```mathematica
In[21]:= ClearAll["Global`*"]

In[22]:= lerp[t_, a_, b_] := a * (1 - t) + b * t

In[23]:= cubicInterp[x_] := 3 * x^2 - 2 x^3

In[24]:= quinticInterp[x_] := 6 * x^5 - 15 * x^4 + 10 * x^3

In[25]:= bilinear = lerp[u[y], lerp[u[x], a, b], lerp[u[x], c, d]]
```

Out[25]= $\left(a\,(1 - u[x]) + b\,u[x]\right)\,(1 - u[y]) + \left(c\,(1 - u[x]) + d\,u[x]\right)\,u[y]$

```mathematica
In[26]:= expansion = Expand[bilinear]
```

Out[26]= $a - a\,u[x] + b\,u[x] - a\,u[y] + c\,u[y] + a\,u[x]\times u[y] - b\,u[x]\times u[y] - c\,u[x]\times u[y] + d\,u[x]\times u[y]$

```mathematica
In[27]:= Total@MonomialList[bilinear, {u[x], u[y]}]
```

Out[27]= $a + \left(-a + b\right)\,u[x] + (-a + c)\,u[y] + \left(a - b - c + d\right)\,u[x]\times u[y]$

```mathematica
In[28]:= dx = Simplify[D[expansion, x]] /. {u → cubicInterp}
```

Out[28]= $\left(6\,x - 6\,x^2\right)\,\left(-a + b + \left(a - b - c + d\right)\,\left(3\,y^2 - 2\,y^3\right)\right)$

```mathematica
In[29]:= dy = Simplify[D[expansion, y]] /. {u → cubicInterp}
```

Out[29]= $\left(-a + c + \left(a - b - c + d\right)\,\left(3\,x^2 - 2\,x^3\right)\right)\,\left(6\,y - 6\,y^2\right)$

```mathematica
In[30]:= ClearAll["Global`*"]

In[31]:= cubicInterp[x_] := 3 * x^2 - 2 x^3

In[32]:= quinticInterp[x_] := 6 * x^5 - 15 * x^4 + 10 * x^3

In[33]:= lerp[t_, a_, b_] := a * (1 - t) + b * t

In[34]:= trilinear = lerp[u[z], lerp[u[y], lerp[u[x], a, b], lerp[u[x], c, d]],
           lerp[u[y], lerp[u[x], e, f], lerp[u[x], g, h]]]
```

Out[34]= $\left(\left(a\,(1 - u[x]) + b\,u[x]\right)\,(1 - u[y]) + \left(c\,(1 - u[x]) + d\,u[x]\right)\,u[y]\right)\,(1 - u[z]) +$
$\left(\left(e\,(1 - u[x]) + f\,u[x]\right)\,(1 - u[y]) + \left(g\,(1 - u[x]) + h\,u[x]\right)\,u[y]\right)\,u[z]$

```mathematica
In[35]:= expansion = Expand[trilinear]
```

Out[35]= $a - a\,u[x] + b\,u[x] - a\,u[y] + c\,u[y] + a\,u[x]\times u[y] - b\,u[x]\times u[y] - c\,u[x]\times u[y] +$
$d\,u[x]\times u[y] - a\,u[z] + e\,u[z] + a\,u[x]\times u[z] - b\,u[x]\times u[z] - e\,u[x]\times u[z] +$
$f\,u[x]\times u[z] + a\,u[y]\times u[z] - c\,u[y]\times u[z] - e\,u[y]\times u[z] + g\,u[y]\times u[z] -$
$a\,u[x]\times u[y]\times u[z] + b\,u[x]\times u[y]\times u[z] + c\,u[x]\times u[y]\times u[z] - d\,u[x]\times u[y]\times u[z] +$
$e\,u[x]\times u[y]\times u[z] - f\,u[x]\times u[y]\times u[z] - g\,u[x]\times u[y]\times u[z] + h\,u[x]\times u[y]\times u[z]$

```mathematica
In[36]:= Total@MonomialList[trilinear, {u[x], u[y], u[z]}]
```

Out[36]= $a + \left(-a + b\right)\,u[x] + (-a + c)\,u[y] +$
$\left(a - b - c + d\right)\,u[x]\times u[y] + (-a + e)\,u[z] + \left(a - b - e + f\right)\,u[x]\times u[z] +$
$(a - c - e + g)\,u[y]\times u[z] + \left(-a + b + c - d + e - f - g + h\right)\,u[x]\times u[y]\times u[z]$

In[37]:= `dx = Simplify[D[expansion, x]] /. {u → cubicInterp}`

Out[37]= $\left(6\,x - 6\,x^2\right)\,\left(-a + b + \left(a - b - e + f\right)\,\left(3\,z^2 - 2\,z^3\right) +$
$\left(3\,y^2 - 2\,y^3\right)\,\left(a - b - c + d + \left(-a + b + c - d + e - f - g + h\right)\,\left(3\,z^2 - 2\,z^3\right)\right)\right)$

In[38]:= `dy = Simplify[D[expansion, y]] /. {u → cubicInterp}`

Out[38]= $\left(6\,y - 6\,y^2\right)\,\left(-a + c + \left(a - c - e + g\right)\,\left(3\,z^2 - 2\,z^3\right) +$
$\left(3\,x^2 - 2\,x^3\right)\,\left(a - b - c + d + \left(-a + b + c - d + e - f - g + h\right)\,\left(3\,z^2 - 2\,z^3\right)\right)\right)$

In[39]:= `dz = Simplify[D[expansion, z]] /. {u → cubicInterp}`

Out[39]= $\left(-a + e + \left(a - c - e + g\right)\,\left(3\,y^2 - 2\,y^3\right) +$
$\left(3\,x^2 - 2\,x^3\right)\,\left(a - b - e + f + \left(-a + b + c - d + e - f - g + h\right)\,\left(3\,y^2 - 2\,y^3\right)\right)\right)\,\left(6\,z - 6\,z^2\right)$