Management Science

Individual Assignment 1

Due on December 14 at 23:59

Instructions

1. Submit a pdf file answering all 3 questions and a notebook with your code.
2. Your code **should be included in the pdf using screenshots**: you should not assume we will read the notebook, we will only execute it if we need to.
3. Handwritten equations are fine, as long as they are clear.
4. Points awarded are not necessarily proportional to the effort needed to answer a question.
5. Any questions on the assignment should be posted on the corresponding discussion forum in Canvas and not be sent via email. This way if I release any additional information it will be common for all.

Question 1 RSM's swimming coach is setting up the 4×50 medley relay team. He has to assign one swimmer to each of the swimming styles: butterfly stroke (BU), backstroke (BA), breaststroke (BR), and freestyle (FR). Currently there are 5 swimmers in the team and the expected times (in seconds) for each swimmer and each swimming style is given in the table bellow:

		BA	BR	BU	FR
Nadine	(N)	40.1	41.7	41.1	33.3
Mary	(M)	39.6	37.6	31.9	31.2
Lina	(L)	35.3	37.9	36.2	33.1
Asma	(A)	30.6	34.1	33.3	29.1
Carol	(C)	36.9	37.2	29.0	31.6

Formulate a binary program to decide which swimmer to assign to each swimming style to minimize the expected total time on the medley relay event. Solve in Gurobi and report the optimal solution. (**10 points**)

Question 2 An airline company wants to solve the capacity allocation problem for one of its flights. Specifically, for a flight one month from today, there are 100 seats available. Customers that attempt to book a ticket in the last 10 days are in general willing to pay a higher price. Thus, to maximize its revenues, the company charges a price of \$150 during period 1 (first 20 days until departure) and a price of \$500 during period 2. The company must set a booking limit $\boldsymbol{\alpha}$, on the maximum number of tickets it will accept to sell during the first period. If it knew that exactly x customers will arrive during period 2, of course it would set the booking limit to $100-x$. The issue is that the number of customers cannot be known a priori. The company assumes that the number of lower paying customers that arrive in period 1 and the number of high paying customers that arrive in period 2 follow Poison distributions with $\lambda_1 = 75$ and $\lambda_2 = 50$ respectively.

A) Standard Capacity allocation problem

Use simulation to determine the optimal booking policy to maximize expected revenue: for every booking limit between 20 and 80, simulate 10000 paths and compute the expected revenue. Plot the expected revenue as a function of the booking limit. (**30 points**)

Hint: You should simulate the number of customers that arrive in period 1, then decide the number of tickets sold in period 1(given the booking limit) and calculate remaining capacity, then simulate customers that arrive in period 2 and calculate the number of tickets sold in period 2.

B) Callable products

The airline is considering using *callable products* (see attached paper for your interest only). The idea is that together with the booking limit $\boldsymbol{\alpha}$, it decides on a refund compensation $\boldsymbol{p}$ than can be anything between \$100 and \$300. Each customer arriving in period 1 decides whether they want to give the airline the option to sell them back the ticket at period 2 for a price of $\boldsymbol{p}$. Each period 1 customer has a probability of $\frac{\boldsymbol{p}-100}{200}$ to prefer this option to a standard guaranteed ticket. Then the company can decide to how many of those customers to sell back at a later stage, depending on the number of high paying customers arriving at period 2.

Note that the company sells on total up to a maximum of $\boldsymbol{\alpha}$ tickets to period 1 customers regardless of the number that accept a refundable ticket.

Simulate the system for all combinations of $\boldsymbol{\alpha}$ in $[20, 80]$ and $\boldsymbol{p}$ in $[100, 300]$. What is the optimal revenue? Plot the optimal revenue as a function of the optimal booking limit (assuming that the refund price is set optimally for the specific booking limit), in the same plot with the revenue from question A. What is the value of the *callable products* scheme? How does the optimal booking limit compare to the one from part **A**. (**10 points**)

Hint: After calculating the total number of tickets $\boldsymbol{n}_1$ to be sold in period 1 you can sample from the binomial distribution to calculate the number of these tickets that were sold with an option to refund, where the parameters will be $\boldsymbol{n}_1$ and the probability of a customer preferring the refund given the offered refund price.

Question 3

A company must decide the quantity of a product to be produced and sent to each of its warehouses. At a later time, a shipping schedule from the warehouses to its distribution centers must be determined, when demand at the distribution centers becomes known. The cost of producing each product is \$10. For each item sold at a distribution center there is a profit of \$20. It is not necessary to satisfy all the demand at the distribution centers, or send to a distribution center all the products that were sent to a warehouse. Note, however, that the product is perishable, and if not sold it must be discarded without a recovery of the production costs or the transportation costs from the production facility to the warehouse.

A) Small problem

There are 2 warehouses, 4 distribution centers and 3 scenarios for the demands. The transportation costs t_wd from the production facilities to the warehouses and from the warehouses to the production facilities and the demands are

	Transportation Costs (\$)				
	PF	Distribution Centers			
Warehouse		1	2	3	4
A	1	1.5	2	2	3
B	1.5	2	1.2	1.1	2.5

	prob	Demand (1000s)			
		Distribution Centers			
Scenario		1	2	3	4
a	0.3	5	3	7.8	4
b	0.2	3	1	6	3
c	0.5	4.2	4.1	3	5.4

A1 Write a stochastic program to maximize the expected profit. (**20 points**)

A2 Solve the problem in Gurobi. How much should you send to each Warehouse? What is the expected profit. (**15 points**)

B) Realistic size problem There are 50 potential locations to rent a warehouse, each warehouse that is rented has a fixed cost of (\$)100,000. There are 100 distribution centers and 100 scenarios with probabilities 1% each. The transportation costs and demands are given in the files *transportationcost.csv*, *demand.csv* respectively, where the production facility appears with the name *PFacility*.

B1 Write your stochastic program in mathematical notation and incorporate the new feature with the fixed cost for the warehouses you are using. (**5 points**)

B2 Solve the problem in Gurobi. How many warehouses are you using? How much should you send to each Warehouse? What is the expected profit. Plot a histogram of profits for the different scenarios of your optimal solution. What is the worst case profit among all scenarios? (**5 points**)

B3 Change your stochastic program (code only, no need to re-write the model) to optimize for the worst case rather than for expected value. How many warehouses are you using? How much should you send to each warehouse? What is the expected profit? Plot a histogram of profits for the different scenarios of your optimal solution. What is the worst case profit among all scenarios? (**5 points**)

Hint: Introduce a (first stage) variable WP representing the worst case second stage profit (revenue minus warehouse-distribution center transportation). Then your objective function should be equal to the first stage costs plus WP. Introduce second stage constraints that ensure that WP is smaller or equal to the second stage profit under all scenarios.