

Management Science

Sensitivity Analysis: Practice Problems

Question 1: The company you are working for manufactures printed circuit boards of three types. The production of each board requires certain labor time and certain machine time. Current orders for the boards exceed company's production capabilities and therefore some boards must be purchased from the competition. The table below summarizes the current demand, the costs of making and buying each board, and the required and available resources.

| | Types of boards | | | |
|------------------------|-----------------|------|------|-----------------|
| | A | B | C | |
| Demand | 100 | 250 | 300 | |
| Cost to make per unit | \$47 | \$23 | \$62 | |
| Cost to buy per unit | \$56 | \$31 | \$72 | Available hours |
| Labor hours per unit | 5 | 1 | 5 | 2000 |
| Machine hours per unit | 3 | 1 | 2 | 800 |

You were asked to formulate and solve the linear programming problem that could help decide how many boards of each type should be made internally and how many should be bought from the competition to minimize the total cost while satisfying the demand and the resource constraints. Formulate an LP model for this problem and solve it in Gurobi. Then, answer the following questions using the sensitivity analysis report. Consider each question separately from the others.

1. Would the optimal solution change if the cost of buying A increased to \$60? How would this change affect the total cost?

2. Would the optimal solution change if the cost of buying B increased to \$60? How would this change affect the total cost?
3. How would the total cost change if 20 more machine hours were available? Would this change affect the optimal solution?
4. How would the total cost change if 60 more machine hours were available? Would this change affect the optimal solution?
5. The new company policy states that at least 70% of boards overall should be produced internally. Write in mathematical format a new linear constraint that will enforce this restriction. Decide without resolving the problem what will happen with the optimal solution if this constraint is added and the problem is resolved.
6. The company might need to make at least a small number of units of product A. Decide without resolving the problem how the total cost would change if the company were forced to make 1 unit of product A.

Question 2: A company wants to ship oranges from its warehouses in Tampa, Miami, and Fresno to markets in New York, Philadelphia, Chicago, and Boston. Oranges are shipped in full truckloads. All available supply should be shipped. Shipping costs per truckload (in \$1000) and supply and demand (both in truckloads) are:

| | To | | | | |
|-------------------|---------------------|-------------------------|--------------------|-------------------|---------------|
| From | 1 – New York | 2 – Philadelphia | 3 – Chicago | 4 – Boston | Supply |
| 1 – Tampa | 9 | 14 | 12 | 17 | 200 |
| 2 – Miami | 11 | 10 | 6 | 10 | 200 |
| 3 – Fresno | 12 | 8 | 15 | 7 | 200 |
| Demand | 130 | 170 | 200 | 150 | |

Additionally, the following constraints need to be satisfied

- Shipping from Miami to Chicago is not allowed
- At most 5 truckloads must be shipped from Tampa to Boston
- At least 2 truckloads must be shipped from Fresno to Chicago

Formulate an LP model for this problem and solve it in Gurobi. Then, answer the following questions using the sensitivity analysis report. Consider each question separately from the others.

1. Would the optimal solution change if the cost of transportation from Tampa to Chicago increased to 14? What effect would it have on the total cost of transportation?
2. Would the optimal solution change if the cost of transportation from Tampa to Chicago increased to 17? What effect would it have on the total cost of transportation?
3. How would the total cost change if the demand in New York increased by 50 truckloads? Does this change in demand affect the optimal solution?
4. How would the total cost change if the supply in Fresno decreased by 50 truckloads?
5. How would the total cost change if the minimum shipment from Fresno to Chicago were 4 truckloads (instead of 2)?
6. How much would the cost of shipping from Tampa to Philadelphia have to be reduced in order for the shipping to become cost-effective?

Question 3: A toy manufacturer makes small cars, trikes (3-wheel bikes), and scooters for children. The current weekly production plan was set based on the analysis made by the manager and it is to make 60 cars, 50 trikes, and 50 scooters per week. The following table summarizes data for the products:

| | Cars | Trikes | Scooters |
|----------------------------|------|--------|----------|
| Demand (units per week) | 100 | 70 | 60 |
| Selling price (per unit) | \$50 | \$40 | \$45 |
| Material (pounds per unit) | 20 | 10 | 15 |
| Labor (minutes per unit) | 15 | 20 | 10 |

Material costs \$1.20 per pound and currently at most 2800 pounds can be purchased from the supplier per week. One full-time worker who works 40 hours per week makes all the toys. His salary is \$500 per week, paid fully even if the worker time is not fully utilized. Other operating expenses for the toy manufacturer amount to \$3,600 per week.

You are a consultant to the manager who is asked the following questions:

1. Is the company currently profitable? Calculate the total profit achieved with the current weekly production plan.
2. Considering the current material and labor restrictions, can we increase the number of units produced of any product without decreasing the number of units of any other product?
3. What are the optimal production plan and the optimal total profit?
4. What is the percentage increase of total profit between the current production plan and the optimal production plan?

Answer the following questions using the sensitivity analysis report. Consider each question separately from the others. (After you have answered a question, you can modify and resolve your model to see if your answer is correct.)

5. If the company had an opportunity to buy 500 pounds per week of extra material from another supplier for \$1.30 per pound, should the company consider it?
6. If the worker could work 5 hours of overtime per week for an additional \$100 per week, should the company consider it?

7. Given the optimal production plan and the current resource restrictions, should the company work on improving the demand for cars, trikes, or scooters? How much would the extra demand of each of these products improve the total profit?
8. The company is considering introducing a new product, a small wagon. Each wagon will use 10 pounds of material and 5 minutes of labor time. Demand has not been studied yet, but the company estimates that wagons could sell for \$25 per unit. If wagons were introduced and the problem was resolved, would the optimal solution and the optimal profit change?

Question 4: If your home or office is carpeted, there's a good chance that carpet came from Dalton, Georgia—also known as the “Carpet Capital of the World.” Manufacturers in the Dalton area produce more than 70% of the total output of the \$9 billion worldwide carpet industry. Competition in this industry is intense and forces producers to strive for maximum efficiency and economies of scale. It also forces producers to continually evaluate investments in new technology. Kamm Industries is one of the leading carpet producers in the Dalton area. Its owner, Geoff Kamm, has asked for your assistance in planning the production schedule for the next quarter (13 weeks). The company has orders for 15 different types of carpets that the company can either produce on 2 types of looms: Dobbie looms and Pantera looms. Pantera looms produce standard tufted carpeting. Dobbie looms can also produce standard tufted carpeting but also allow the incorporation of designs (such as flowers or corporate logos) into the carpeting. The following table summarizes the orders for each type of carpet that must be produced in the coming quarter along with their production rates and costs on each type of loom, and the cost of subcontracting each order. Note that the first 4 orders involved special production requirements that can only be achieved on a Dobbie loom or via subcontracting. Assume that any portion of an order may be subcontracted.

| | Demand | Dobbie | | Pantera | | Subcontract |
|--------|---------|--------|---------|---------|---------|-------------|
| Carpet | Yds | Yd/Hr | Cost/Yd | Yd/Hr | Cost/Yd | Cost/Yd |
| 1 | 14,000 | 4.510 | \$2.66 | na | na | \$2.77 |
| 2 | 52,000 | 4.796 | 2.55 | na | na | 2.73 |
| 3 | 44,000 | 4.629 | 2.64 | na | na | 2.85 |
| 4 | 20,000 | 4.256 | 2.56 | na | na | 2.73 |
| 5 | 77,500 | 5.145 | 1.61 | 5.428 | \$1.60 | 1.76 |
| 6 | 109,500 | 3.806 | 1.62 | 3.935 | 1.61 | 1.76 |
| 7 | 120,000 | 4.168 | 1.64 | 4.316 | 1.61 | 1.76 |
| 8 | 60,000 | 5.251 | 1.48 | 5.356 | 1.47 | 1.59 |
| 9 | 7,500 | 5.223 | 1.50 | 5.277 | 1.50 | 1.71 |
| 10 | 69,500 | 5.216 | 1.44 | 5.419 | 1.42 | 1.63 |
| 11 | 68,500 | 3.744 | 1.64 | 3.835 | 1.64 | 1.80 |
| 12 | 83,000 | 4.157 | 1.57 | 4.291 | 1.56 | 1.78 |
| 13 | 10,000 | 4.422 | 1.49 | 4.558 | 1.48 | 1.63 |
| 14 | 381,000 | 5.281 | 1.31 | 5.353 | 1.30 | 1.44 |
| 15 | 64,000 | 4.222 | 1.51 | 4.288 | 1.50 | 1.69 |

Kamm currently owns and operates 15 Dobbie looms and 80 Pantera looms. To maximize efficiency and keep pace with demand, the company operates 24 hours a day, 7 days a week. Each machine is down for routine maintenance for approximately 2 hours per week. Create an LP model for this problem that can be used to determine the optimal production plan, solve it in Gurobi and answer the following questions. You can load the data from the file

q4.csv

1. What is the optimal production plan and associated cost?
2. What would happen to the total cost if one of the Dobbie machines broke and could not be used at all during the quarter?
3. What would happen to the total cost if an additional Dobbie machine was purchased and available for the quarter?
4. What would happen to the total cost if one of the Pantera machines broke and could not be used at all during the quarter?

5. What would happen to the total cost if an additional Pantera machine was purchased and available for the quarter?
6. Explain the shadow prices and the values in the Allowable Increase column of the Sensitivity Report for the products that are being outsourced.
7. How much money does it cost to produce carpet order 2? How much would the total cost decrease if that order were eliminated? Explain.
8. If the carpets in orders 5 through 15 all sell for the same amount, which type of carpet should Kamm encourage its sales force to sell more of? Why?
9. If the cost of buying the carpet in order 1 increased to \$2.80 per yard, would the optimal solution change? Why?
10. If the cost of buying the carpet in order 15 decreased to \$1.65 per yard, would the optimal solution change? Why?