Recursion Theorem Notes

Geoffrey Matthews

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Self Reference

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- Print the following twice, the second time in quotations:
  “Print the following twice, the second time in quotations:”

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- Print the following twice, the second time in quotations:
  “Print the following twice, the second time in quotations:”
- “yields falsehood when appended to its own quotation.”
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Quine Pages

- `https://www.nyx.net/~gthompso/quine.htm`
- `http://www.madore.org/~david/computers/quine.html`

A Quine Turing Machine

- Consider the TM B:

  B: on input $\langle M \rangle$:
  - Create TM A:

    A: on input w:
    - Erase w
    - Write $\langle M \rangle$ on the tape.
    - Simulate what's on the tape.
  - Write $\langle A \rangle$ on the tape.

- Now consider TM A:

  A: on input w:
  - Erase w
  - Write $\langle B \rangle$ on the tape.
  - Simulate what's on the tape.

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  - Simulate what's on the tape.

- Note that B could do other things before writing A and halting.

Recursion theorem makes undecidability easier to prove

$$Halt = \{\langle M, w\rangle : M \text{ is a TM that terminates on } w\}$$

Assume there is a TM H that decides this language.
Construct the following TM Q:

Q : On input w:

- Obtain description of self, $\langle Q\rangle$.
- Run H on $\langle Q, w\rangle$.
- If H accepts, loop forever, else halt.