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# Methods for calculating phase angle from measured whole body bioimpedance modulus

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**Abstract.** Assuming the Cole equation we have developed a method to calculate the Cole parameters ( $R_0$ ,  $R_\infty$ ,  $\alpha$ ,  $\tau_Z$ ) and the phase angle from four frequency measurements of impedance modulus values. The values obtained compare well with impedance measurements obtained using the Solatron 1294/1260 as obtained when making whole body measurements on five persons. We have also performed calculations using an algorithm based on the Kramers-Kronig approach. The results which are presented show that it is possible to obtain complete body impedance data combining relatively simple measurements with advanced calculation using a laptop. This extends the potential of portable equipment, since the measurements will require less instrumentation.

## 1. Introduction

In bioimpedance measurements we generally need both the impedance modulus and the phase angle values. This is provided using the more advanced, and also more expensive equipment, like the Solatron 1260/1294 system that we have used.

For many measurements somewhat simpler and transportable equipment is used and the measurements can be limited to an impedance modulus measurement only. This however is a strong limitation for the use of the results since the phase angle values are needed for many further calculations.

This paper discusses how impedance modulus measurements together with established principles and formulas can be used to calculate the complete bioimpedance characteristics.

First, both impedance modulus and phase angle are measured using the Solatron 1260/1294 system. The phase angle value is used as a reference for the calculations.

A least squared method is used to calculate the Cole parameters, which then can be used to find the real and imaginary impedance components and from them the phase angle.

In the second method we employ the Kramers-Kronig [1] relation to calculate the phase angle values.

The measurements and calculations are performed for measurements on five persons. The phase angle values thus obtained correspond well with the Solatron 1260/1294 system measurements.

## 2. Methods for calculation of phase angle

We have used two methods to obtain the phase angle from the impedance modulus, one is based on a Cole equation model while the other is based on an numerical adaption of the Kramers-Kronig equation.

### 2.1. Cole equation model

From earlier experiments we have seen that all measurements produce circular arcs in the complex impedance plane (Wessel plot) above 1kHz [2]. We can use this to make a model of the Cole equation to calculate the phase angle from the impedance modulus and also the Cole parameters ( $R_0$ ,  $R_\infty$ ,  $\alpha$ ,  $\tau_z$ ) using non-linear least squares method on the Cole equation model. We implemented this using the Python programming language, and using four frequencies; 5kHz, 50kHz, 100kHz and 200kHz. Starting from the Cole equation:

$$Z = R_\infty + \frac{R_0 - R_\infty}{1 + (j\omega\tau_z)^\alpha} \quad \text{Eq. 1}$$

And so introducing the relation:

$$j^\alpha = \cos\left(\alpha \frac{\pi}{2}\right) + j \sin\left(\alpha \frac{\pi}{2}\right) \quad \text{Eq. 2}$$

The modulus  $|Z|$  can then be expressed as:

$$|Z| = \sqrt{\frac{k_1^\alpha R_\infty^2 + 2R_\infty k_1^{\alpha/2} \cdot k_2 R_0 + R_0^2}{1 + 2k_1^{\alpha/2} k_2 + k_1^\alpha}} \quad \text{Eq. 3}$$

Where  $k_1$  and  $k_2$  are given by:

$$k_1 = (2\pi f)^2 \cdot \tau_z^2 = \omega^2 \tau_z^2 \quad k_2 = \cos\left(\frac{\pi \alpha}{2}\right)$$

The expression for  $|Z|$  is determined by the Cole parameters  $R_0$ ,  $R_\infty$ ,  $\alpha$  and  $\tau_z$ . Thus by making measurements of  $|Z|$  at four different frequencies we get four independent equations for determination of the Cole parameters. These can be solved using numerical methods.

### 2.2. Kramers-Kronig equation

From the Kramers-Kronig theory we have that if the real part of a linear network function of frequency is known over the complete frequency spectrum, it is possible to calculate the imaginary part [1]. We can then use the following relation to calculate the phase angle at a given frequency ( $\omega$ ) from the modulus.

$$\varphi(\omega_1) = \frac{2\omega}{\pi} \int_0^\infty \ln \frac{|Z(\omega)|}{\omega^2 - \omega_1^2} d\omega \quad \text{Eq. 4}$$

This equation can be used for analytical evaluation, but since we are using numerical evaluation we need to make the following sum approximation from Eq. 4:

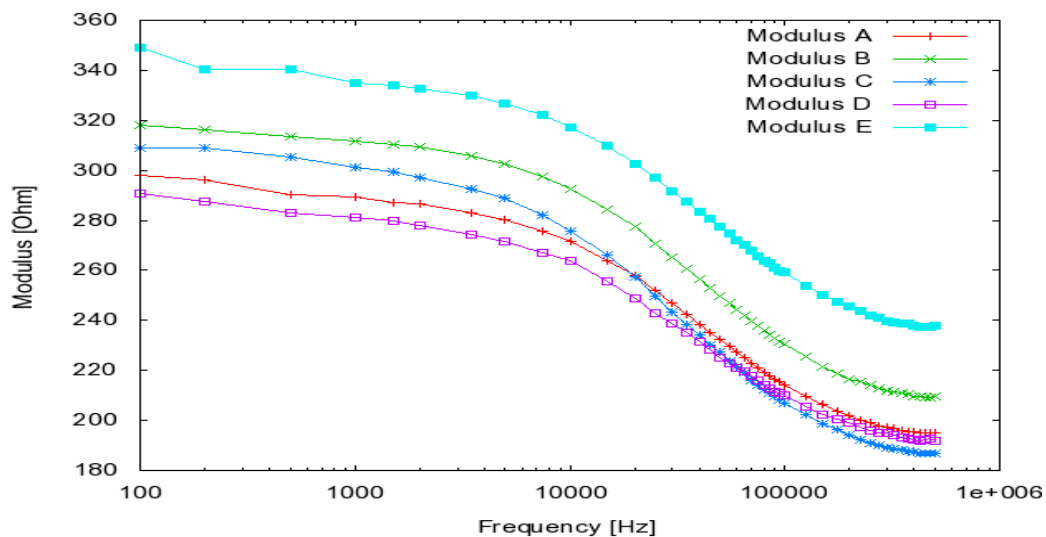
$$\varphi(\omega_1) = \frac{2\omega}{\pi} \sum_{i=0}^n \ln \frac{|Z(\omega)|}{\omega_i^2 - \omega_1^2} \Delta\omega = \frac{2\omega}{\pi} \sum_{i=0}^n \frac{\ln(|Z(\omega_i)|) - \ln(|Z(\omega_1)|)}{\omega_i^2 - \omega_1^2} \Delta\omega \quad \text{Eq. 5}$$

This sum requires equal and small steps in frequencies, and since we did not do the measurements with these small steps in the frequencies we performed a cubic-spline interpolation using Python. We then obtained a linear distribution of data points, with a step in frequency between each data point of 100Hz.

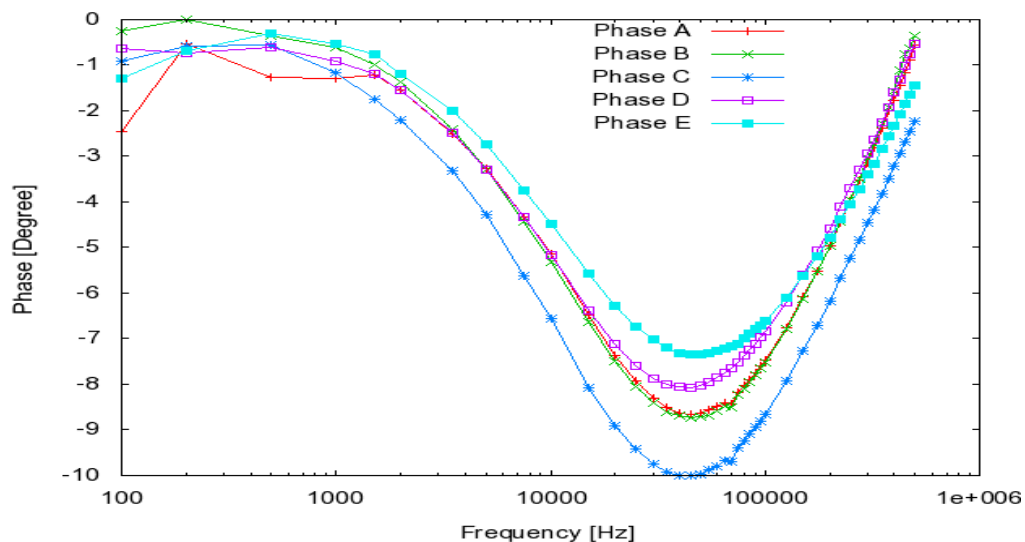
### 3. Measurements

Using a Solatron 1260/1294 we used a 4-electrode setup to do whole body measurements on five persons, using measurement frequencies from 100Hz to 500kHz. For the Cole equation model we only used four of the frequencies (5, 50, 100 and 200kHz), while for the Kramers-Kronig equation we have used data from all the frequencies.

Two electrodes, one current carrying (CC) and one pickup (PU), were placed on the right leg with 7 cm between, and similarly on the right arm. The measurement was done using 800mV AC, this amplitude was determined after doing an amplitude sweep from 80mV to 2V using a whole body measurement with a 2-electrode setup to determine the maximum safe amplitude within the linear range. The measured results are shown in figures 1 and 2.



**Figure 1.** Five whole body impedance modulus measurements



**Figure 2.** Five whole body phase angle measurements

#### 4 Comparison of measured and calculated results

Table 1 shows the measured and calculated phase angles and also the percentage error results for five different persons A to E at the frequencies 5, 50, 100 and 200kHz. Column M represents the values measured by the Solatron system, here considered as reference values while C and K are the values calculated using the Cole and Kramers-Kronig approaches respectively.

	A			B			C			D			E		
f [kHz]	M [°]	C [°]	K [°]	M [°]	C [°]	K [°]	M [°]	C [°]	K [°]	M [°]	C [°]	K [°]	M [°]	C [°]	K [°]
5	-3.8	-3.52	-3.57	-3.83	-3.89	-3.65	-4.69	-4.58	-4.71	-3.83	-3.97	-3.62	-3.34	-3.47	-3.16
50	-8.46	-8.63	-7.84	-8.53	-8.66	-7.93	-9.61	-10.28	-9.28	-7.92	-7.86	-7.39	-7.92	-7.86	-6.79
100	-7.45	-7.73	-6.90	-7.49	-7.94	-6.83	-8.47	-8.83	-7.86	-6.90	-6.99	-6.27	-6.90	-6.99	-5.80
200	-5.27	-5.83	-4.96	-5.27	-6.25	-4.93	-6.32	-6.43	-5.58	-4.94	-5.41	-4.50	-4.94	-5.41	-4.36
	C [%]		K [%]		C [%]		K [%]		C [%]		K [%]		C [%]		K [%]
5	-7.37		-6.05		+1.57		-4.70		-2.35		-0.43		+3.66		-5.48
50	+2.01		-7.33		+1.52		-7.03		+6.97		-3.43		-0.76		-6.69
100	+10.74		-7.38		+6.01		-9.08		+4.25		-7.91		+1.30		-9.13
200	+10.63		-5.88		+18.60		-15.94		+1.74		-11.71		+1.82		-8.91

**Table 1.** Measured (M), calculated using Cole (C) and calculated phase angles and using Kramers-Kronig (K) for the five whole-body measurements on persons A-E.

The calculated values correspond well with those measured, with the best fit obtained for the Cole equation approach. The results for the Kramers-Kronig approach all have less negative values (up to 1°) for the phase angle than the measured. This is caused by the fact that the calculation only used values up to 500kHz resulting in a truncation of the summation [3]. Increasing the highest frequency should improve the results.

#### 5 Conclusion

It has been shown that the bioimpedance phase angle can be calculated from simple bioimpedance modulus measurements with good accuracy using both a Cole equation approach giving the Cole parameters and a Kramers-Kronig direct calculation of the phase angle. The best results are obtained using the Cole equation, while the Kramers-Kronig method results in a reduced value due to the truncation of the expansion.

This shows that portable measurement equipment with limited functionality may give complete bioimpedance results when used in combination with a laptop for computation.

#### References

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