Lab Project: GPS Systems

General Information: Our modern GPS systems use linear algebra to quickly determine our position in space-time. Here we get a glimpse of the mathematics involved in this technology. A *highly* suggested resource for this project is:

> Kalman, D. (2002). An Underdetermined Linear System for GPS. *The College Mathematics Journal*, 33(5), 384–390. https://doi.org/10.2307/1559010.

Instructions:

Computing GPS in $\mathbb{R}^3$ requires a system of four equations of the form,

$$(x - a_i)^2 + (y - b_i)^2 + (z - c_i)^2 = d_i^2,$$

where (x, y, z) are the unknown GPS coordinates (in units of earth radii) of the object at time t (in milliseconds), (a_i, b_i, c_i) are the coordinates of the ith satellite with signal emanating at time t_i, and d_i is the distance from the object to the ith satellite. We can calculate d_i using the the equation $distance = rate \times time$ and the fact that the speed of light is 0.047 (in units of earth radii per millisecond) as $d_i = 0.047(t - t_i)$.

Satellite (i)	Position (a_i, b_i, c_i)	Initial Time (t_i)
1	(1,2,0)	11.99
2	(2,0,2)	8.23
3	(1,1,1)	33.30
4	(2,1,0)	10.47

Based on the above satellite information, determine the coordinates (x, y, z) of the object at time t such that the object is on the surface of the earth (i.e. $x^2 + y^2 + z^2 = 1$).