

Some simple LR parse tables

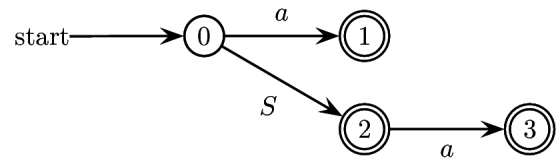
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Left recursion: $S \rightarrow Sa \mid a$

Stack	Input
0	a a a a \$
0 a 1	a a a \$
0 S 2	a a a \$
0 S 2 a 3	a a \$
0 S 2	a a \$
0 S 2 a 3	a \$
0 S 2	a \$
0 S 2 a 3	\$
0 S 2	\$

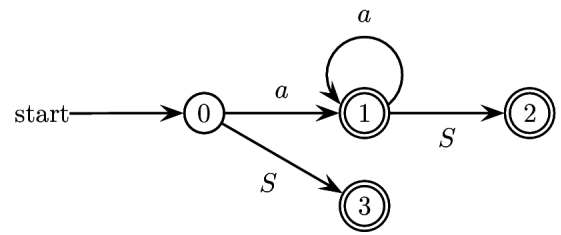
	a	$\$$	S
0	1		2
1	$S \rightarrow a$		
2	3	accept	
3	$S \rightarrow Sa$	$S \rightarrow Sa$	



Right recursion: $S \rightarrow aS \mid a$

Stack	Input
0	a a a a \$
0 a 1	a a a \$
0 a 1 a 1	a a \$
0 a 1 a 1 a 1	a \$
0 a 1 a 1 a 1 a 1	\$
0 a 1 a 1 a 1 S 2	\$
0 a 1 a 1 S 2	\$
0 a 1 S 2	\$
0 S 3	\$

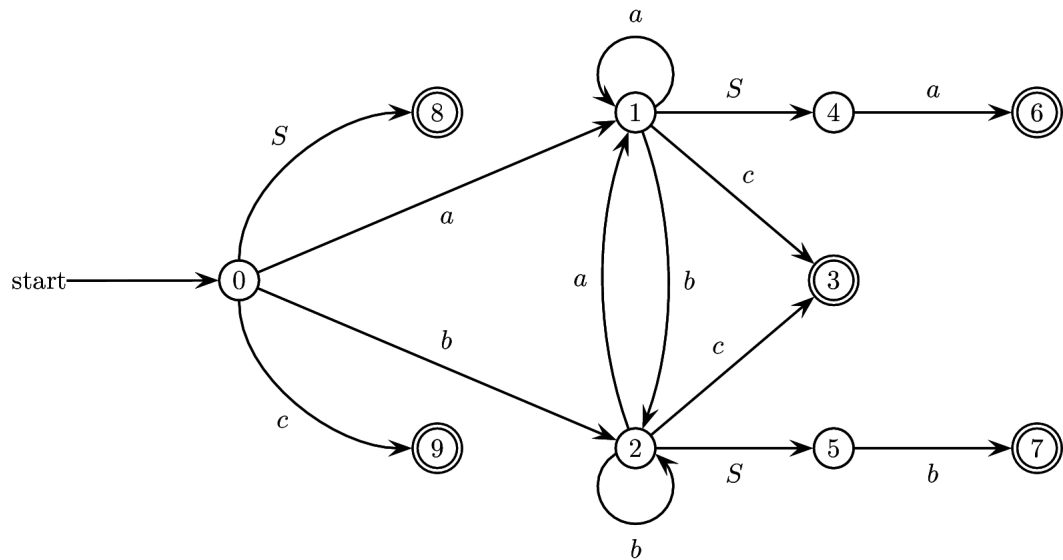
	a	$\$$	S
0	1		3
1	1	$S \rightarrow a$	2
2		$S \rightarrow aS$	
3		accept	



Middle recursion: $S \rightarrow aSa \mid bSb \mid c$

Stack	Input
0	a b c b a \$
0 a 1	b c b a \$
0 a 1 b 2	c b a \$
0 a 1 b 2 c 3	b a \$
0 a 1 b 2 S 5	b a \$
0 a 1 b 2 S 5 b 7	a \$
0 a 1 S 4	a \$
0 a 1 S 4 a 6	\$
0 S 8	\$

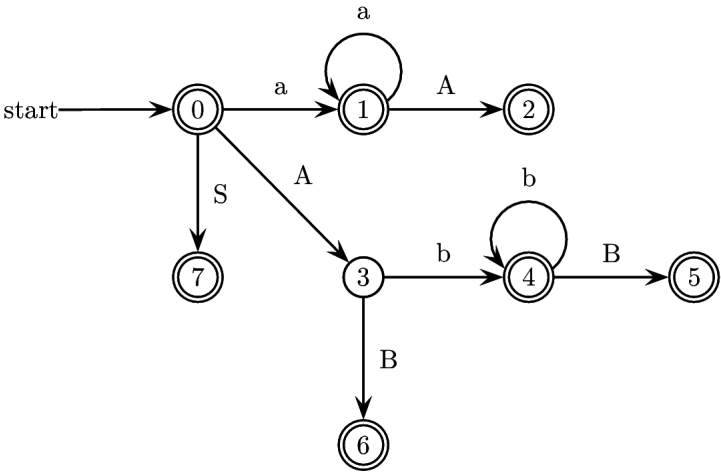
	a	b	c	\$	S
0	1	2	9		8
1	1	2	3		4
2	1	2	3		5
3	$S \rightarrow c$	$S \rightarrow c$			
4	6				
5		7			
6	$S \rightarrow aSa$	$S \rightarrow aSa$		$S \rightarrow aSa$	
7	$S \rightarrow bSb$	$S \rightarrow bSb$		$S \rightarrow bSb$	
8				accept	
9				$S \rightarrow c$	



$$\begin{aligned}
 S &\rightarrow AB \\
 A &\rightarrow aA \mid \epsilon \\
 B &\rightarrow bB \mid b
 \end{aligned}$$

Stack	Input
0	a a b b b \$
0 a 1	a b b b \$
0 a 1 a 1	b b b \$
0 a 1 A 2	b b b \$
0 A 3	b b b \$
0 A 3 b 4	b b \$
0 A 3 b 4 b 4	b \$
0 A 3 b 4 b 4 b 4	\$
0 A 3 b 4 b 4 B 5	\$
0 A 3 b 4 B 5	\$
0 A 3 B 6	\$
0 S 7	\$

	a	b	\$	S	A	B
0	1	$A \rightarrow \epsilon$		7	3	
1	1	$A \rightarrow \epsilon$			2	
2		$A \rightarrow aA$				
3		4				6
4		4	$B \rightarrow b$			5
5			$B \rightarrow bB$			
6			$S \rightarrow AB$			
7			accept			



Stack	Input
0	b b b \$
0 A 3	b b b \$
0 A 3 b 4	b b \$
0 A 3 b 4 b 4	b \$
0 A 3 b 4 b 4 b 4	\$
0 A 3 b 4 b 4 B 5	\$
0 A 3 b 4 B 5	\$
0 A 3 B 6	\$
0 S 7	\$

Same number of *as* and *bs*, Part I

$$S \rightarrow \epsilon \mid aB \mid bA$$

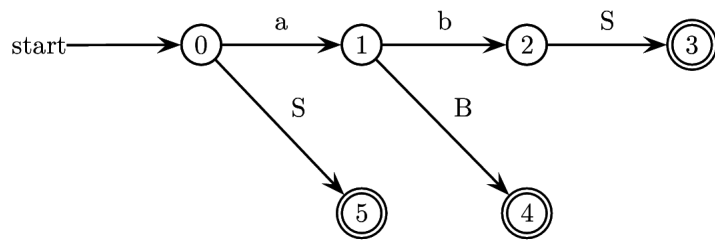
$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow abS \Rightarrow ab$$

Stack	Input
0	a b \$
0 a 1 b 2	\$
0 a 1 b 2 S 3	\$
0 a 1 B 4	\$
0 S 5	\$

	a	b	\$	S	A	B
0	1			5		
1		2				4
2			$S \rightarrow \epsilon$	3		
3			$B \rightarrow bS$			
4			$S \rightarrow aB$			
5			accept			



Same number of *as* and *bs*, Part II

$$S \rightarrow \epsilon \mid aB \mid bA$$

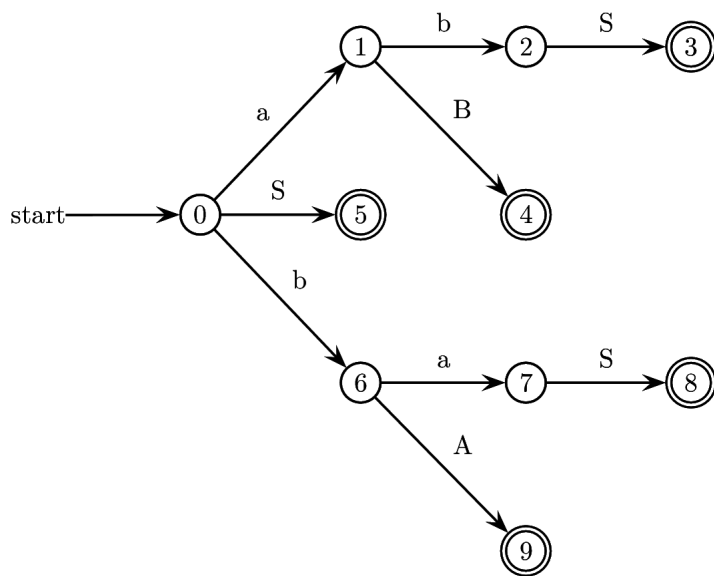
$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow bA \Rightarrow baS \Rightarrow ba$$

Stack	Input
0	b a \$
0 b 6	a \$
0 b 6 a 7	\$
0 b 6 a 7 S 8	\$
0 b 6 A 9	\$
0 S 5	\$

	a	b	\$	S	A	B
0	1	6		5		
1		2				4
2			$S \rightarrow \epsilon$	3		
3			$B \rightarrow bS$			
4			$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			



Same number of *as* and *bs*, Part III

$$S \rightarrow \epsilon \mid aB \mid bA$$

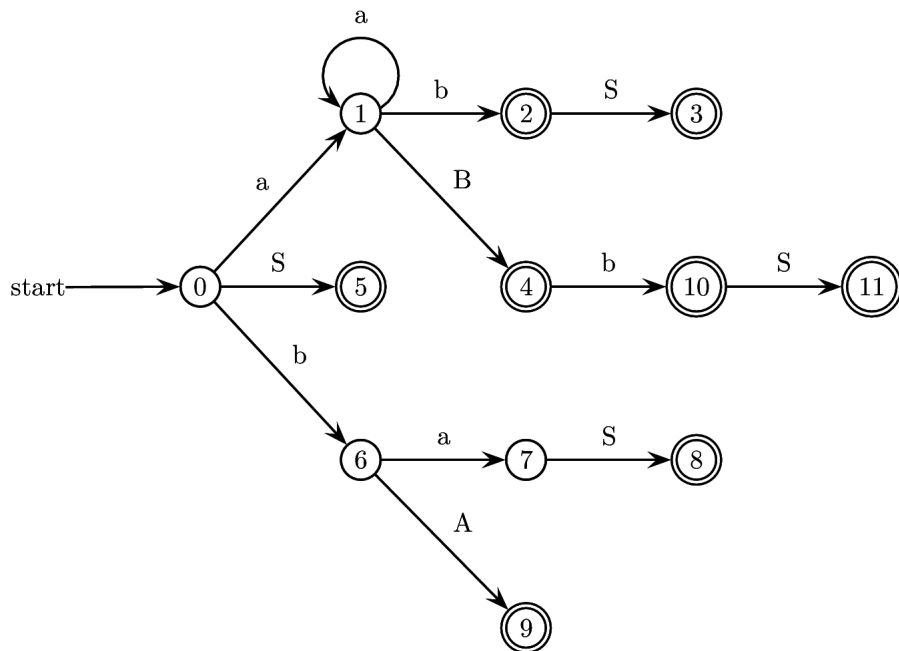
$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b \$
0 a 1	a b b \$
0 a 1 a 1	b b \$
0 a 1 a 1 b 2	b \$
0 a 1 a 1 b 2 S 3	b \$
0 a 1 a 1 B 4	b \$
0 a 1 a 1 B 4 b 10	\$
0 a 1 a 1 B 4 b 10 S 11	\$

	a	b	\$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		10	$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			
10			$S \rightarrow \epsilon$	11		
11			$B \rightarrow bS$			



Same number of *as* and *bs*, Part IV

$$S \rightarrow \epsilon \mid aB \mid bA$$

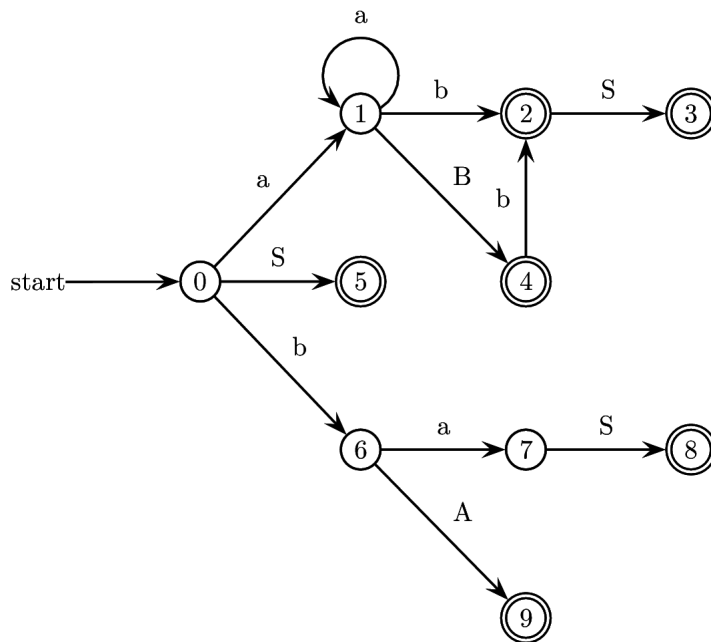
$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b \$
0 a 1	a b b \$
0 a 1 a 1	b b \$
0 a 1 a 1 b 2	b \$
0 a 1 a 1 b 2 S 3	b \$
0 a 1 a 1 B 4	b \$
0 a 1 a 1 B 4 b 2	\$
0 a 1 a 1 B 4 b 2 S 3	\$

	a	b	\$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		2	$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			



Same number of *as* and *bs*, Part V

$$S \rightarrow \epsilon \mid aB \mid bA$$

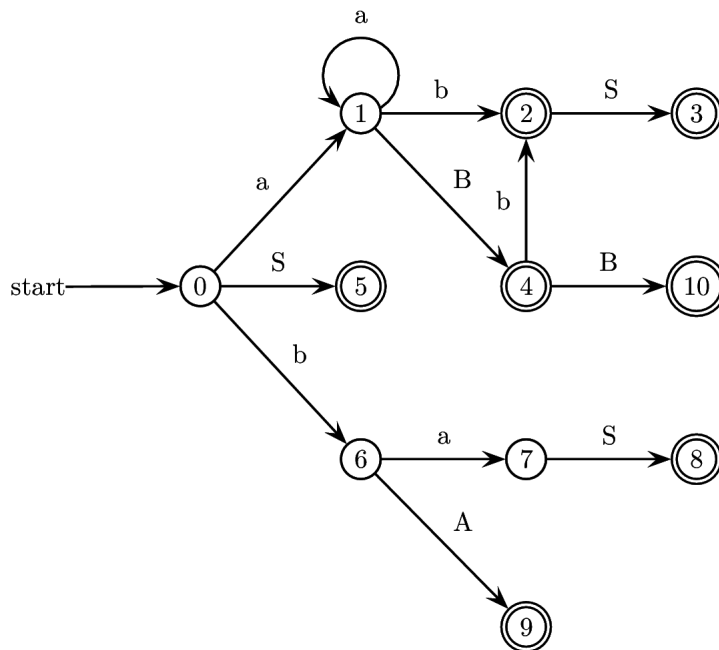
$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b \$
0 a 1	a b b \$
0 a 1 a 1	b b \$
0 a 1 a 1 b 2	b \$
0 a 1 a 1 b 2 S 3	b \$
0 a 1 a 1 B 4	b \$
0 a 1 a 1 B 4 b 2	\$
0 a 1 a 1 B 4 b 2 S 3	\$
0 a 1 a 1 B 4 B 10	\$

	a	b	\$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		2	$S \rightarrow aB$			10
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			
10			$B \rightarrow aBB$			



Same number of *as* and *bs*, Part VI

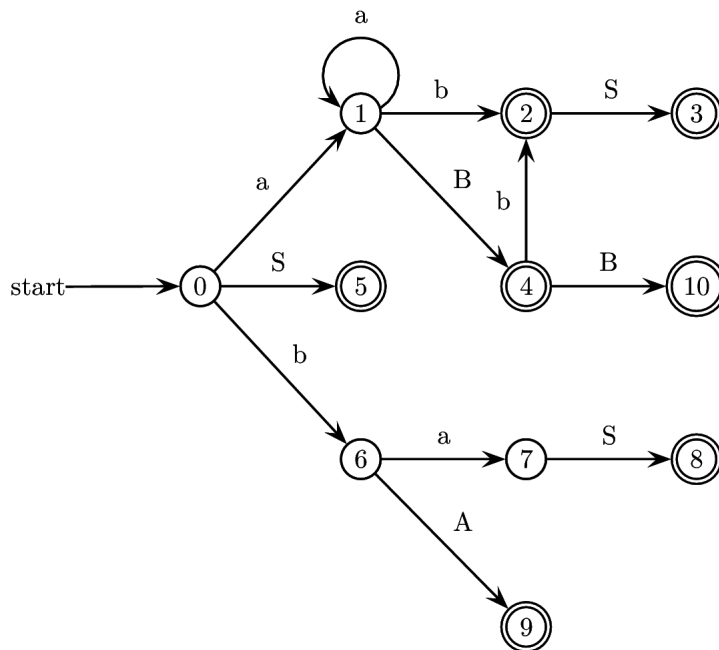
$$S \rightarrow \epsilon \mid aB \mid bA$$

$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input		a	b	\$	S	A	B
0	a a b b \$	0	1	6		5		
0 a 1	a b b \$	1	1	2				4
0 a 1 a 1	b b \$	2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
0 a 1 a 1 b 2	b \$	3		$B \rightarrow bS$	$B \rightarrow bS$			
0 a 1 a 1 b 2 S 3	b \$	4		2	$S \rightarrow aB$			10
0 a 1 a 1 B 4	b \$	5			accept			
0 a 1 a 1 B 4 b 2	\$	6	7				9	
0 a 1 a 1 B 4 b 2 S 3	\$	7			$S \rightarrow \epsilon$	8		
0 a 1 a 1 B 4 B 10	\$	8			$A \rightarrow aS$			
0 a 1 B 4	\$	9			$S \rightarrow bA$			
0 S 5	\$	10			$B \rightarrow aBB$			



Same number of *as* and *bs*, Part VII

$$S \rightarrow \epsilon \mid aB \mid bA$$

$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow bA \Rightarrow bbAA \Rightarrow bbAaS \Rightarrow bbAa \Rightarrow bbaSa \Rightarrow bbaa$$

Stack	Input		a	b	\$	S	A	B
0	b b a a \$	0	1	6		5		
0 b 6	b a a \$	1	1	2				4
0 b 6 b 6	a a \$	2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
0 b 6 b 6 a 7	a \$	3		$B \rightarrow bS$	$B \rightarrow bS$			
0 b 6 b 6 a 7 S 8	a \$	4		2	$S \rightarrow aB$			10
0 b 6 b 6 A 9	a \$	5			accept			
0 b 6 b 6 A 9 a 7	\$	6	7				9	
0 b 6 b 6 A 9 a 7 S 8	\$	7	$S \rightarrow \epsilon$		$S \rightarrow \epsilon$	8		
0 b 6 b 6 A 9 A 11	\$	8	$A \rightarrow aS$		$A \rightarrow aS$			
0 b 6 A 9	\$	9	7		$S \rightarrow bA$			
0 S 5	\$	10			$B \rightarrow aBB$			
		11			$A \rightarrow bAA$			

By symmetry:

