Homework HW2 – SSSP

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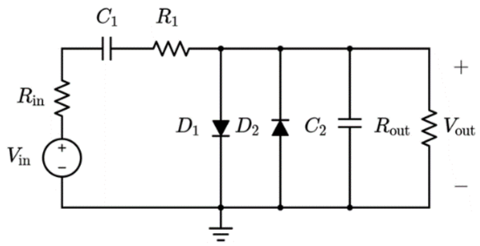
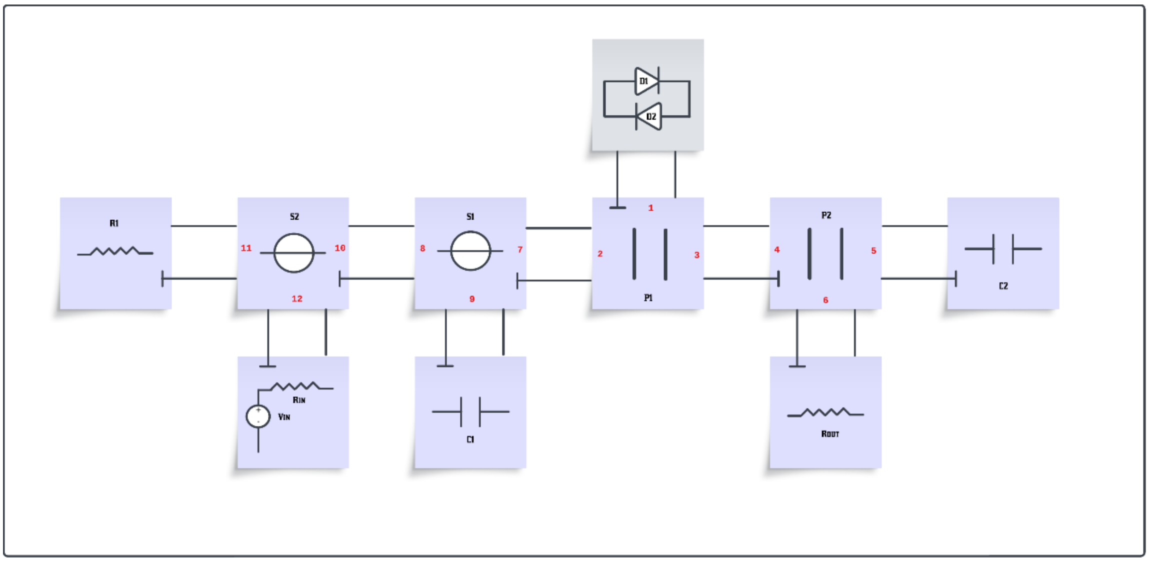
1 INTRODUCTION

In this homework, we implemented the Wave Digital Filter (*WDF*) of the Clipping Stage of the *MXR Distortion+*. Given the analog circuit, the corresponding WDF was derived from the analog electrical circuit as shown below:





2 WDF SCHEME DESCRIPTION

- **Root**: The root of the *WDF* is the nonlinear port elements, which in this case are the two parallel diodes (D_1, D_2).
- **Parallel Junctions (P_1 and P_2)**:
  - P_1: Connected to the root, P_2 , and the series junction S_1.
  - P_2: Connected to capacitor C_2 and the output resistor R_{out}. This forms a parallel between C_2 and R_{out}.
- **Series Junctions (S_1 and S_2)**:
  - S_1: Connected to P_1 and has C_1 and S_2 attached.
  - S_2: Connected to R_1 and the input V_{in} and R_{in}.

3 SETTING OF FREE PARAMETERS

The adaptation conditions for the ports are as follows:

$$Z_{11} = R_1 \qquad Z_{12} = R_{in} \qquad Z_9 = \frac{T_s}{2*C_1} \qquad Z_6 = R_{out} \qquad Z_5 = \frac{T_s}{2*C_2}$$

To make the ports of the adaptors reflection-free, the following calculations are made:

•*For S2:* $Z_{10} = Z_{11} + Z_{12}$. •*For S1:* $Z_8 = Z_{10}$, $Z_7 = Z_8 + Z_9$. •*For P2:* $Z_4 = \frac{Z_5 * Z_6}{Z_5 + Z_6}$. • *For P1:* $Z_3 = Z_4$, $Z_2 = Z_7$, $Z_1 = \frac{Z_3 * Z_2}{Z_3 + Z_2}$.

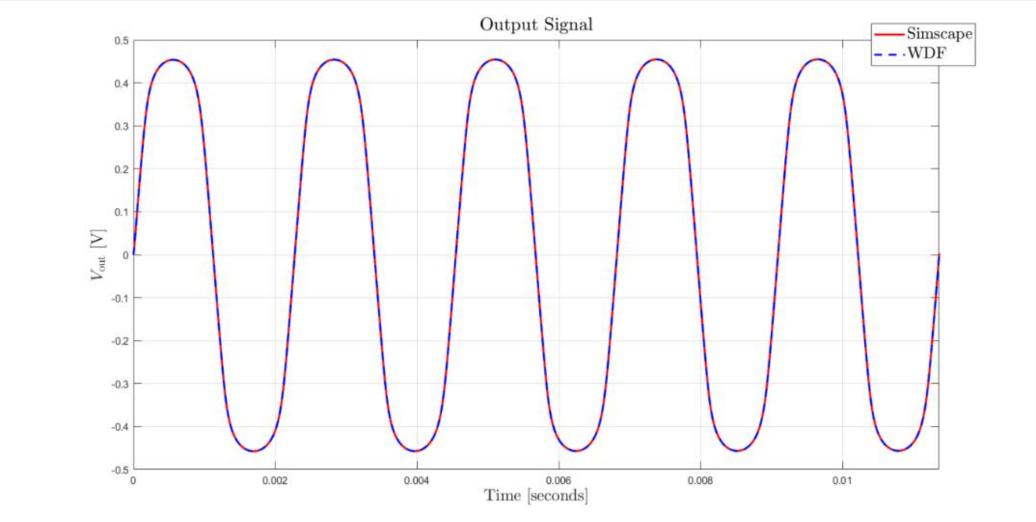
The Scattering Matrices are computed as follows:

```
% Series adaptor S2 (port 10, 11, 12)
gammaSer2 = Z11/(Z11+Z12);
Sser2 = [       0          ,        -1        ,       -1       ;
          -gammaSer2       ,  (1-gammaSer2)  ,  -gammaSer2     ;
        (gammaSer2-1)      ,  (gammaSer2-1)  ,   gammaSer2     ];

% Series adaptor S1 (port 7, 8, 9)
gammaSer1 = Z8/(Z8+Z9);
Sser1 = [      0           ,        -1        ,       -1       ;
           -gammaSer1      ,  (1-gammaSer1)  ,  -gammaSer1     ;
           (gammaSer1-1)   ,  (gammaSer1-1)  ,   gammaSer1     ];
```

```
% Parallel adaptor P2 (port 4, 5, 6)
gammaPar2 = Z5/(Z5+Z6);
Spar2 = [   0     ,  (1-gammaPar2) ,    gammaPar2    ;
            1     ,   -gammaPar2   ,    gammaPar2    ;
            1     ,  (1-gammaPar2) ,  (gammaPar2-1)  ];

% Parallel adaptor P1 (port 1, 2, 3)
gammaPar1 = Z2/(Z2+Z3);
Spar1 = [   0     ,  (1-gammaPar1) ,    gammaPar1     ;
            1     ,   -gammaPar1   ,    gammaPar1     ;
            1     ,  (1-gammaPar1) ,  (gammaPar1-1)   ];
```

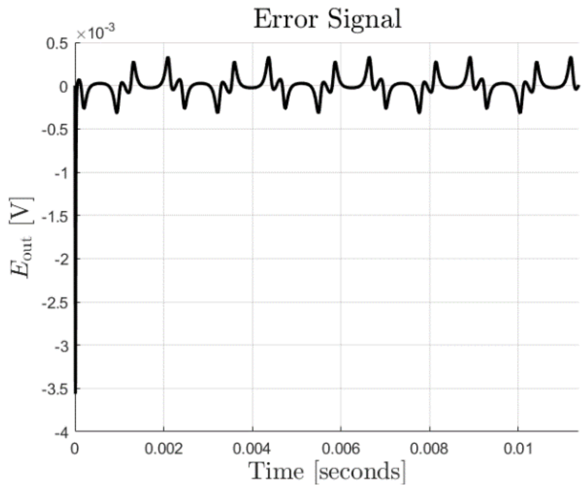
4 PLOTS

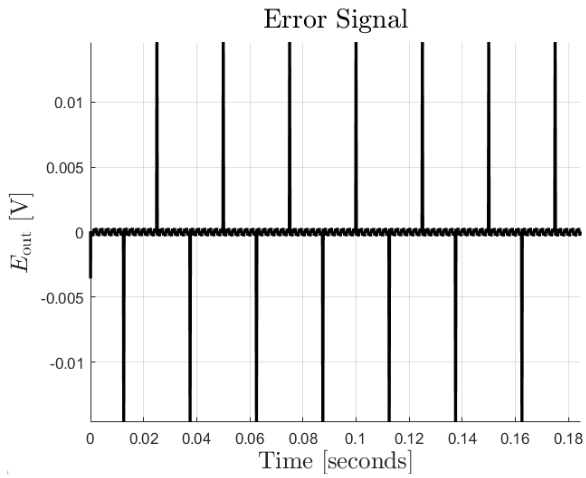
The plot comparing the output signal with the ground-truth signal is shown below:



The output signal matches the ground truth output signal.

The **MSE** value is **2.0941e-07** and the plot of the error signal is:





On the right image, the error is plotted over some periods of the original input signal. What stands out in this plot are the spikes, which are the parts of the error signal that are more pronounced. These spikes occur approximately every 0.01257 seconds, but (*in this case*) they do not significantly affect the final result as they are limited to about ±0.0146V.

5 CONCLUSION

The implemented WDF for the *MXR Distortion+* Clipping Stage successfully models the analog circuit, with a very low MSE value indicating high accuracy. The use of reflection-free ports ensures stability and correct wave interaction.