Question 1:

Northeastern Airlines is considering the purchase of new long-, medium-, and short-range jet passenger airplanes. The purchase price would be \$67 million for each long-range plane, \$50 million for each medium-range plane, and \$35 million for each short-range plane. The board of directors has authorized a maximum commitment of \$1.5 billion for these purchases. Regardless of which airplanes are purchased, air travel of all distances is expected to be sufficiently large that these planes would be utilized at essentially maximum capacity. It is estimated that the net annual profit (after capital recovery costs are subtracted) would be \$4.2 million per long-range plane, \$3 million per medium-range plane, and \$2.3 million per short-range plane. It is predicted that enough trained pilots will be available to the company to crew 30 new airplanes. If only short-range planes were purchased, the maintenance facilities would be able to handle 40 new planes. However, each medium-range plane is equivalent to $1\frac{2}{3}$ short-range planes, and each long-range plane is equivalent to $1\frac{2}{3}$ short-range planes in terms of their use of the maintenance facilities. The information given here was obtained by a preliminary analysis of the problem. A more detailed analysis will be conducted subsequently. However, using the preceding data as a first approximation, management wishes to know how many planes of each type should be purchased to maximize profit.

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$$\begin{array}{ll} \max & 4.2L + 3M + 2.3S \\ s.t. & 67L + 50M + 35S \leq 1500 \\ & L + M + S \leq 30 \\ & 1.666L + 1.666M + S \leq 40 \\ & L, M, S \geq 0 \\ & L, M, S \quad integer \end{array}$$

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\end{aligned}
$$

```python
types=["longrange","mediumrange","shortrange"]

price={"longrange":67,"mediumrange":50,"shortrange":35}  #in millions
profit={"longrange":4.2,"mediumrange":3,"shortrange":2.3}
maintananceNeeded={"longrange":5/3,"mediumrange":5/3,"shortrange":1}
maintananceAvail=40
pilots=30
budget=1500 #in millions


model=gp.Model()
x=model.addVars(types,vtype=gp.GRB.INTEGER,obj=profit,name=" ")
model.ModelSense=gp.GRB.MAXIMIZE
model.addConstr(gp.quicksum(x[t] for t in types)<=pilots)
model.addConstr(gp.quicksum(price[t]*x[t] for t in types)<=budget)
model.addConstr(gp.quicksum(maintananceNeeded[t]*x[t] for t in types)<=maintananceAvail)

model.optimize()

model.printAttr("X")
print ("total profit", model.ObjVal)
```

```
Optimal solution found (tolerance 1.00e-04)
Best objective 9.560000000000e+01, best bound 9.560000000000e+01, gap 0.0000%

    Variable            X
-------------------------
 [longrange]           14
 [shortrange]           16
total profit 95.6
```

Question 2: Radford Castings can produce brake shoes on six different machines. The following table summarizes the manufacturing costs associated with producing the brake shoes on each machine along with the available capacity on each machine. Note that the fixed cost should be paid only if the machine is used. If the company has received an order for 1800 brake shoes, how should it schedule these machines to minimize the total cost? Formulate a model that can be used to solve this problem and solve it in Gurobi.

Machine	Fixed Cost	Variable Cost	Capacity
1	$1000	$21	500
2	$950	$23	600
3	$875	$25	750
4	$850	$24	400
5	$800	$20	600
6	$700	$26	800

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5	$800	$20	600
6	$700	$26	800

$$\min \quad 1000y_1 + 21x_1 + 950y_2 + 23x_2 + 875y_3 + 25x_3 + 850y_4 + 24x_4 + 800y_5 + 20x_5 + 700y_6 + 26x_6$$

$$s.t. \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1800$$

$$x_1 \le 500y_1 \qquad x_2 \le 600y_2$$

$$x_3 \le 750y_3 \qquad x_4 \le 400y_4$$

$$x_5 \le 600y_5 \qquad x_6 \le 800y_6$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \quad \text{integer}$$

$$y_1, y_2, y_3, y_4, y_5, y_6 \quad \text{binary}$$

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4	$850	$24	400
5	$800	$20	600
6	$700	$26	800

```
fixedCosts=[1000,950,875,850,800,700]
variableCosts=[21,23,25,24,20,26]
capacity=[500,600,750,400,600,800]
totalOrders=1800
n=len(fixedCosts)

model=gp.Model()
x=model.addVars(n,vtype=gp.GRB.INTEGER,obj=variableCosts,name="x")
y=model.addVars(n,vtype=gp.GRB.BINARY,obj=fixedCosts,name="y")
model.addConstr(gp.quicksum(x[i] for i in range(n))==totalOrders)
model.addConstrs(x[i]<=capacity[i]*y[i] for i in range(n))
model.optimize()
model.printAttr("X")
print ("total profit", model.ObjVal)
```

$$\min \quad 1000y_1 + 21x_1 + 950y_2 + 23x_2 + 875y_3 + 25x_3 + 850y_4 + 24x_4 + 800y_5 + 20x_5 + 700y_6 + 26x_6$$

$$s.t. \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1800$$

$x_1 \leq 500y_1 \qquad x_2 \leq 600y_2$

$x_3 \leq 750y_3 \qquad x_4 \leq 400y_4$

$x_5 \leq 600y_5 \qquad x_6 \leq 800y_6$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

$x_1, x_2, x_3, x_4, x_5, x_6$ integer

$y_1, y_2, y_3, y_4, y_5, y_6$ binary

```
    Variable            X
-------------------------
        x[0]          500
        x[1]          600
        x[3]          100
        x[4]          600
        y[0]            1
        y[1]            1
        y[3]            1
        y[4]            1
total profit 42300.0
```

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1. There must be at least as many shows scheduled that are categorised as public interest as there are shows scheduled that are categorised as containing violence.
2. If "Focus on Science — The Fusion Issue" is scheduled, then either "Jake" or "L. A. Law" (or both) must be scheduled as well.
3. WCBN-TV cannot schedule both "Focus on Science" and "Urban Action for Education", as both of these shows are considered a bit on the dry side.
4. If WCBN-TV schedules two or more shows in the comedy category, then they must schedule at least one show in the drama category.
5. If WCBN-TV schedules more than three shows in the "contains violence" category, they will lose an estimated £4 million in advertising revenues from family-oriented sponsors.

TV show	Advertising revenue	Public interest	violence	Comedy	Drama
Cheers	6		yes	yes	
Dynasty	10		yes		yes
L.A. Law	9	yes	yes		yes
Jake	4		yes		yes
Bob Newhart	5			yes	
News Special–the Middle East	2	yes	yes		
Focus on Sciense:The Fusion Issue	6	yes			yes
Magnificent Beaches	7			yes	
Urban Action for Education	8	yes			

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D	Dynasty	10		yes		yes
L	L.A. Law	9	yes	yes		yes
J	Jake	4		yes		yes
B	Bob Newhart	5			yes	
N	News Special–the Middle East	2	yes	yes		
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$$\max \quad 6C + 10D + 9L + 4J + 5B + 2N + 6F + 7M + 8U$$

$$s.t. \quad C + D + L + J + B + N + F + M + U = 5$$

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$$s.t. \quad C + D + L + J + B + N + F + M + U = 5$$

$$L + N + F + U \geq C + D + L + J + N \quad (1)$$

$$J + L \geq F \quad (2) \qquad F + U \leq 1 \quad (3)$$

5. If WCBN-TV schedules more than three shows in the “contains violence” category, they will lose an estimated £4 million in advertising revenues from family-oriented sponsors.

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$$\max \quad 6C + 10D + 9L + 4J + 5B + 2N + 6F + 7M + 8U$$

$$s.t. \quad C + D + L + J + B + N + F + M + U = 5$$

$$L + N + F + U \geq C + D + L + J + N \quad (1)$$

$$J + L \geq F \quad (2) \qquad F + U \leq 1 \quad (3)$$

$$if \quad C + B + M \geq 2 \quad then \quad D + L + J + F \geq 1$$

Not a linear constraint yet.

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$$\max \quad 6C + 10D + 9L + 4J + 5B + 2N + 6F + 7M + 8U$$

$$s.t. \quad C + D + L + J + B + N + F + M + U = 5$$

$$L + N + F + U \geq C + D + L + J + N \quad (1)$$

$$J + L \geq F \quad (2) \qquad F + U \leq 1 \quad (3)$$

$$if \quad C + B + M \geq 2 \quad then \quad D + L + J + F \geq 1$$

Not a linear constraint yet.

$$C + B + M \leq 1 \quad or \quad D + L + J + F \geq 1$$

Not a linear constraint yet.

$$y \in \{0,1\}, \quad C + B + M \leq 1 + 2 \cdot y, \quad D + L + J + F \geq y$$

✓

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$$\max \quad 6C + 10D + 9L + 4J + 5B + 2N + 6F + 7M + 8U - 4z$$

$$s.t. \quad C + D + L + J + B + N + F + M + U = 5$$

$$L + N + F + U \geq C + D + L + J + N \quad (1)$$

$$J + L \geq F \quad (2) \qquad F + U \leq 1 \quad (3)$$

$$y \in \{0,1\}, \quad C + B + M \leq 1 + 2 \cdot y, \quad D + L + J + F \geq y \quad (4)$$

if $C + D + L + J + N \geq 3$ then decrease revenue by 4

Introduce a binary variable z to indicate whether the constraint

$$C + D + L + J + N \leq 2$$

is violated. Violating the constraint is permitted but then the objective must be penalized by 4.

$$z \in \{0,1\}, \quad C + D + L + J + N \leq 2 + 3z$$

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$$\max \quad 6C + 10D + 9L + 4J + 5B + 2N + 6F + 7M + 8U - 4z$$

$$s.t. \quad C + D + L + J + B + N + F + M + U = 5$$

$$L + N + F + U \geq C + D + L + J + N \quad (1)$$

$$J + L \geq F \quad (2) \qquad F + U \leq 1 \quad (3)$$

$$y \in \{0,1\}, \quad C + B + M \leq 1 + 2 \cdot y, \quad D + L + J + F \geq y \quad (4)$$

$$z \in \{0,1\}, \quad C + D + L + J + N \leq 2 + 3z \quad (5)$$

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$$\max \quad 6C + 10D + 9L + 4J + 5B + 2N + 6F + 7M + 8U - 4z$$

$$s.t. \quad C + D + L + J + B + N + F + M + U = 5$$

$$L + N + F + U \geq C + D + L + J + N \quad (1)$$

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M	Magnificent Beaches	7			yes	
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```
shows=["C","D","L","J","B","N","F","M","U"]
revenues={"C":6,"D":10,"L":9,"J":4,"B":5,"N":2,"F":6,"M":7,"U":8}
publicInterest=["L","N","F","U"]
violence=["C","D","L","J","N"]
comedy=["C","B","M"]
drama=["D","L","J","F"]
bigM=len(shows) #using bigM to deactivate constraints.
                #Being lazy always use the same bigM
model=gp.Model()
model.ModelSense=gp.GRB.MAXIMIZE
x=model.addVars(shows,obj=revenues,vtype=gp.GRB.BINARY,name="Show")
#exactly 5 shows
model.addConstr(gp.quicksum(x[s] for s in shows)==5)
#constraint 1
model.addConstr(gp.quicksum(x[p] for p in publicInterest)>=
                gp.quicksum(x[v] for v in violence))
#constraint 2
model.addConstr(x["J"]+x["L"]>= x["F"])
#constraint 3
model.addConstr(x["F"]+x["U"]<= 1)
#contraint 4
y=model.addVar(vtype=gp.GRB.BINARY,name="at_least_2_commedy")
model.addConstr(gp.quicksum(x[c] for c in comedy)<=1+bigM*y)
model.addConstr(gp.quicksum(x[d] for d in drama)>=y)
#constraint 5, notice the objective coefficient in the z indicator
z=model.addVar(vtype=gp.GRB.BINARY,obj=-4,name="at_least_3_violence")
model.addConstr(gp.quicksum(x[v] for v in violence)<=2+bigM*z)
```

```
optimal value 39.0

   Variable            X
-------------------------
    Show[D]            1
    Show[L]            1
    Show[B]            1
    Show[M]            1
    Show[U]            1
at_least_2_commedy        1
```

Question 4:

The Belmont Bank is considering placing ATM machines in the town centres of some of the following six communities: Arlington, Belmont, Cambridge, Lexington, Somerville and Winchester. The bank would like to purchase the minimum number of ATM machines needed to ensure that at least one ATM machine within a ten-minute drive from the centre of each of these six communities. The times required to drive between the communities are as follows:

The data is also in Q4_data.csv

Town	Arlington	Belmont	Cambridge	Lexington	Concord	Winchester
Arlington	0	5	10	15	20	15
Belmont	5	0	8	10	15	12
Cambridge	10	8	0	15	20	10
Lexington	15	10	15	0	10	12
Concord	20	15	20	10	0	12
Winchester	15	12	10	12	12	0

$$\min \quad A + B + C_a + L + C_o + W$$

$$\begin{aligned}
s.t. \quad & A + B + C_a \geq 1 \quad (A) \\
& A + B + C_a + L \geq 1 \quad (B) \\
& A + B + C_a + W \geq 1 \quad (C_a) \\
& B + L + C_o \geq 1 \quad (L) \\
& L + C_o \geq 1 \quad (C_o) \\
& C_a + W \geq 1 \quad (W) \\
& A, B, C_a, L, C_o, W \quad binary
\end{aligned}$$

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Winchester	15	12	10	12	12	0

$$\min \quad A + B + C_a + L + C_o + W$$

$$s.t. \quad A + B + C_a \geq 1 \quad (A)$$

$$\cancel{A + B + C_a + L \geq 1 \quad (B)}$$

$$\cancel{A + B + C_a + W \geq 1 \quad (C_a)}$$

$$\cancel{B + L + C_o \geq 1 \quad (L)}$$

$$L + C_o \geq 1 \quad (C_o)$$

$$C_a + W \geq 1 \quad (W)$$

$$A, B, C_a, L, C_o, W \quad binary$$

Simplification possible but not necessary.

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$$\min \quad A + B + C_a + L + C_o + W$$

$$\begin{aligned}
s.t. \quad & A + B + C_a \geq 1 \quad (A) \\
& A + B + C_a + L \geq 1 \quad (B) \\
& A + B + C_a + W \geq 1 \quad (C_a) \\
& B + L + C_o \geq 1 \quad (L) \\
& L + C_o \geq 1 \quad (C_o) \\
& C_a + W \geq 1 \quad (W) \\
& A, B, C_a, L, C_o, W \quad binary
\end{aligned}$$

```python
import pandas as pd
df=pd.read_csv("Q4_data.csv")
towns=list(df.Town)
distances={}
for t1 in towns:
    for t2 in towns:
        distances[(t1,t2)]=float(df[df.Town==t1][t2])

model=gp.Model()
x=model.addVars(towns,vtype=gp.GRB.BINARY,obj=1,name=" ")
model.addConstrs(
    gp.quicksum(x[t1] for t1 in towns if (distances[(t1,t2)]<=10)) >=1
        for t2 in towns)
model.optimize()
print("Total Machines required",model.objval)
model.printAttr("X")
```

```
Total Machines required 2.0

    Variable            X
-------------------------
 [Cambridge]            1
 [Lexington]            1
```

Question 5: Quantas Airline is configuring a new aircraft and must decide how many First-Class seats, Business-Class seats, and Economy-Class seats to have in the aircraft. The aircraft can have 40 rows of seats, each of which has to be fully First-Class (2 seats per row), Business-Class (4 seats per row), or Economy-Class (6 seats per row). There must be at least three rows of each class. It is also required that the number of Economy-Class seats is at least two times larger than the number of all other seats. Quantas does not want to share the actual profit from each type of ticket. All they agreed to share is that the profit on a First-Class ticket is 4 times the profit of an Economy-Class ticket and the profit of a Business-Class ticket is 3 times the profit of an Economy-Class ticket.

1. Assume that Quantas can sell enough tickets to occupy all seats. Formulate a model for this problem that can find the number of rows of seats for each class that will maximize the total profit per flight. Solve it in Gurobi.

Question 5: Quantas Airline is configuring a new aircraft and must decide how many First-Class seats, Business-Class seats, and Economy-Class seats to have in the aircraft. The aircraft can have 40 rows of seats, each of which has to be fully First-Class (2 seats per row), Business-Class (4 seats per row), or Economy-Class (6 seats per row). There must be at least three rows of each class. It is also required that the number of Economy-Class seats is at least two times larger than the number of all other seats. Quantas does not want to share the actual profit from each type of ticket. All they agreed to share is that the profit on a First-Class ticket is 4 times the profit of an Economy-Class ticket and the profit of a Business-Class ticket is 3 times the profit of an Economy-Class ticket.

1. Assume that Quantas can sell enough tickets to occupy all seats. Formulate a model for this problem that can find the number of rows of seats for each class that will maximize the total profit per flight. Solve it in Gurobi.

$$
\begin{aligned}
\max \quad & 2 \cdot 4F + 4 \cdot 3B + 6E \\
s.t. \quad & F + B + E = 40 \\
& 6E \geq 2 \cdot 4B + 2 \cdot 2F \\
& F, B, E \geq 3 \\
& F, B, E \quad integer
\end{aligned}
$$

```
totalRows=40
ticketTypes=["First","Business","Economy"]
profitPerTicket={"First":4,"Business":3,"Economy":1}
seatsPerRow={"First":2,"Business":4,"Economy":6}

model=gp.Model()
x=model.addVars(ticketTypes,vtype=gp.GRB.INTEGER,lb=3,name="Rows")
model.setObjective(gp.quicksum(profitPerTicket[t]*seatsPerRow[t]*x[t]
                        for t in ticketTypes),gp.GRB.MAXIMIZE)
model.addConstr(gp.quicksum(x[t]for t in ticketTypes)==totalRows)
economySeats=seatsPerRow["Economy"]*x["Economy"]
otherSeats=gp.quicksum(seatsPerRow[t]*x[t]
                    for t in ticketTypes if t!="Economy")
model.addConstr(economySeats>=2*otherSeats)

model.optimize()
print("optimal value",model.objval)
model.printAttr("X")
```

```
optimal value 336.0

    Variable            X
-------------------------
 Rows[First]            3
Rows[Business]          15
Rows[Economy]           22
```

Question 5: Quantas Airline is configuring a new aircraft and must decide how many First-Class seats, Business-Class seats, and Economy-Class seats to have in the aircraft. The aircraft can have 40 rows of seats, each of which has to be fully First-Class (2 seats per row), Business-Class (4 seats per row), or Economy-Class (6 seats per row). There must be at least three rows of each class. It is also required that the number of Economy-Class seats is at least two times larger than the number of all other seats. Quantas does not want to share the actual profit from each type of ticket. All they agreed to share is that the profit on a First-Class ticket is 4 times the profit of an Economy-Class ticket and the profit of a Business-Class ticket is 3 times the profit of an Economy-Class ticket.

2. Instead of assuming unlimited demand, Quantas would like to consider demand forecast for one week. The aircraft has to be configured once and then fly once every day from Melbourne to Kuala Lumpur and back. The expected demand for each class in each direction – on each working day and on each day of the weekend – is provided in the table below. Note that profits can only be collected for sold tickets, and for each class the number of sold tickets cannot be higher than the number of seats in the aircraft and cannot be higher than the demand for those seats. Formulate a model for this problem that will maximize the total profit. Solve it in Gurobi.

Class	First	Business	Economy
Demand per flight on each working day	4	35	80
Demand per flight on Saturday and Sunday	7	18	120

decisions

$Rows \quad : F, B, E$

WD tickets (per day) : F_W, B_W, E_W

S/S tickets (per day) : F_S, B_S, E_S

Question 5: Quantas Airline is configuring a new aircraft and must decide how many First-Class seats, Business-Class seats, and Economy-Class seats to have in the aircraft. The aircraft can have 40 rows of seats, each of which has to be fully First-Class (2 seats per row), Business-Class (4 seats per row), or Economy-Class (6 seats per row). There must be at least three rows of each class. It is also required that the number of Economy-Class seats is at least two times larger than the number of all other seats. Quantas does not want to share the actual profit from each type of ticket. All they agreed to share is that the profit on a First-Class ticket is 4 times the profit of an Economy-Class ticket and the profit of a Business-Class ticket is 3 times the profit of an Economy-Class ticket.

$$\max \quad 4\cdot(10F_W+4F_S)+3\cdot(10B_W+4B_S)+(10E_W+4E_S)$$

$$s.t. \quad F+B+E=40$$

$$6E \geq 2\cdot 4B + 2\cdot 2F$$

$$F_W \leq 2F, \quad F_S \leq 2F, \quad B_W \leq 4B, \quad B_S \leq 4B, \quad E_W \leq 6E, \quad E_S \leq 6E$$

$$F_W \leq 4, \quad F_S \leq 7, \quad B_W \leq 35, \quad B_S \leq 18, \quad E_W \leq 80, \quad E_S \leq 120$$

$$F,B,E \geq 3$$

$$F,B,E \quad integer$$

2. Instead of assuming unlimited demand, Quantas would like to consider demand forecast for one week. The aircraft has to be configured once and then fly once every day from Melbourne to Kuala Lumpur and back. The expected demand for each class in each direction – on each working day and on each day of the weekend – is provided in the table below. Note that profits can only be collected for sold tickets, and for each class the number of sold tickets cannot be higher than the number of seats in the aircraft and cannot be higher than the demand for those seats. Formulate a model for this problem that will maximize the total profit. Solve it in Gurobi.

Class	First	Business	Economy
Demand per flight on each working day	4	35	80
Demand per flight on Saturday and Sunday	7	18	120

```
totalRows=40
ticketTypes=["First","Business","Economy"]
profitPerTicket={"First":4,"Business":3,"Economy":1}
seatsPerRow={"First":2,"Business":4,"Economy":6}
weekendDemands={"First":7,"Business":18,"Economy":120}
workingDayDemands={"First":4,"Business":35,"Economy":80}
workingDayFlights=10
weekendFlights=4

model=gp.Model()
x=model.addVars(ticketTypes,vtype=gp.GRB.INTEGER,lb=3,name="Rows")
wdt=model.addVars(ticketTypes,vtype=gp.GRB.INTEGER,name="WD")#working day
sst=model.addVars(ticketTypes,vtype=gp.GRB.INTEGER,name="SS")#saturdaysunday day

model.setObjective(gp.quicksum(profitPerTicket[t]*(workingDayFlights*wdt[t]+weekendFlights*sst[t])
                              for t in ticketTypes),gp.GRB.MAXIMIZE)
model.addConstr(gp.quicksum(x[t]for t in ticketTypes)==totalRows)
economySeats=seatsPerRow["Economy"]*x["Economy"]
otherSeats=gp.quicksum(seatsPerRow[t]*x[t]
                       for t in ticketTypes if t!="Economy")
model.addConstr(economySeats>=2*otherSeats)
model.addConstrs(wdt[t]<=workingDayDemands[t] for t in ticketTypes)
model.addConstrs(wdt[t]<=seatsPerRow[t]*x[t] for t in ticketTypes)
model.addConstrs(sst[t]<=weekendDemands[t] for t in ticketTypes)
model.addConstrs(sst[t]<=seatsPerRow[t]*x[t] for t in ticketTypes)

model.optimize()
print("optimal value",model.objval)
model.printAttr("X")
```

```
optimal value 2818.0

    Variable            X
-------------------------
 Rows[First]            4
Rows[Business]           9
Rows[Economy]          27
   WD[First]            4
WD[Business]           35
 WD[Economy]           80
   SS[First]            7
SS[Business]           18
 SS[Economy]          120
```