CSCI 301, Lab # 2

Winter 2018

Goal: The purpose of this lab is write some code in which functions are both passed as parameters, and returned as values.

Due: Your program, named `lab02.rkt`, must be submitted to Canvas before midnight, Monday, Jan 29.

Unit tests: At a minimum, your program must pass the unit tests found in the file `lab02-test.rkt`. Place this file in the same folder as your program, and run it; there should be no output.

Program: Write a SCHEME procedure called `I` that does approximate numeric integration, as a companion to the `D` procedure from the lecture notes:

```
(define D
  (lambda (f)
    (let* ((delta 0.00001)
           (two-delta (* 2 delta)))
      (lambda (x)
        (/ (- (f (+ x delta))
              (f (- x delta)))
           two-delta)))))
```

This was found using the numeric approximation to the derivative:

$$\frac{d}{dx}f(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

This can be made more accurate simply by making Δx smaller, so long as we don't get too small for our computer representation.

We have a numeric approximation for the *definite* integral, as follows:

$$\int_a^b f dx = \sum_{i=0}^{n} f(a + i\Delta x) \tag{1}$$

where $a + n\Delta x = b$. This is the "add up the skinny rectangles" definition of the definite integral.

However, we don't have a simple definition of the *indefinite* integral, sometimes called the antiderivative, like we do for the derivative. There's no way to give a simple numerical definition, for example, for I in the following equation:

$$\int f(x)dx = I(f) \tag{2}$$

The indefinite integral of a function, however, can be defined as a function which, when plugged into a *definite* integral, with limits a and b, will return a value. In fact, it's quite simple:

$$\int_a^b f dx = I(b) - I(a) \tag{3}$$

This gives us our key. We will define a Scheme function `i`, for integrate, which, when given a function as an argument, `(i f)`, returns a function of two parameters, `a` and `b`. When *this returned function* is given two numbers, it computes the definite integral of `f` from `a` to `b`, using the summation of skinny rectangles technique.

Here are some examples of how it should work. Since

$$\int_2^4 2x\ dx = 4^2 - 2^2 = 12 \tag{4}$$

then, approximately,

```
((i (lambda (x) (* 2 x))) 2 4)
    => 12
((i (d (lambda (x) (* x x)))) 2 4)
    => 12
```

Note that we can't give a tolerance for the error for our integral function, since we don't really know how big the error will be when we add up the skinny rectangles. Further, greater accuracy can only be achieved with more terms in the summation, which (unlike the derivative) will make our integral slower and slower. For this assignment, we'll make a decent compromise and let the number of rectangles be fixed at $n = 100,000$, which should give us pretty good accuracy without taking too long.