# University of Waterloo, Winter 2023
# AMATH741/CM750/CS778: Numerical Solution of Partial Differential Equations

**Instructor:** Prof. Hans De Sterck, **email:** hdesterck@uwaterloo.ca
**Lectures:** Tuesdays and Thursdays 10:00-11:20am, MC 4058
**TA:** Yanming Kang, **email:** y53kang@uwaterloo.ca
**Instructor Office hours:** TBD, MC 6032

**Course description and objectives:**
Many problems in science, engineering and finance can be formulated in terms of partial differential equations (PDEs). Since analytical solutions are not available in general, it is necessary to use numerical methods to approximate the solution. This course will cover the basic techniques for solving PDEs numerically.
The goal of the course is threefold. You will receive a solid introduction to the theory of numerical methods for partial differential equations (with derivations of the methods and some proofs). You will learn to implement the computational methods efficiently in Matlab or another language of your choice, and you will apply the methods to problems in several fields, for example, fluid mechanics, diffusion processes and wave phenomena.
This is a graduate-level numerical methods course that will be useful for students from Applied Mathematics, Computational Mathematics, Computer Science, Data Science, Quantitative Finance, and other programs in Mathematics, Science and Engineering.

**Prerequisites:** Some previous experience with numerical computation and programming, and some background in PDEs, is desirable. (Also: it is not recommended to take this course if you have previously taken a numerical PDEs course, such as AMATH442/642 at University of Waterloo.)

**Tentative outline:**
1. Finite Difference (FD) Methods (4 weeks)
2. Finite Volume (FV) Methods (4 weeks)
3. Finite Element (FE) Methods (4 weeks)

**References:**
- Numerical Mathematics, Quarteroni, Sacco and Saleri, Springer, 2007
  (You can download pdf files for this book on campus at link.springer.com.
  For off-campus access, you first have to login via the library website, see
  https://login.proxy.lib.uwaterloo.ca/login
  which will allow you to access the book's proxy URL
  https://link-springer-com.proxy.lib.uwaterloo.ca/book/10.1007/b98885 .)
- Numerical partial differential equations: Finite Difference Methods, J.W. Thomas, Springer, 2004. (FD) (also available from link.springer.com)
- The mathematical theory of finite element methods, Brenner and Scott, Springer, 1994. (FE, theoretical) (also available from link.springer.com)
- A first course in the numerical analysis of differential equations, Iserles, Cambridge University Press, 1997. (FD and FE, Chapters 7-14)

- Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics, by Dietrich Braess, Cambridge University Press, 2001. (FE, theoretical)
- An introduction to the finite element method, Reddy, McGraw-Hill, 1993. (FE, comprehensive introduction with engineering applications)
- Finite volume methods for hyperbolic problems, Leveque, Cambridge, 2002. (FV)
- Numerical Methods for Conservation Laws, Hesthaven, SIAM, 2018.

**Course Website:** The LEARN system will be used extensively for all course communications. Crowdmark and LEARN will be used for assignment submissions. Piazza will be used as a student/instructor forum.

**Assignments:**

- There will be four assignments of equal value, with a mix of theoretical questions and programming questions. Assignments will be submitted electronically on Crowdmark (for the written or graphical assignment answers) and on LEARN (for the computer code files). Marked assignments will be given back via Crowdmark. Some problems may not be marked (solutions will be provided for these problems). ***You can take one three calendar-day extension on assignments (no questions asked), but you have to notify the instructor of this ahead of the assignment due date.***
- Matlab is the recommended computer language for the computational assignment questions, but you can choose to use a different language like python or Julia or C or C++ (contact the instructor if you want to choose a language not listed; Maple or Mathematica are not suitable). Instructions about installing Matlab on your computer and accessing the Waterloo campus licence, or running Matlab remotely, will be given in Assignment 0 (not to be handed in).
- Matlab is recommended because some of the questions may be written in part assuming you use Matlab, and for some questions some short Matlab code fragments may be provided on LEARN for download. So if you do not have much programming experience, Matlab is recommended. But there should not be an issue if you prefer to use another language that you are experienced in: it should not be difficult to translate the few code snippets that will be provided in Matlab to another language like python, and you can consult with other students (e.g., on piazza) about translating the Matlab that is provided to python or another language.

| **Assignments** | **Handout date (tentative)** | **Due date (tentative)** |
|---|---|---|
| Assignment 1 | Thu Jan 19 | Fri Feb 3 |
| Assignment 2 | Thu Feb 9 | Tue Feb 28 |
| Assignment 3 | Thu Mar 2 | Fri Mar 17 |
| Assignment 4 | Thu Mar 23 | Mon Apr 10 |

**Final Grade:** 50% Assignments, 50% Final Exam.
(The final exam will be a closed-book 2.5-hour exam held during the exam period. You will be allowed to bring one sheet of paper on which you can write formulas (two-sided).

***The date for the Final Exam is currently set for Monday April 24, 12:30-3:00pm***, but remains to be confirmed.)

**Late Assignments/Make-Up Exams:** Beyond the above-mentioned extension on one assignment, there will be no make-up exams and no extensions for assignments except under extenuating (and documented) circumstances. **If you are ill**, please be prepared to provide a note from the health center or your doctor, see https://uwaterloo.ca/campus-wellness/health-services/student-medical-clinic/verification-illness-services (but if you are affected by Covid-19 or influenza-like symptoms, you can self-declare, see https://uwaterloo.ca/quest/help/students/how-do-i/self-declare-absence-graduate-students). If your instructor decides that your circumstances warrant special accommodation, your final grade will be calculated based on your performance on the remaining tests/assignments. When possible, advance notice must be given.

**Grade Appeals:** We will make every effort to be fair and consistent in the marking. Grade appeals must be submitted to the TA in writing within one week from the date the assignment was returned. For this reason, it is important that you look over all returned assignments on the day they are returned.

**How the class will be taught:**

- **Handwritten pdf "board notes"** will be provided in advance of the lectures and will be discussed during the lectures. These notes contain all the primary course material. *You will be asked to read and study the relevant section of the board notes before the corresponding classroom session.* In the classroom session the instructor will go over the board notes highlighting the big lines and the important or tricky parts (at a faster speed than a regular board lecture), and students will have ample opportunity to ask questions about the board notes and about other aspects of the course (like assignment questions etc.). Class sessions are scheduled for a length of up to 80min, but they make take less time, since you will have studied the material beforehand, and some sessions will treat topics that are shorter than other sessions.
- This way of teaching can easily be live-streamed (with recordings posted), either from the classroom or the instructor's home, in case covid-19 disruptions demand changes in course delivery.
- **Typed pdf course notes** will also be posted on the course website, to complement the hand-written pdf board notes and classroom sessions.

**Academic Integrity**: All submitted assignment solutions should be strictly your own work. You are allowed to discuss theoretical and computational assignment problems with your classmates at a general level (but not step-by-step). You are not allowed to show parts of your written assignment to another student. You are not allowed to copy any material from another student or from any other source. For programming questions, you are allowed to discuss issues you encounter with others, but you are not allowed to show others your computer code or copy computer code from others. You are not allowed to copy solutions or computer code from online resources. All assignment material you submit (including written documents, program code and graphical output) should be strictly your own work.

Compliance will be actively monitored. Instances of suspected cheating will be dealt with seriously, in accordance with Faculty and University policies.

More generally, in order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check the Office of Academic Integrity for more information.]

**Grievance:** A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4. When in doubt, please be certain to contact the department's administrative assistant who will provide further assistance.

**Discipline:** A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for his/her actions. [Check the Office of Academic Integrity for more information.] A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate associate dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline. For typical penalties, check Guidelines for the Assessment of Penalties.

**Appeals:** A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals.

**Note for students with disabilities:** AccessAbility Services, located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.

**Mental Health**: If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, we strongly encourage you to seek support.
UWaterloo Resources

- Campus Wellness: *https://uwaterloo.ca/campus-wellness/students*
- Counselling Services: counselling.services@uwaterloo.ca, *519-888-4567 ext 32655*
- Health Services: 519-888-4096.

Off-Campus Resources

- Good2Talk (24/7): Free confidential help line for post-secondary students. Phone: 1-866-925-5454, https://good2talk.ca/

- Here 24/7: Mental Health and Crisis Service Team. Phone: 1-844-437-3247 or 519-821-3582, https://here247.ca/
- OK2BME: set of support services for lesbian, gay, bisexual, transgender or questioning teens in Waterloo. Phone: 519-884-0000, https://ok2bme.ca/

**Diversity:** It is our intent that students from all diverse backgrounds and perspectives be well served by this course, and that students' learning needs be addressed both in and out of class. We recognize the immense value of the diversity in identities, perspectives, and contributions that students bring, and the benefit it has on our educational environment. Your suggestions are encouraged and appreciated. Please let us know ways to improve the effectiveness of the course for you personally or for other students or student groups. In particular:

- We will gladly honour your request to address you by an alternate/preferred name or gender pronoun. Please advise us of this preference early in the semester so we may make appropriate changes to our records.
- We will honour your religious holidays and celebrations. Please inform us of these at the start of the course.
- We will follow AccessAbility Services guidelines and protocols on how to best support students with different learning needs.

**Contingency plans for possible covid-19 disruptions:**

- **lectures:**
  - if a situation arises where the instructor needs to isolate, or if on-campus classes need to be cancelled, the on-campus lectures will be replaced by live-streamed online-only lectures at the originally scheduled times (with recordings made available)
  - in case the instructor is unable to teach, lecture recordings from previous years may be provided
  - if a student needs to isolate, lecture recordings from previous years may be made available upon request
  - under some (exceptional) circumstances, live-streamed teaching from the lecture hall may be considered (with recordings made available)
- **final exam**:
  - in case on-campus exams are not possible during the final exam period, the final exam will be held in a live-proctored online format during a scheduled time slot (90min) in the final exam period, where the instructor and TA will proctor the exam in a zoom session, using the online Crowdmark system for exam submission (you would dial into zoom using your laptop and laptop camera; we would not use external services such as ProctorU); the live-proctored exam format was used for this course during the past two years and worked well; alternatives would be available where needed, e.g., for students who face technological restrictions (for example, a fully oral exam)
  - if a student needs to isolate during the timeslot of the on-campus final exam, an alternative exam format will be provided (this may include a live-proctored zoom exam simultaneous with the on-campus exam or an oral exam)

**Remote Teaching and Learning: STUDENT NOTICE OF RECORDING**

Activities for this course involve recording, in partial fulfillment of the course learning outcomes. You will receive notification of recording via at least one of the following mechanisms: within the Learning Management System (LEARN), a message from your course instructor, course syllabus/website, or other means. Some technologies may also provide a recording indicator. Images, audio, text/chat messaging that have been recorded may be used and/or made available by the University to students, instructor and TA of AMATH741/CM750/CS778 for the purpose of helping in the study of the course materials. If an oral exam is needed, audio of the exam will be recorded for record-keeping, unless the student declines recording. Recordings will be managed according to the University records classification scheme, WatClass, and will be securely destroyed when no longer needed by the University. Your personal information is protected in accordance with the Freedom of Information and Protection of Privacy Act, as well as University policies and guidelines and may be subject to disclosure where required by law.

The University will use reasonable means to protect the security and confidentiality of the recorded information, but cannot provide a guarantee of such due to factors beyond the University's control, such as recordings being forwarded, copied, intercepted, circulated, disclosed, or stored without the University's knowledge or permission, or the introduction of malware into computer system which could potentially damage or disrupt the computer, networks, and security settings. The University is not responsible for connectivity/technical difficulties or loss of data associated with your hardware, software or Internet connection.

By engaging in course activities that involve recording, you are consenting to the use of your appearance, image, text/chat messaging, and voice and/or likeness in the manner and under the conditions specified herein. (In the case of a live stream event, if you choose not to have your image or audio recorded, you may disable the audio and video functionality (see: *Student privacy during live events*). Instructions to participate using a pseudonym instead of your real name are included where the feature exists; however, you must disclose the pseudonym to your instructor in advance in order to facilitate class participation.) This notice serves as confirmation of your understanding that you can choose to participate in any course component without being recorded.

You are not permitted to disclose the link to/URL of an event or an event session recording or copies of recording to anyone, for any reason. Recordings are available only to authorized individuals who have been directly provided the above instructions/link for their use. Recordings for personal use, required to facilitate your learning and preparation of personal course/lecture notes, should not be shared with others without the permission of the instructor or event coordinator. Review the University's guidelines for faculty, staff and students entering relationships with external organizations offering access to course materials for more information on your obligations with respect to keeping copies of course materials. For more information about accessibility, connect with AccessAbility Services.