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2° Prova

Questão 1: Calcule as derivadas das funções:

a) $f(x) = \sqrt{5x}\, \cos(3x)$;

$$(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$= \frac{5}{2\sqrt{5x}} \cdot \cos(3x) + \sqrt{5x} \cdot (-\operatorname{sen}(3x) \cdot 3)$$

$$= \frac{5\cos(3x)}{2\sqrt{5x}} + \sqrt{5x} \cdot (-3\operatorname{sen}(3x))$$

$$= \frac{5\cos(3x)}{2\sqrt{5x}} - \sqrt{5x} \cdot 3\operatorname{sen}(3x)$$

$$= \frac{5\cos(3x)}{2\sqrt{5x}} - 3\sqrt{5x}\operatorname{sen}(3x)$$

$$(f(x) \cdot g(x)) = \frac{5\cos(3x) - 30x \cdot \operatorname{sen}(3x)}{2\sqrt{5x}} \;_{//}$$

b) $f(x) = e^{\operatorname{sen}(x^2)}$

$$f'(x) = e^{g(x)}$$

$$f'(x) = g'(x) \cdot e^{g(x)}$$

$$= \cos(x^2) \cdot 2x \cdot e^{\operatorname{sen}(x^2)}$$

$$f'(x) = 2e^{\operatorname{sen}(x^2)}\, x \cdot \cos(x^2) \;_{//}$$

c) $f(x) = (2x^5 + x^3 + 51)^{\frac{1}{2}}$ $\qquad g(x) = 2x^5 + x^3 + 51$

$$f'(x) = f'(g(x)) \cdot g(x)$$

$$f'(x) = g^{\frac{1}{2}} \cdot (2x^5 + x^3 + 51)$$

$$= \frac{1}{2} g^{\frac{1}{2}-1} \cdot (2x^5 + x^3 + 51)$$

$$= \frac{1}{2} g^{-\frac{1}{2}} \cdot (2 \cdot 5x^4 + 3x^2) \quad \rightarrow \text{continua}$$

continuação da 1. c.

$$f(x) = \frac{1}{2} \cdot (2x^5 + x^3 + 51)^{-\frac{1}{2}} \cdot (10x^4 + 3x^2)$$

$$= \frac{1}{2} \cdot \frac{1}{(2x^5 + x^3 + 51)^{\frac{1}{2}}} \cdot (10x^4 + 3x^2)$$

$$= \frac{1}{2(2x^5 + x^3 + 51)^{\frac{1}{2}}} \cdot (10x^4 + 3x^2)$$

$$f'(x) = \frac{10x^4 + 3x^2}{2(2x^3 + x^3 + 51)^{\frac{1}{2}}} \rightarrow f'(x) = \frac{10x^4 + 3x^2}{2\sqrt{2x^5 + x^3 + 51}}$$ //

Questão 2 - Determine y' dadas as equações:

a) $\sin(xy) + x^2 y = e^x + 21$ $\quad y' = \frac{dy}{dx}$

⇒ Derivando

$$\Rightarrow \frac{d(\sin(xy))}{dx} + \frac{d(x^2y)}{dx} = \frac{d(e^x)}{dx} + \frac{d(21)}{dx}$$

derivada de uma constante é 0.

$\hookrightarrow g = xy$

$$\Rightarrow d(\sin(g)) \cdot \frac{d(xy)}{dx} + \frac{d(x^2)}{dx} \cdot y + x^2 \cdot \frac{d(y)}{dx} = \frac{d(e^x)}{dx} + 0$$

$$\Rightarrow \cos(g) \cdot \left(y + x \cdot \frac{d(y)}{dx}\right) + 2xy + x^2 \cdot 1 \cdot \frac{dy}{dx} = \frac{d(e^x)}{dx}$$

$$\Rightarrow \cos(xy) \cdot \left(y + x \cdot \frac{d(y)}{dy} \cdot \frac{dy}{dx}\right) + 2xy + x^2 \cdot \frac{dy}{dx} = e^x$$

$$\Rightarrow \cos(xy) \cdot \left(y + x \cdot 1 \cdot \frac{dy}{dx}\right) + 2xy + x^2 \cdot \frac{dy}{dx} = e^x$$

$$\Rightarrow \cos(xy) \cdot y + \cos(xy) \cdot x \cdot \frac{dy}{dx} + 2xy + x^2 \cdot \frac{dy}{dx} = e^x$$

$$\Rightarrow 3x^2y^3 + 3x^3y^2 \cdot \frac{d(y)}{dy} \cdot \frac{dy}{dx} = y + x \cdot \frac{dy}{dx}$$

$$\Rightarrow 3x^2y^3 + 3x^3y^2 \cdot 1 \cdot \frac{dy}{dx} = y + x \cdot \frac{dy}{dx}$$

$$\Rightarrow 3x^3y^2 \cdot \frac{dy}{dx} - x \cdot \frac{dy}{dx} = y - 3x^2y^3$$

$$\Rightarrow (3x^3y^2 - x) \cdot \frac{dy}{dx} = y - 3x^2y^3$$

→ continua

continuação 2.a

$\Rightarrow y' = \dfrac{y - 3x^2 y^3}{3x^3 y^2 - x}$

$y' = \dfrac{-y \cdot (-1 + 3x^2 y^2)}{x \cdot (3x^2 y^2 - 1)}$

$y' = \dfrac{-y}{x}$ #

b) $x^3 y^3 = xy$

$3x^2 \cdot y^3 + x^3 \cdot 2y^2 \cdot y' = y' \cdot x + y \cdot 1$

$3x^2 y^3 + x^3 \cdot 2y^2 y' = y' \cdot x + y$

$x^3 2y^2 y' - xy' = y - 3x^2 y^3$

$y'(x^3 2y^2 - x) = y - 3x^2 y^3$

$y' = \dfrac{y - 3x^2 y^3}{x^3 2y^2 - x}$ //