Quantitative Finance Cheat Sheet

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Contents

1 Time Value of Money

- **Future Value**
  - *Equation*: $FV = PV \times (1 + r)^n$
  - *Variables*: FV = Future Value, PV = Present Value, r = Annual Interest Rate, n = Number of Years
  - *Definition*: The value of a current amount of money at some point in the future, considering a certain rate of return.
  - *Example*: If you invest \$100 at an annual interest rate of 5%, the future value after one year would be $\$100 \times (1 + 0.05)^1 = \$105$.
- **Present Value**
  - *Equation*: $PV = \frac{FV}{(1+r)^n}$
  - *Variables*: PV = Present Value, FV = Future Value, r = Annual Discount Rate, n = Number of Years
  - *Definition*: The current value of a future amount of money, discounted back to the present at a specific rate.
  - *Example*: If you are to receive \$105 one year from now and the annual discount rate is 5%, the present value would be $\frac{\$105}{(1+0.05)^1} = \$100$.
- **Continuous Compounding**
  - *Equation*: $A = Pe^{rt}$
  - *Variables*: A = Amount after time t, P = Principal Amount, r = Annual Interest Rate, t = Time in Years
  - *Definition*: Compounding interest continuously over time.

– *Example*: If you invest \$100 at an annual interest rate of 5% compounded continuously, the amount after one year would be $\$100 \times e^{0.05\times1}$.

sectionStatistical Concepts

- **Mean**
  – *Equation*: $\mu = \frac{\sum_{i=1}^{n} x_i}{n}$
  – *Variables*: μ = Mean, x_i = Individual Data Points, n = Number of Data Points
  – *Definition*: The average of a set of numbers.
  – *Example*: The mean of $2, 4, 6$ is $\frac{2+4+6}{3} = 4$.
- **Variance**
  – *Equation*: $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$
  – *Variables*: σ^2 = Variance, x_i = Individual Data Points, μ = Mean, n = Number of Data Points
  – *Definition*: A measure of how spread out the values in a data set are around the mean.
  – *Example*: The variance of $2, 4, 6$ is $\frac{(2-4)^2+(4-4)^2+(6-4)^2}{3} = \frac{8}{3}$.
- **Standard Deviation**
  – *Equation*: $\sigma = \sqrt{\sigma^2}$
  – *Variables*: σ = Standard Deviation, σ^2 = Variance
  – *Definition*: The square root of the variance, indicating the average distance of each data point from the mean.
  – *Example*: The standard deviation of $2, 4, 6$ is $\sqrt{\frac{8}{3}} \approx 1.63$.
- **Multi-Asset Variance**
  – *Equation*: $\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$
  – *Variables*: σ_p^2 = Portfolio Variance, w_i, w_j = Weights of Assets i and j, σ_{ij} = Covariance between Assets i and j
  – *Definition*: A measure of the risk or volatility of a portfolio containing multiple assets.
  – *Example*: In a two-asset portfolio with weights $w_1 = 0.6$ and $w_2 = 0.4$ and covariance $\sigma_{12} = 0.002$, the multi-asset variance would be $0.6^2 \times \sigma_1^2 + 0.4^2 \times \sigma_2^2 + 2 \times 0.6 \times 0.4 \times 0.002$.

2 Option Pricing Models

- **Black-Scholes Model**
  – *Equation*: $C = S_0 e^{-qt} N(d_1) - X e^{-rt} N(d_2)$
  – *Variables*: C = Option Price, S_0 = Current Stock Price, X = Strike Price, t = Time to Expiry, r = Risk-free Rate, q = Dividend Yield
  – *Definition*: A mathematical model for pricing European-style options.
  – *Example*: To price a European call option with a stock price of \$100, strike price of \$100, time to expiration of 1 year, risk-free rate of 5%, and implied volatility of 20%, you would use the Black-Scholes equation.

- **Binomial Model**
  - *Equation*: $C = \frac{1}{(1+r)^T} \sum_{i=0}^{T} \binom{T}{i} p^i (1-p)^{T-i} \max(S_0 u^i d^{T-i} - X, 0)$
  - *Variables*: C = Option Price, S_0 = Current Stock Price, X = Strike Price, T = Time Steps, r = Risk-free Rate, p = Probability of Up Move, u = Up Factor, d = Down Factor
  - *Definition*: A model that calculates the price of an option by constructing a risk-neutral binomial tree.
  - *Example*: In a one-step binomial model with stock price $100, up factor 1.1, down factor 0.9, risk-free rate 5%, and strike price $100, the option price can be calculated using the binomial formula.
- **Trinomial Model**
  - *Equation*: Similar to the Binomial Model but with three possible moves (up u, down d, or stay s).
  - *Variables*: u = Up Factor, d = Down Factor, s = Stay Factor
  - *Definition*: An extension of the binomial model to include the possibility of the stock price staying the same.
  - *Example*: Useful for pricing American options where the option can be exercised at any time before expiration.
- **Numerical Methods**
  - *Equation*: Various, such as Finite Difference Methods $V_{xx} + V_t + rSV_s - rV = 0$
  - *Variables*: V = Option Price, S = Stock Price, r = Risk-free Rate, t = Time
  - *Definition*: Techniques for solving differential equations to find the option price.
  - *Example*: Used when the option has features that make analytical solutions difficult.
- **Heston Model**
  - *Equation*: Uses Stochastic Volatility $dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^1$
  - *Variables*: dS_t = Change in Stock Price, v_t = Volatility, W_t^1 = Wiener Process
  - *Definition*: A model accounting for volatility as a random process.
  - *Example*: Suitable for pricing options on assets that exhibit volatility clustering.
- **Merton Model**
  - *Equation*: Incorporates Jump Diffusion $dS_t = (\mu - \lambda k) S_t dt + \sqrt{v_t} S_t dW_t^1 + J dN_t$
  - *Variables*: dS_t = Change in Stock Price, J = Jump Size, N_t = Poisson Process, λ = Jump Intensity, k = Expected Jump Size
  - *Definition*: A model that incorporates sudden price jumps in addition to the usual random movement.
  - *Example*: Used for pricing options on stocks that may have large price jumps, like during earnings announcements.
- **FFT (Fast Fourier Transform)**
  - *Equation*: $C(K) = e^{-\alpha K} \frac{1}{\pi} \int_0^\infty e^{-ui} \phi(u - \alpha) du$
  - *Variables*: $C(K)$ = Option Price at Strike K, $\phi(u)$ = Characteristic Function, α = Dampening Factor
  - *Definition*: A computational technique to price options quickly.
  - *Example*: Useful for pricing a large number of options with different strikes or maturities simultaneously.

3 Portfolio Theory

- **Portfolio Return**
  - *Equation*: $R_p = w_1 R_1 + w_2 R_2 + \cdots + w_n R_n$
  - *Variables*: R_p = Portfolio Return, w_i = Weight of Asset i, R_i = Return of Asset i
  - *Definition*: The expected return of a portfolio, calculated as a weighted sum of the individual asset returns.
  - *Example*: If you have a portfolio with 50% in Stock A with a return of 10% and 50% in Stock B with a return of 20%, the portfolio return would be $0.5 \times 0.1 + 0.5 \times 0.2 = 0.15$ or 15%.
- **Portfolio Variance**
  - *Equation*: $\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}$
  - *Variables*: σ_p^2 = Portfolio Variance, w_i, w_j = Weights of Assets i and j, σ_{ij} = Covariance between Assets i and j
  - *Definition*: A measure of the risk or volatility of a portfolio.
  - *Example*: If a portfolio contains two assets with variances σ_1^2 and σ_2^2, and a correlation coefficient ρ, the portfolio variance would be $w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2$.
- **CAPM (Capital Asset Pricing Model)**
  - *Equation*: $R_i = R_f + \beta_i(R_m - R_f)$
  - *Variables*: R_i = Expected Return of Asset i, R_f = Risk-free Rate, β_i = Beta of Asset i, R_m = Market Return
  - *Definition*: A model that describes the relationship between the expected return of an asset and its risk, measured by beta.
  - *Example*: If the risk-free rate is 2%, the market return is 8%, and the asset's beta is 1.2, the expected return would be $2\% + 1.2 \times (8\% - 2\%)$.
- **Fama-French Three-Factor Model (FF)**
  - *Equation*: $R_i = R_f + \beta_1(R_m - R_f) + \beta_2 SMB + \beta_3 HML$
  - *Variables*: SMB = Size Factor, HML = Value Factor, β_2, β_3 = Factor Loadings
  - *Definition*: An extension of CAPM that includes size and value factors in addition to the market risk factor.
  - *Example*: Used to evaluate the performance of asset pricing models.
- **Modigliani-Miller Theorem**
  - *Equation*: $V_{\text{unlevered}} = V_{\text{levered}}$
  - *Variables*: $V_{\text{unlevered}}$ = Value of Unlevered Firm, V_{levered} = Value of Levered Firm
  - *Definition*: States that the value of a firm is independent of its capital structure in a frictionless market.
  - *Example*: Whether a company is financed by debt or equity should not affect its overall value according to the theorem.
- **Modigliani and Modigliani Measure (M2)**
  - *Equation*: $M^2 = R_p - R_f + (R_m - R_f)$

- *Variables*: R_p = Portfolio Return, R_f = Risk-free Rate, R_m = Market Return
- *Definition*: A measure to compare the risk-adjusted returns of various portfolios.
- *Example*: If a portfolio has a return of 12%, the risk-free rate is 2%, and the market return is 8%, the M2 measure would be $12\% - 2\% + (8\% - 2\%)$.

4 Risk Measures

- **Value-at-Risk (VaR)**
  - *Definition*: The maximum potential loss over a specific time horizon at a given confidence level.
  - *Example*: A 5% one-day VaR of $1 million means there is a 5% chance that the portfolio will fall in value by more than $1 million over a one-day period.
  - *Methods*:
    * *Historical Simulation*: Looks at past data to see how badly things could have gone.
    * *Parametric VaR*: Uses statistical models to estimate how bad future losses could be.
    * *Monte Carlo Simulation*: Uses computer simulations to model possible future scenarios and estimate worst-case losses.
  - *Multi-Asset VaR Equation*: $\text{VaR} = \sqrt{\mathbf{w}^\top \Sigma \mathbf{w}}$
  - *Variables*: $\mathbf{w}$ = Portfolio Weights Vector, Σ = Covariance Matrix
- **Expected Shortfall (ES)**
  - *Equation*: $\text{ES} = -E[X|X \leq -\text{VaR}]$
  - *Definition*: Another name for CVaR, focuses on the tail risk of distribution.
  - *Example*: Computed as the average of losses that occur in the worst $1 - \alpha$
- **Credit Value-at-Risk (CVaR)**
  - *Equation*: $\text{CVaR} = \text{EL} + \text{UL}$
  - *Variables*: EL = Expected Loss, UL = Unexpected Loss
  - *Definition*: A risk measure that estimates the potential loss due to credit events, such as default or changes in credit rating.
  - *Example*: If the expected loss is $500,000 and the unexpected loss is $300,000, then the CVaR would be $800,000.
- **Liquidity-Adjusted Value-at-Risk (LVaR)**
  - *Equation*: $\text{LVaR} = \text{VaR} + \text{Liquidity Cost}$
  - *Definition*: VaR adjusted for the liquidity of the assets in the portfolio.
  - *Example*: If the VaR is $1 million and the estimated liquidity cost is $200,000, then the LVaR would be $1.2 million.
- **Earnings at Risk (EaR)**
  - *Equation*: $\text{EaR} = \text{Potential Change in Earnings due to Price Changes}$
  - *Definition*: Measures the potential change in earnings (or cash flows) due to changes in market variables.

- *Example*: If interest rates rise by 1%, the EaR might quantify the reduction in the company's earnings.

- **Sharpe Ratio**
  - *Equation*: Sharpe Ratio $= \frac{R_p - R_f}{\sigma_p}$
  - *Variables*: R_p = Portfolio Return, R_f = Risk-Free Rate, σ_p = Portfolio Standard Deviation
  - *Definition*: A measure for calculating risk-adjusted return.
  - *Example*: If the portfolio return is 15%, the risk-free rate is 5%, and the portfolio standard deviation is 10%, then the Sharpe Ratio would be $\frac{0.15-0.05}{0.10} = 1$.

5 Stochastic Calculus

- **Brownian Motion**
  - *Equation*: $dW_t \sim N(0, dt)$
  - *Variables*: dW_t = Change in Brownian Motion, dt = Time Increment
  - *Definition*: A stochastic process that describes random motion, often used as a fundamental building block in financial modeling.
  - *Example*: In financial markets, Brownian Motion can model stock price movements.

- **Ito's Lemma**
  - *Equation*: $df(t, S_t) = \left(\frac{\partial f}{\partial t} + \mu S_t \frac{\partial f}{\partial S_t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 f}{\partial S_t^2} \right) dt + \sigma S_t \frac{\partial f}{\partial S_t} dW_t$
  - *Variables*: $f(t, S_t)$ = Function of Time and Stock Price, μ = Drift Rate, σ = Volatility, S_t = Stock Price at Time t
  - *Definition*: A fundamental theorem in stochastic calculus.
  - *Example*: If $f(t, S_t) = S_t^2$, then using Ito's Lemma we can find $df(t, S_t)$.

- **Geometric Brownian Motion (GBM)**
  - *Equation*: $dS_t = \mu S_t dt + \sigma S_t dW_t$
  - *Variables*: dS_t = Change in Stock Price, S_t = Stock Price, μ = Drift, σ = Volatility, dt = Time Increment
  - *Definition*: A stochastic process that is often used to model stock prices.
  - *Example*: Used in the Black-Scholes formula for option pricing.

- **Jump Diffusion**
  - *Equation*: $dS_t = \mu S_t dt + \sigma S_t dW_t + dJ_t$
  - *Variables*: dJ_t = Jump at Time t
  - *Definition*: Extends GBM by including jumps in the stock price.
  - *Example*: Used for modeling stock prices that can experience sudden, significant changes.

Thank You