

# Some L<sup>A</sup>T<sub>E</sub>X examples

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## 1 Mechanics

This file contains some examples to get you started using L<sup>A</sup>T<sub>E</sub>X to typeset mathematics. It is the premiere software for technical publications. Good places to get started with tutorials:

- <http://www.latex-tutorial.com/>
- <http://www.stdout.org/~winston/latex/latexsheet.pdf>

To compile a L<sup>A</sup>T<sub>E</sub>X file, `myfile.tex` to `myfile.pdf`, in the labs, simply enter the following command in a terminal window:

```
pdflatex myfile.tex
```

or use a GUI such as TexWorks or TexStudio.

You can also get your L<sup>A</sup>T<sub>E</sub>X processed online, for example, at

- <https://www.overleaf.com/>
- [www.sharelatex.com](http://www.sharelatex.com)

## 2 Some example text

Here is some inline math:  $\sum_{i=1}^n i^2$  and here is the same thing with display math:

$$\sum_{i=1}^n i^2$$

Here is a set of equations lined up nicely:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b) \\ &= a(a+b) + b(a+b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

You can talk about the real numbers,  $\mathbb{R}$ , the integers  $\mathbb{Z}$ , the rational numbers  $\mathbb{Q}$ , and the natural numbers,  $\mathbb{N}$ , using nice fonts. Notice how I made new commands for some of these in the preamble, to simplify typing. Here is an enumerated list:

1.  $\mathcal{P}(\{1, 2, 3\}) \subseteq \mathcal{P}(\{1, 2, 3, 4\})$
2.  $\bigcup_{i \in \mathbb{N}} i^2 = \{0, 1, 4, 9, \dots\}$
- 3.

$$\bigcap_{i \in \mathbb{N}} i^2 \neq \{0, 1, 4, 9, \dots\}$$

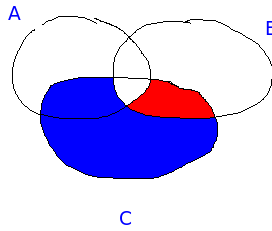
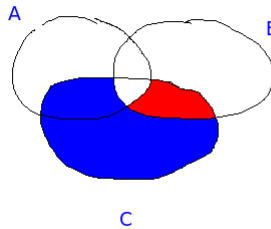


Figure 1: A diagram of some sets.

### 3 Figures

You can also include and scale figures. I drew the picture shown in Figure 1 with a simple paint program, saved it as a `.png` file, and imported it into this document.



You can also include figures inline, like this:

but it looks weird some-

times.

Later on, we'll see how to make spectacular diagrams using the `tikz` package.

## 4 Solutions to Some Exercises From the Book of Proof

**Note:** These problems are all solved in the book. I include their solutions here to demonstrate how to typeset them in  $\text{\LaTeX}$ .

### Chapter 1 Exercises

#### Section 1.1

1.  $\{5x - 1 : x \in \mathbb{Z}\} = \{\dots, -11, -6, -1, 4, 9, 14, 19, 24, 29, \dots\}$

13.  $\{x \in \mathbb{Z} : |6x| < 5\} = \{0\}$

#### Section 1.2

1. (a)  $A \times B = \{(1, a), (1, c), (2, a), (2, c), (3, a), (3, c), (4, a), (4, c)\}$

#### Section 1.4

A. Find the indicated sets.

3.  $\mathcal{P}(\{\{a, b\}, \{c\}\}) = \{\emptyset, \{\{a, b\}\}, \{\{c\}\}, \{\{a, b\}, \{c\}\}\}$

B. Suppose that  $|A| = m$  and  $|B| = n$ . Find the indicated cardinalities.

13.  $|\mathcal{P}(\mathcal{P}(\mathcal{P}(A)))| = 2^{2^{(2^m)}}$

15.  $|\mathcal{P}(A \times B)| = 2^{mn}$

### Section 1.5

3. Suppose  $A = \{0, 1\}$  and  $B = \{1, 2\}$ . Find:

(a)  $A \cup B = \{1, 3, 4, 5, 6, 7, 8, 9\}$

(b)  $A \cap B = \{4, 6\}$

### Section 1.6

1. Suppose  $A = \{4, 3, 6, 7, 1, 9\}$  and  $B = \{5, 6, 8, 4\}$  have the universal set  $U = \{n \in \mathbb{Z} : 0 \leq n \leq 10\}$

$$\overline{A \cap B} = \{0, 1, 2, 3, 4, 6, 7, 9, 10\}$$

### Section 1.8

5(a)  $\bigcup_{i \in \mathbb{N}} [i, i+1] = [1, \infty)$  or:

$$\bigcup_{i \in \mathbb{N}} [i, i+1] = [1, \infty)$$

5(b)  $\bigcap_{i \in \mathbb{N}} [i, i+1] = \emptyset$  or:

$$\bigcap_{i \in \mathbb{N}} [i, i+1] = \emptyset$$

## Chapter 2 Exercises

**Section 2.2** Express each statement as one of the forms  $P \wedge Q$ ,  $P \vee Q$ , or  $\sim P$ . (I will also accept  $\neg P$ .)

9.  $x \in A - B$

$$(x \in A) \wedge \neg(x \in B)$$

### Section 2.5

5. Write a truth table for  $(P \wedge \neg P) \vee Q$

$P$	$Q$	$(P \wedge \neg P)$	$(P \wedge \neg P) \vee Q$
$T$	$T$	$F$	<b>T</b>
$T$	$F$	$F$	<b>F</b>
$F$	$T$	$F$	<b>T</b>
$F$	$F$	$F$	<b>F</b>

## Chapter 3 Exercises

### Section 3.3

1. Suppose a set  $A$  has 37 elements. How many subsets of  $A$  have 10 elements? How many subsets have 30 elements? How many have 0 elements?

Answers:  $\binom{37}{10} = 348,330,136$ ;  $\binom{37}{30} = 10,295,472$ ;  $\binom{37}{0} = 1$ .

## Chapter 4 Exercises

7. Suppose  $a, b \in \mathbb{Z}$ . If  $a \mid b$ , then  $a^2 \mid b^2$ .

*Proof.* Suppose  $a \mid b$ .

By definition of divisibility, this means  $b = ac$  for some integer  $c$ .

Squaring both sides of this equation produces  $b^2 = a^2c^2$ .

Then  $b^2 = a^2d$ , where  $d = c^2 \in \mathbb{Z}$ .

By definition of divisibility, this means  $a^2 \mid b^2$ . ■

## Chapter 5 Exercises

**9. Proposition** Suppose  $n \in \mathbb{Z}$ . If  $3 \nmid n^2$ , then  $3 \nmid n$ .

*Proof.* (Contrapositive) Suppose it is not the case that  $3 \nmid n$ , so  $3 \mid n$ . This means that  $n = 3a$  for some integer  $a$ . Consequently  $n^2 = 9a^2$ , from which we get  $n^2 = 3(3a^2)$ . This shows that there is an integer  $b = 3a^2$  for which  $n^2 = 3b$ , which means  $3 \mid n^2$ . Therefore it is not the case that  $3 \nmid n^2$ . ■

**21. Proposition** Let  $a, b \in \mathbb{Z}$  and  $n \in \mathbb{N}$ . If  $a \equiv b \pmod{n}$ , then  $a^3 \equiv b^3 \pmod{n}$ .

*Proof.* (Direct) Suppose  $a \equiv b \pmod{n}$ . This means  $n \mid (a - b)$ , so there is an integer  $c$  for which  $a - b = nc$ . Then:

$$\begin{aligned} a - b &= nc \\ (a - b)(a^2 + ab + b^2) &= nc(a^2 + ab + b^2) \\ a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3 &= nc(a^2 + ab + b^2) \\ a^3 - b^3 &= nc(a^2 + ab + b^2). \end{aligned}$$

Since  $a^2 + ab + b^2 \in \mathbb{Z}$ , the equation  $a^3 - b^3 = nc(a^2 + ab + b^2)$  implies  $n \mid (a^3 - b^3)$ , and therefore  $a^3 \equiv b^3 \pmod{n}$ . ■

## Chapter 9 Exercises

**27.** The equation  $x^2 = 2^x$  has three real solutions.

*Proof.* By inspection,  $x = 2$  and  $x = 4$  are two solutions of this equation. But there is a third solution. Let  $m$  be the real number for which  $m2^m = \frac{1}{2}$ . Then negative number  $x = -2m$  is a solution, as follows.

$$x^2 = (-2m)^2 = 4m^2 = 4 \left( \frac{m2^m}{2^m} \right)^2 = 4 \left( \frac{\frac{1}{2}}{2^m} \right)^2 = \frac{1}{2^{2m}} = 2^{-2m} = 2^x$$

## Chapter 10 Exercises

**1.** For every integer  $n \in \mathbb{N}$ , it follows that

$$1 + 2 + 3 + 4 + \dots + n = \frac{n^2 + n}{2}$$

or

$$\sum_{i=1}^n i = \frac{n^2 + n}{2}$$

**In this proof I use the second notation. The book shows the solution in the first notation.**

*Proof.* we will prove this with mathematical induction.

(1) Observe that if  $n = 1$ , this statement is  $1 = \frac{1^2+1}{2}$ , which is obviously true.

(2) Consider any integer  $k \geq 1$ . We must show that  $S_k$  implies  $S_{k+1}$ . In other words, we must show that if

$$\sum_{i=1}^k i = \frac{k^2 + k}{2}$$

is true, then

$$\sum_{i=1}^{k+1} i = \frac{(k+1)^2 + (k+1)}{2}$$

is also true. We use direct proof.

Suppose  $k \geq 1$  and

$$\sum_{i=1}^k i = \frac{k^2 + k}{2}$$

We observe that

$$\begin{aligned}
 \sum_{i=1}^{k+1} i &= \sum_{i=1}^k i + (k+1) && \text{(isolating the last term in the sum)} \\
 &= \frac{k^2 + k}{2} + (k+1) && \text{(by the inductive hypothesis)} \\
 &= \frac{k^2 + k + 2(k+2)}{2} \\
 &= \frac{k^2 + 2k + 1 + k + 1}{2} \\
 &= \frac{(k+1)^2 + (k+1)}{2}
 \end{aligned}$$

Therefore we have shown that

$$\sum_{i=1}^{k+1} i = \frac{(k+1)^2 + (k+1)}{2}$$

## 5 Some examples typesetting finite automata in L<sup>A</sup>T<sub>E</sub>X

Put the following in your preamble:

```

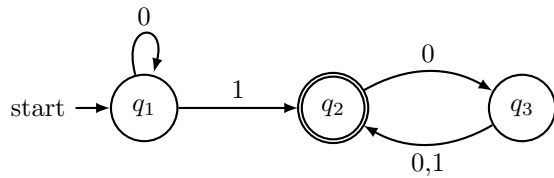
\usepackage{tikz}
\usetikzlibrary{arrows,automata}

```

```

\begin{tikzpicture}[->,>=latex,thick,auto,node distance=2.5cm]
  \node[state,initial] (q1) {$q_1$};
  \node[state,accepting] (q2) [right of=q1] {$q_2$};
  \node[state] (q3) [right of=q2] {$q_3$};
  \path (q1) edge [loop above] node {0} (q1);
  \path (q1) edge node {1} (q2);
  \path (q2) edge [bend left] node {0} (q3);
  \path (q3) edge [bend left] node {0,1} (q2);
\end{tikzpicture}

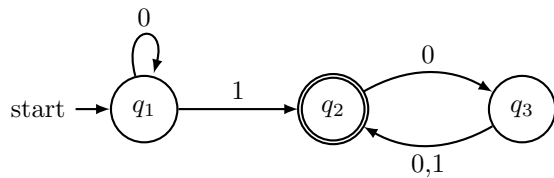
```



```

\begin{tikzpicture}[->,>=latex,thick,auto,node distance=2.5cm]
  \node[state,initial] (q1) {$q_1$};
  \node[state,accepting] (q2) [right of=q1] {$q_2$};
  \node[state] (q3) [right of=q2] {$q_3$};
  \path (q1) edge [loop above] node {0} (q1)
        (q1) edge node {1} (q2)
        (q2) edge [bend left] node {0} (q3)
        (q3) edge [bend left] node {0,1} (q2);
\end{tikzpicture}

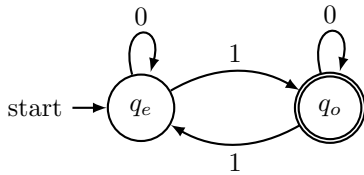
```



```

\begin{tikzpicture}[->,>=latex,thick,auto,node distance=2.5cm]
  \node[state,initial] (e) {$q_e$};
  \node[state,accepting] (o) [right of=e] {$q_o$};
  \path (e) edge [loop above] node {0} (e)
        (e) edge [bend left] node {1} (o)
        (o) edge [bend left] node {1} (e)
        (o) edge [loop above] node {0} (o);
\end{tikzpicture}

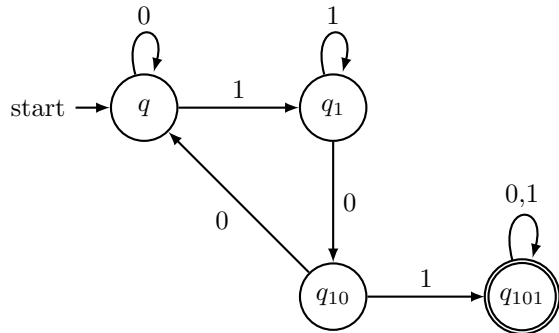
```



```

\begin{tikzpicture}[->,>=latex,thick,auto,node distance=2.5cm]
  \node[state,initial] (q) {$q$};
  \node[state] (1) [right of=q] {$q_1$};
  \node[state] (10) [below of=1] {$q_{10}$};
  \node[state,accepting] (101) [right of=10] {$q_{101}$};
  \path (q) edge [loop above] node {0} (q)
        (q) edge node {1} (1)
        (1) edge [loop above] node {1} (1)
        (1) edge node {0} (10)
        (10) edge node {0} (q)
        (10) edge node {1} (101)
        (101) edge [loop above] node {0,1} (101);
\end{tikzpicture}

```



```

\begin{tikzpicture}[->,>=latex,thick,auto,node distance=2.5cm]
  \node[state,initial] (000) {$q_{000}$};
  \node[state,accepting] (100) [right of=000] {$q_{100}$};
  \node[state] (010) [right of=100] {$q_{010}$};
  \node[state,accepting] (110) [right of=010] {$q_{110}$};
  \node[state] (001) [below of=000] {$q_{001}$};
  \node[state,accepting] (101) [right of=001] {$q_{101}$};
  \node[state] (011) [right of=101] {$q_{011}$};
  \node[state,accepting] (111) [right of=011] {$q_{111}$};
  \path (000) edge [loop above] node {0} (000)
  (000) edge node {1} (001)
  (100) edge node {0} (000)
  (100) edge node [left] {1} (001)
  (010) edge node {0} (100)
  (010) edge [out=-140,in=50] node [left] {1} (101)
  (110) edge [out=90,in=90] node [above] {0} (100)
  (110) edge node [above] {1} (101)
  (001) edge node {0} (010)
  (001) edge [out=-90,in=-90] node {1} (011)
  (101) edge [out=40,in=-130] node [right] {0} (010)
  (101) edge node {1} (011)
  (011) edge node [below] {0} (110)
  (011) edge node {1} (111)
  (111) edge node {0} (110)
  (111) edge [loop below] node {1} (111);
\end{tikzpicture}

```

