## 1.2 First-order PDEs – linear advection equation

**def:** linear advection equation in 1D:

$$\boxed{\frac{\partial u(x,t)}{\partial t} + a\,\frac{\partial u(x,t)}{\partial x} = 0 \quad (1)} \quad (a \in \mathbb{R})$$

**Thm:** $u(x,t) = f(x-at)$ is a solution of (1) for any function $f(s)$ that is sufficiently differentiable

**proof:** let $s = x - at$

let $u(x,t) = f(x-at) = f(s(x,t))$

then $\dfrac{\partial u}{\partial x} = \dfrac{df(s)}{ds}\dfrac{\partial s}{\partial x} = f' \cdot 1$

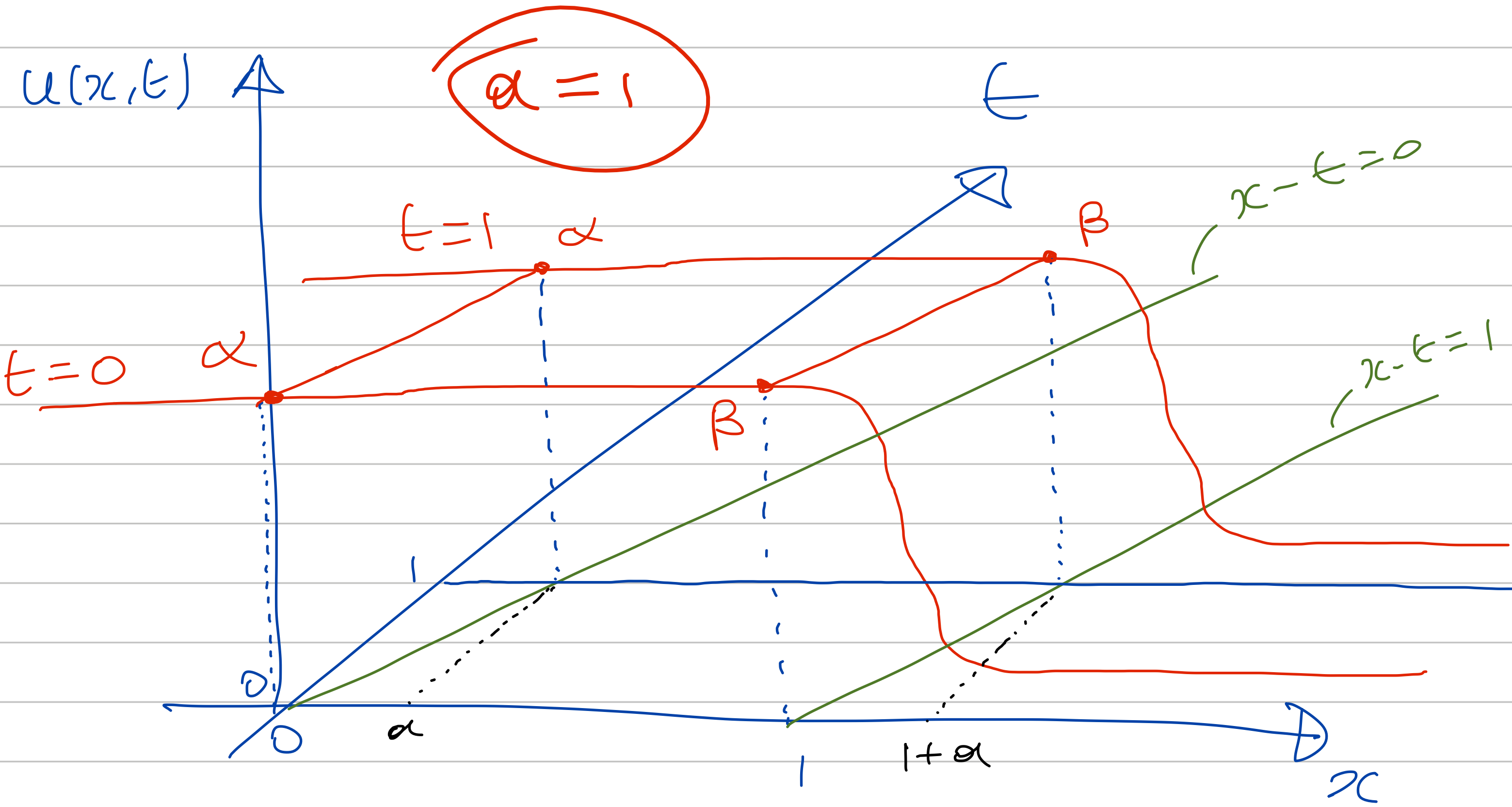
$\dfrac{\partial u}{\partial \cancel{y}\,t} = \dfrac{df(s)}{ds}\dfrac{\partial s}{\partial \cancel{y}\,t} = f' \cdot (-a)$

so $\dfrac{\partial u}{\partial t} + a\dfrac{\partial u}{\partial x} = -af' + af' = 0$ $\square$

"full derivative" (not partial)

( note: $\dfrac{df(s)}{ds} = f'(s) = f'$, since $f$ has only one argument)

$u_t + \alpha u_x = 0 \implies u(x,t) = f(x-\alpha t)$




the $f(s)$ function shifts to the right with speed $\alpha$

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general comment: many PDE problems cannot be solved exactly (in closed form, using elementary functions)

$\implies$ we need numerical methods to approximate the solutions to the PDE problem