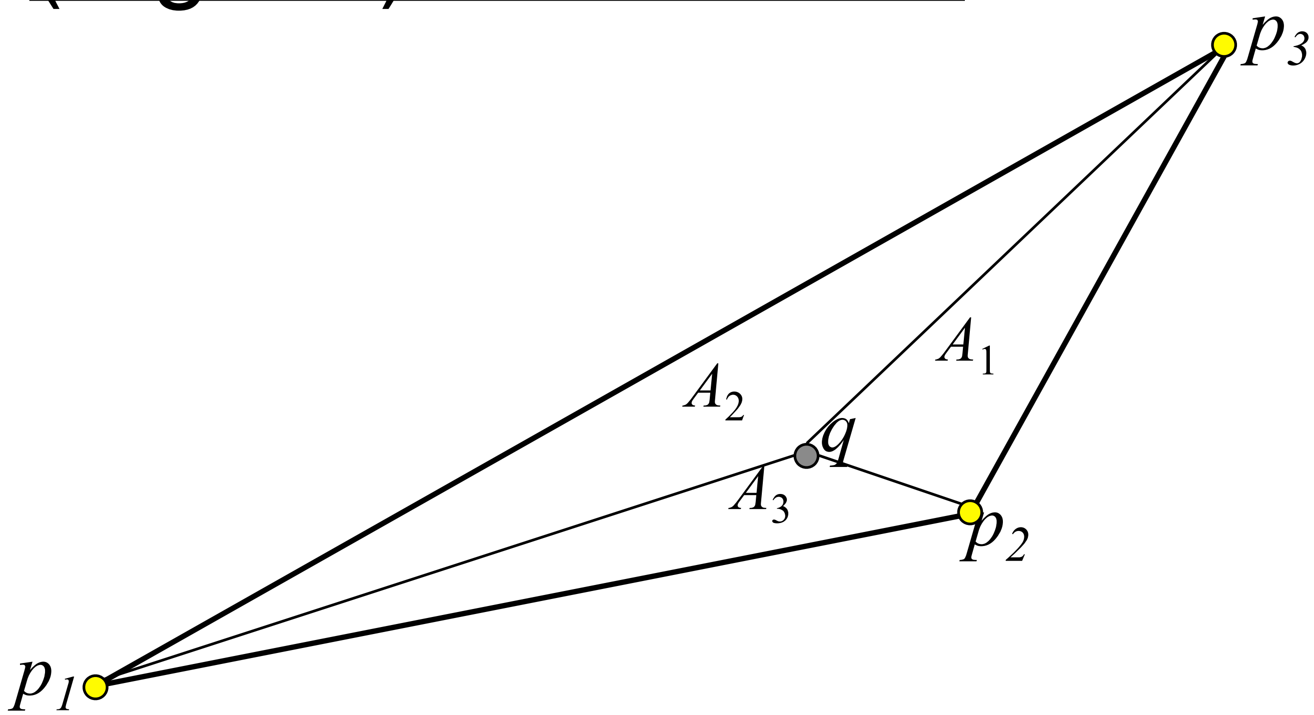
<u>(Signed) Area Ratios</u>:



$$\alpha = \frac{A_1}{A_1 + A_2 + A_3}$$

$$\beta = \frac{A_2}{A_1 + A_2 + A_3}$$

$$\gamma = \frac{A_3}{A_1 + A_2 + A_3}$$

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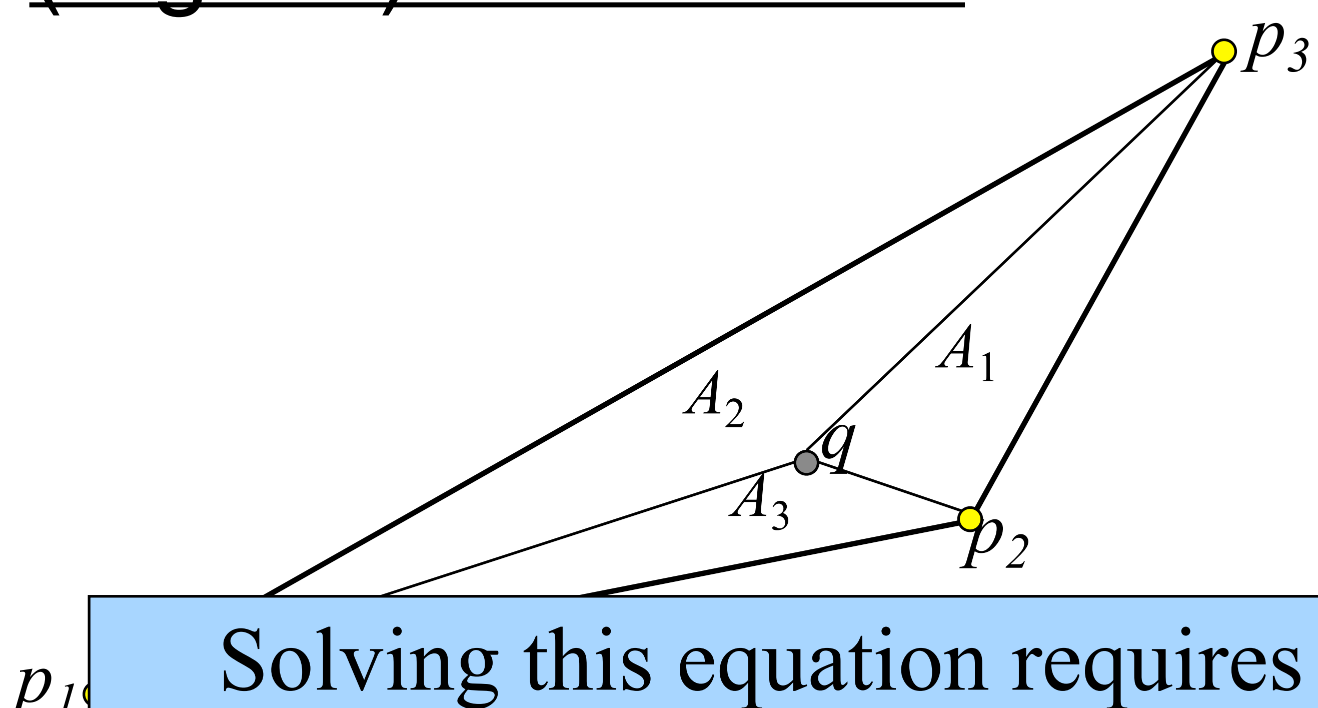

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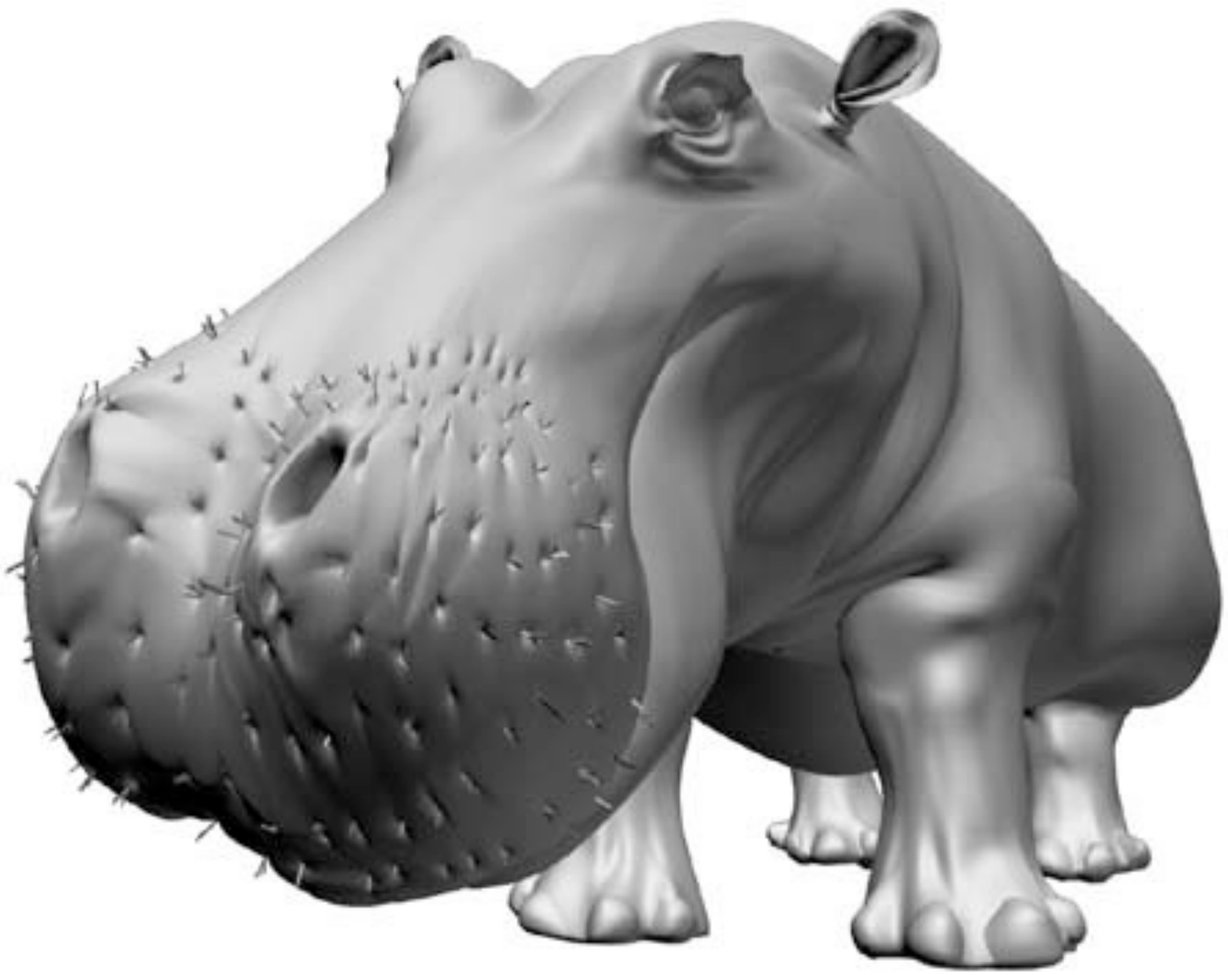
$$\gamma = \frac{A_3}{A_1 + A_2 + A_3}$$

Solving this equation requires computing the areas of three triangles for every point q. (DERIVATION IN CLASS)

Texture Mapping (Briefly, More Later)



J. Birn

Textures

- How can we go about drawing surfaces with complex detail?

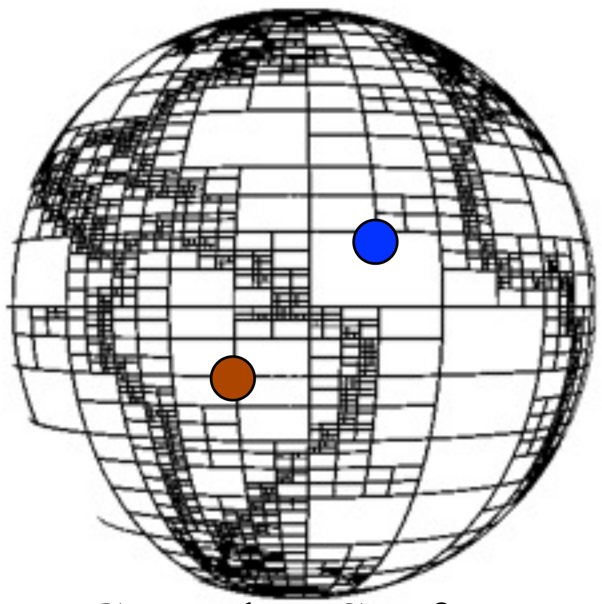
Target Model

Textures

- How can we go about drawing surfaces with complex detail?

Target Model

- We could tessellate the sphere in a complex fashion and then associate the appropriate material properties to each vertex



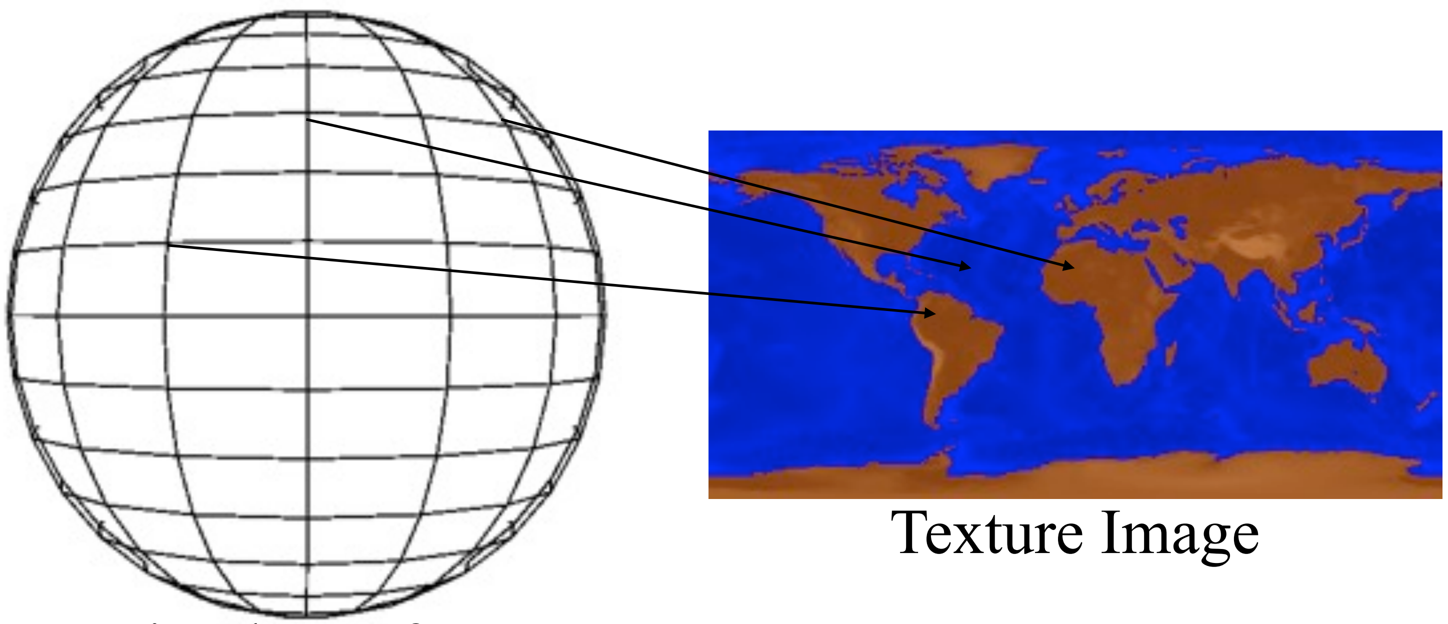
Complex Surface

Textures

- How can we go about drawing surfaces with complex detail?

Target Model

- We could use a simple tessellation and use the location of surface points to look up the appropriate color values

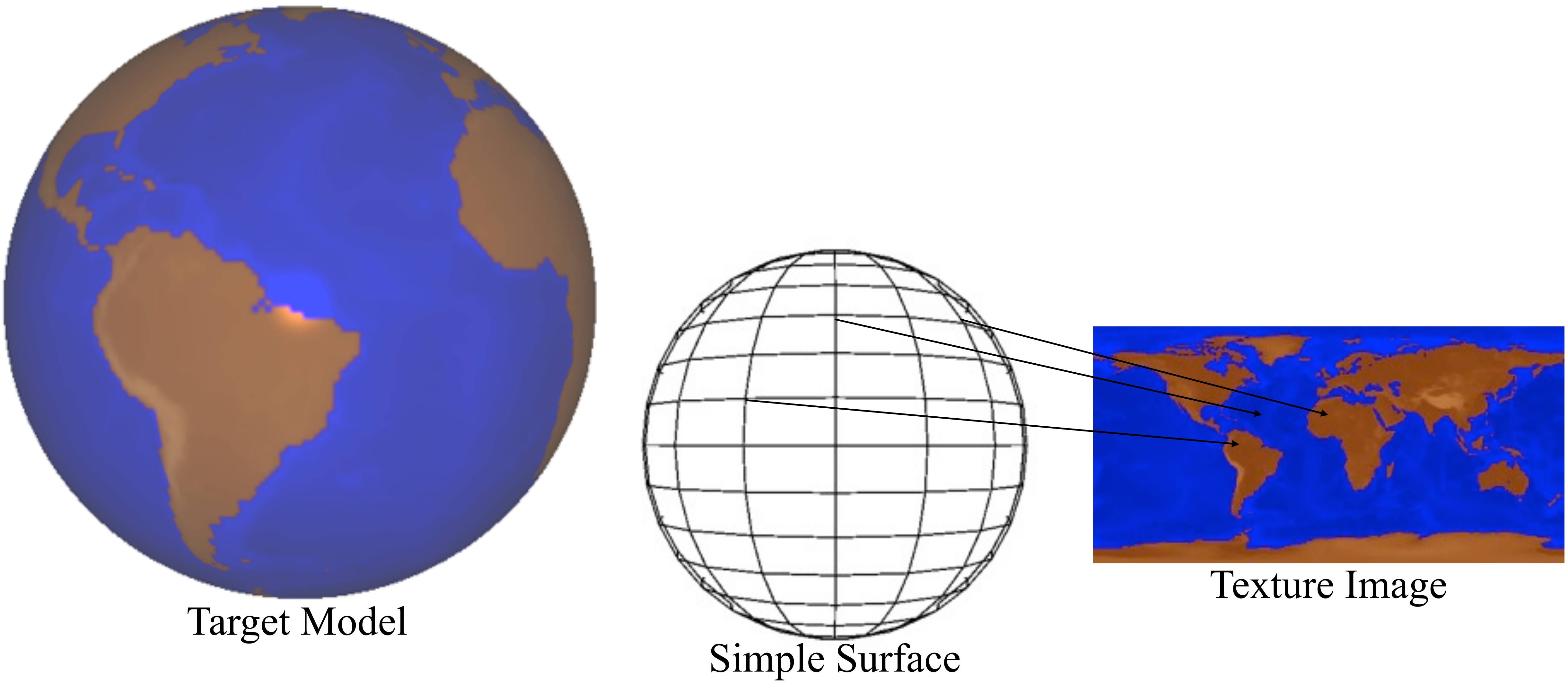


Simple Surface

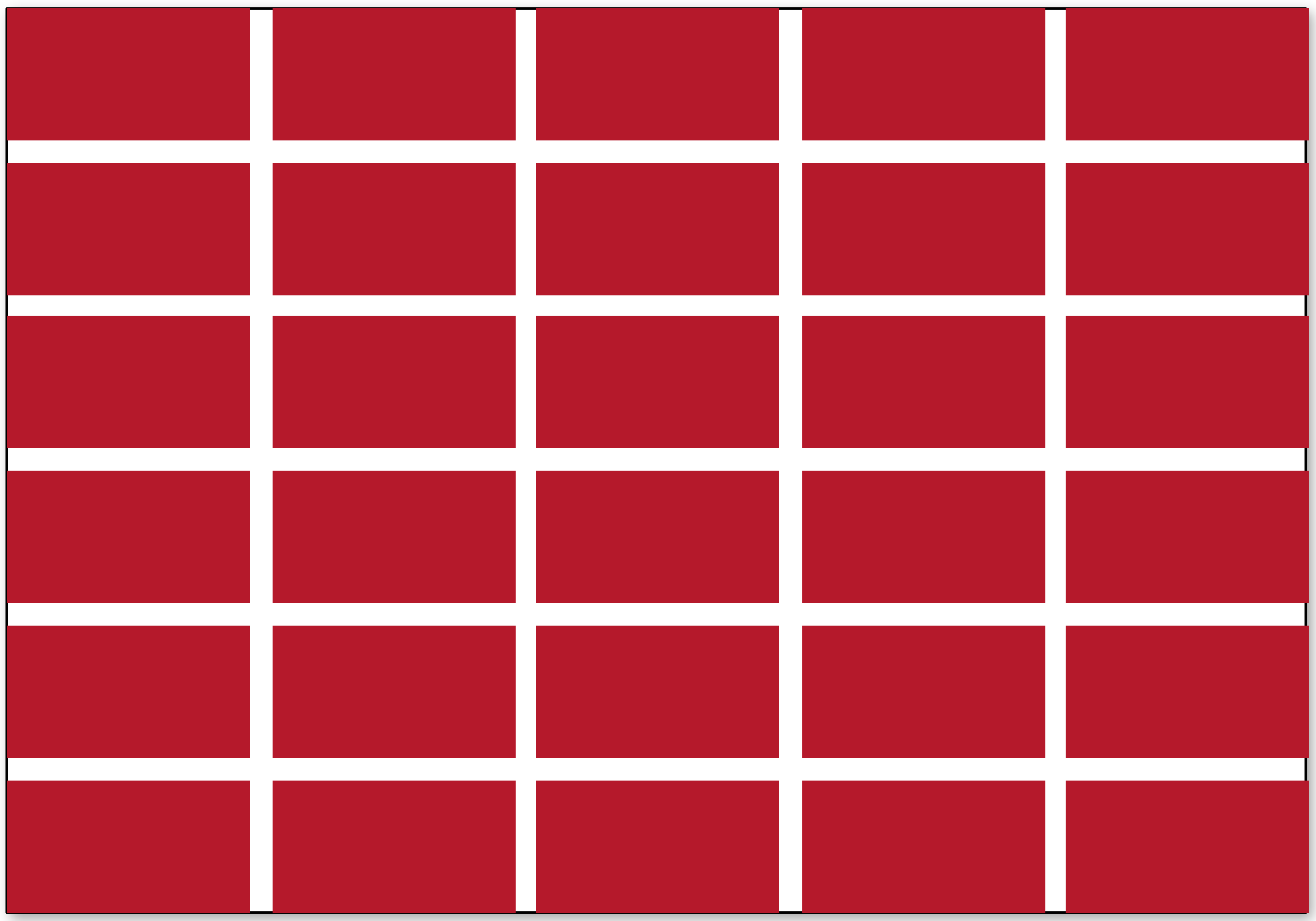
Texture Image

Textures

- Advantages:
  - The 3D model remains simple
  - It is easier to design/modify a texture image than it is to design/modify a surface in 3D.



Another Example: Brick Wall



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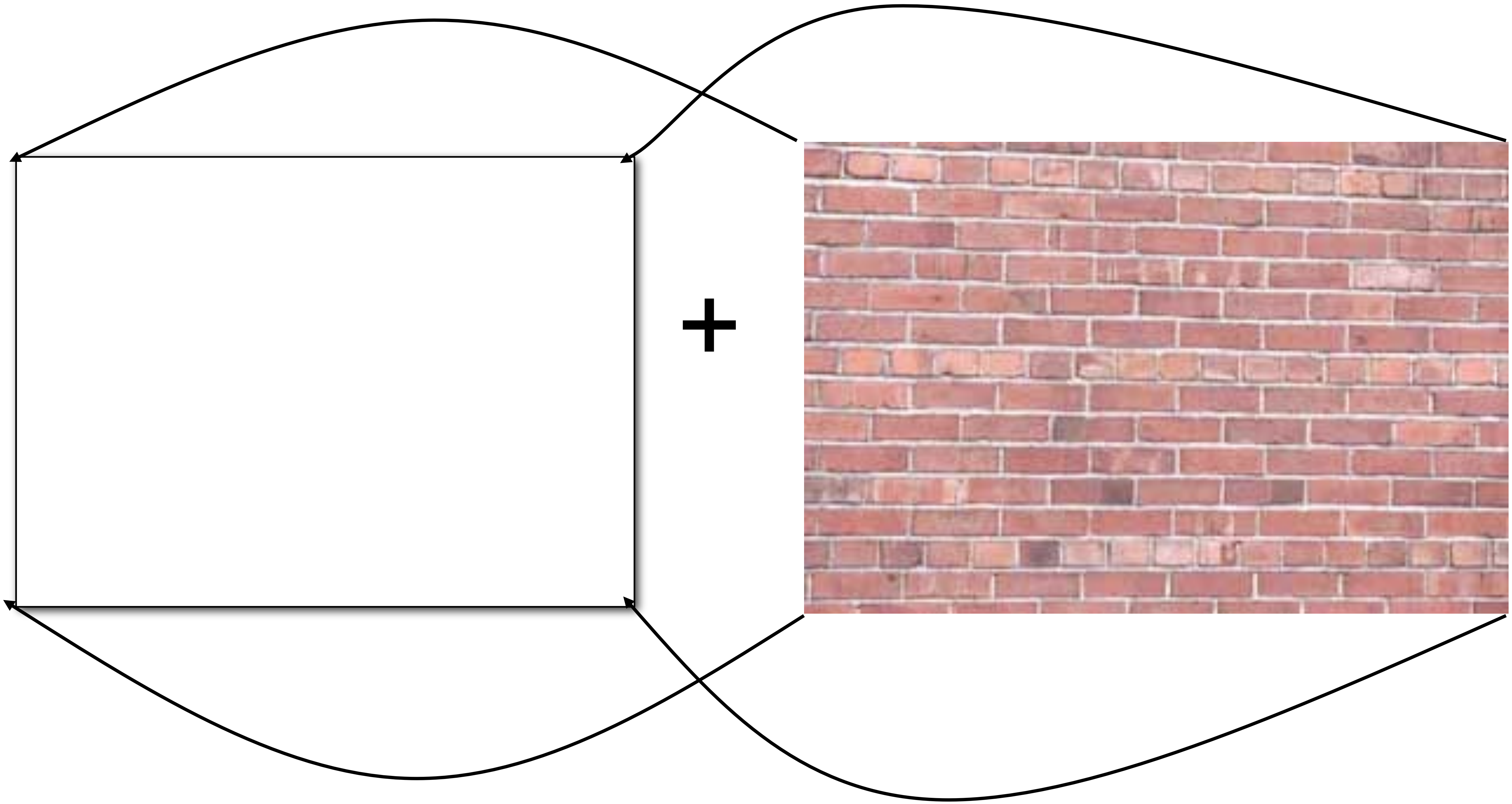
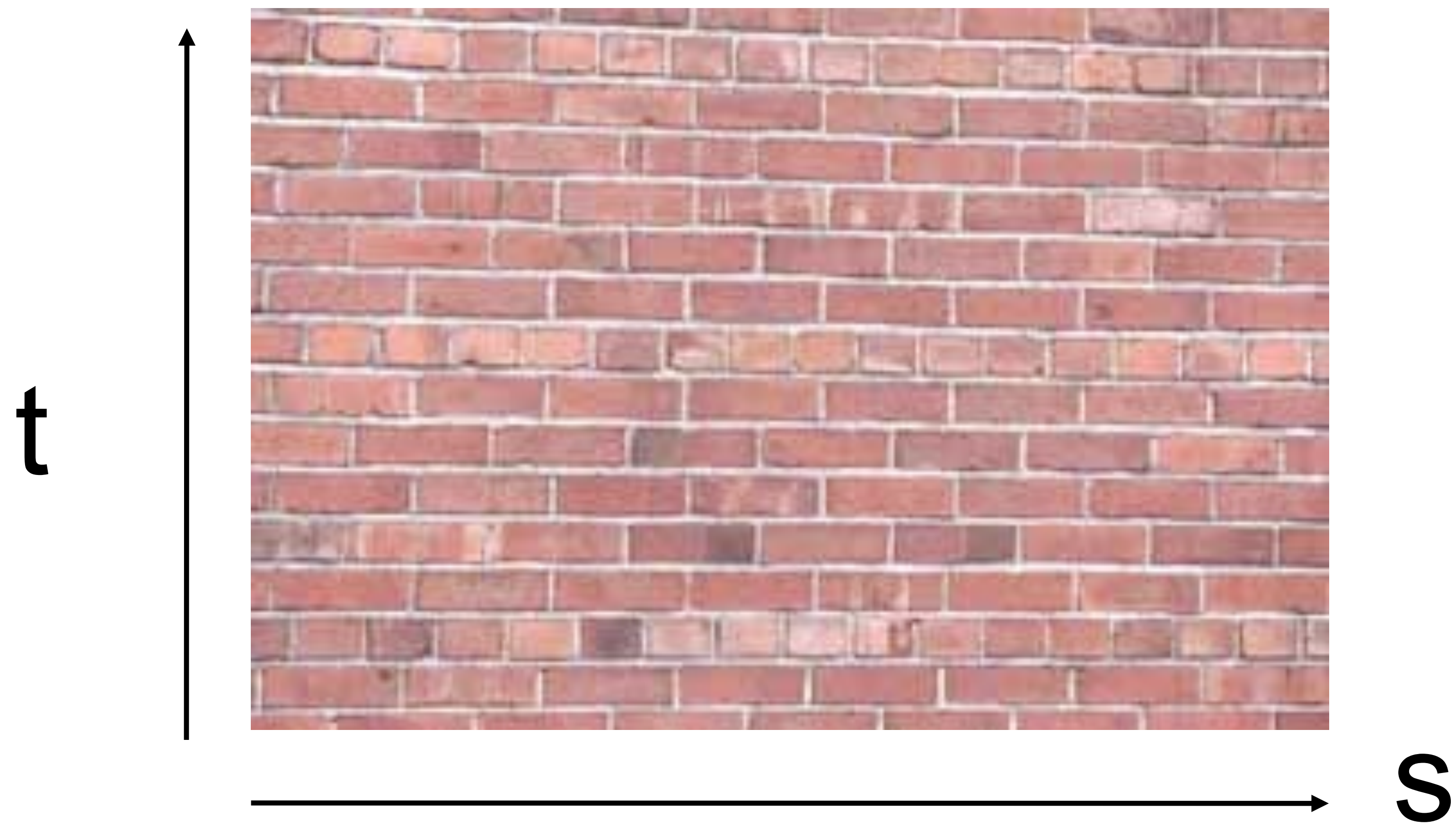


2D Texture

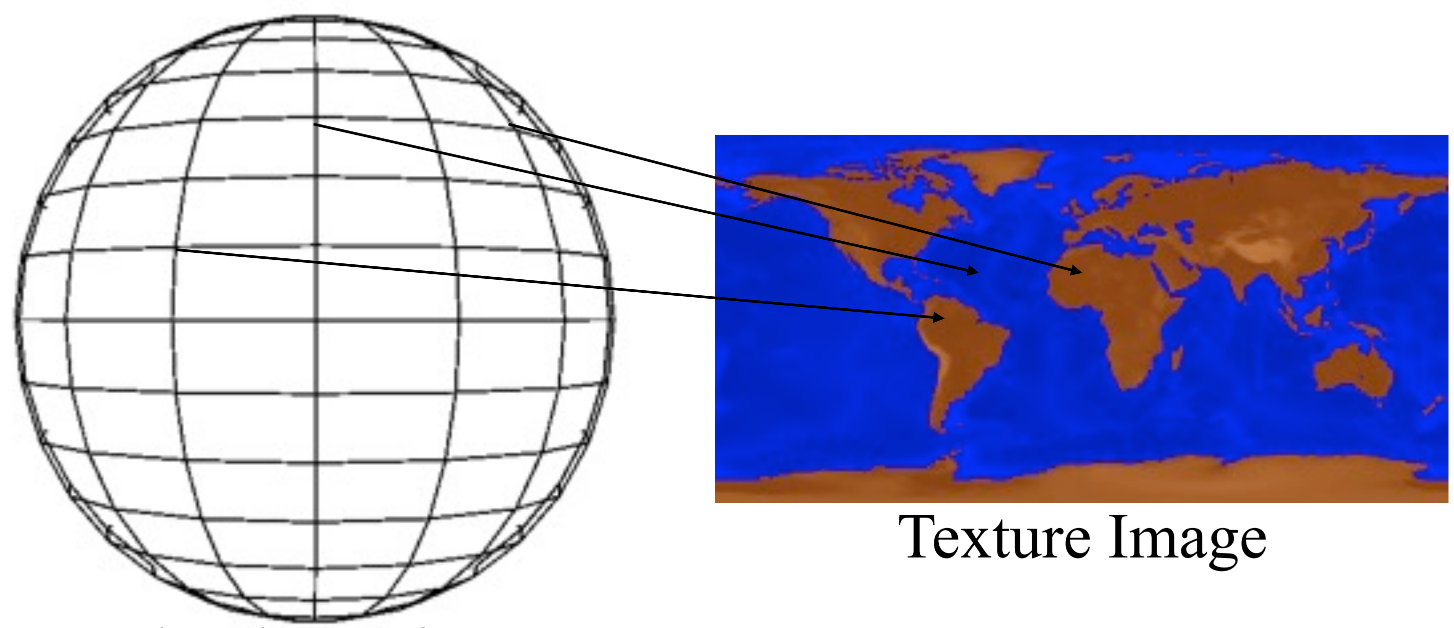
- Coordinates described by variables *s* and *t and* range over interval (0,1)
- Texture elements are called *texels*
- Often 4 bytes (rgba) per texel



Texture Mapping a Sphere

- How do you generate texture coordinates at each intersection point?

Target Model



Simple Surface

Texture Image