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Avaliação II

1) Dê as posições relativas:

a) $r: \begin{cases} x - y - z = 2 \\ x + y - z = 0 \end{cases}$ e $s: \begin{cases} 2x - 3y + z = 5 \\ x + y - 2z = 0 \end{cases}$

$r: 2x - 2z = 2 \div 2$

$x - z = 1$ $\qquad$ $z + 1 + y - z = 0$

$x = z + 1$ $\qquad$ $y = -1$

$r: (x, y, z) = (z+1, -1, z)$

$(x, y, z) = (1, -1, 0) + \lambda(1, 0, 1)$

$s: \begin{cases} 4x - 6y + 2z = 10 \\ x + y - 2z = 0 \end{cases}$

$5x - 5y = 10 \div 5$ $\qquad$ $y + 2 + y - 2z = 0$

$x - y = 2$ $\qquad$ $2y + 2 = 2z$

$x = y + 2$ $\qquad$ $z = y + 1$

$(x, y, z) = (y+2, y, y+1)$

$(x, y, z) = (2, 0, 1) + \lambda(1, 1, 1)$

$A = (1, -1, 0)$

$B = (2, 0, 1)$

$\vec{AB} = B - A = (1, 1, 1)$

$$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Concorrentes"

b) $r: \dfrac{x-1}{2} = y = z$ e

$\pi: X = (3, 0, 1) + \lambda(1, 0, 1) + \mu(2, 2, 0).$

$\vec{r} = (2, 1, 1)$

$\vec{x_1} = (1, 0, 1)$

$\vec{x_2} = (2, 2, 0)$

$$\begin{array}{ccccc} & & 0 & 4 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 & 2 \\ & & 0 & 2 & 2 \end{array}$$

$2 + 2 - 4 = 0 \rightarrow 0 = 0$

Os 3 vetores são LD, logo, a reta é paralela ao plano.

$(1, 0, 0) = (3, 0, 1) + \lambda(1, 0, 1) + \mu(2, 2, 0)$

$(1, 0, 0) = (3 + \lambda + \mu, 2\mu, 1 + \lambda)$

$\begin{cases} 3 + \lambda + \mu = 1 \\ 2\mu = 0 \\ 1 + \lambda = 0 \end{cases}$ $\qquad$ $\mu = 0$, $\lambda = -1$, $3 - 1 = 1$

O sistema não é possível e portanto o ponto de r não está contido no plano, logo, eles são apenas paralelos"

c) $r: \begin{cases} x - y + z = 0 \\ 2x + y - z - 1 = 0 \end{cases}$ e

$\pi: X = (0, \frac{1}{2}, 0) + \lambda(1, \frac{1}{2}, 0) + \mu(0, 1, 1)$

$\begin{cases} x - y + z = 0 \\ 2x + y - z - 1 = 0 \end{cases}$ $\qquad$ $\frac{1}{3} - y + z = 0$

$3x - 1 = 0$ $\qquad$ $y = z + \frac{1}{3}$

$x = \frac{1}{3}$ $\qquad$ $(x, y, z) = (\frac{1}{3}, z + \frac{1}{3}, z)$

$(x, y, z) = (\frac{1}{3}, \frac{1}{3}, 0) + \lambda(0, 1, 1)$

$\vec{r} = (0, 1, 1)$

$\pi_1 = (1, -\frac{1}{2}, 0)$ $\qquad$ $(\frac{1}{3}, \frac{1}{3}, 0) = (0, \frac{1}{2}, 0) + \lambda(1, -\frac{1}{2}, 0)$

$\pi_2 = (0, 1, 1)$ $\qquad$ $+ \mu(0, 1, 1)$

$$\left(\frac{1}{3},\frac{1}{3},0\right) = \left(\lambda, \frac{1}{2}-\frac{\lambda}{2}+\mu, \mu\right)$$

$$\begin{cases} \lambda = \frac{1}{3} \\ \frac{1}{2}-\frac{\lambda}{2}+\mu = \frac{1}{3} \\ \mu = 0 \end{cases} \qquad \frac{1}{2}-\frac{1}{6} = \frac{1}{3} \qquad \frac{1}{3} = \frac{1}{3}$$

a reta r está contida no plano.

2) Calcule as distâncias:

a) $P=(-1,-1,4)$ e $Q=(1,2,-8)$.

$$d(P,Q) = \sqrt{(x_1-x_0)^2+(y-y_0)^2+(z-z_0)^2}$$

$$d(P,Q) = \sqrt{(-1-1)^2+(-1-2)^2+(4+8)^2}$$

$$d(P,Q) = \sqrt{(-2)^2+(-3)^2+(12)^2}$$

$$d(P,Q) = \sqrt{4+9+144}$$

$$d(P,Q) = \sqrt{157}\,//$$

b) $P=(-2,0,1)$ e $r: \begin{cases} x=3\lambda+1 \\ y=2\lambda-2 \\ z=\lambda \end{cases}$

$\lambda = 0$

$\vec{QP} = P-Q = (-3,2,1)$ $\qquad \vec{u}=(3,2,1)$ $\qquad Q=(1,-2,0)$

$6k \quad 2i \quad -3j$

$$\vec{QP}\times\vec{u} = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ -3 & 2 & 1 & -3 & 2 \\ 3 & 2 & 1 & 3 & 2 \end{array}\right|$$

$2i \quad 3j \quad -6k$

$$\vec{QP}\times\vec{u} = \cancel{2i} + \underbrace{3j - 6k - 6k - \cancel{2i} + 3j}_{6j-12k}$$

$$\vec{QP}\times\vec{u} = (0,6,-12)$$

$$d(P,r) = \frac{\|\vec{QP}\times\vec{u}\|}{\|\vec{u}\|}$$

$$d(P,r) = \frac{\sqrt{0^2+6^2+(-12)^2}}{\sqrt{3^2+2^2+1^2}}$$

$$d(P,r) = \frac{\sqrt{36+144}}{\sqrt{9+4+1}} \rightarrow d(P,r) = \frac{\sqrt{180}}{\sqrt{14}}$$

$$d(P,r) = \sqrt{\frac{180}{14}} \rightarrow d(P,r) = \frac{3\sqrt{70}}{7}\,//$$

c) $P=(1,1,\frac{15}{9})$ e $\pi: 4x-6y+12z+21=0$

$$d(P,\pi) = \frac{\left|4.1-6.1+\frac{12.15}{6}+21\right|}{\sqrt{4^2+(-6)^2+12^2}}$$

$$d(P,\pi) = \frac{|49|}{\sqrt{196}} = \frac{49}{14} = \frac{7}{2}\,//$$

d) $r: \frac{x+4}{3} = \frac{y}{4} = \frac{z+5}{-2}$ e

$$s: \begin{cases} x = 21+6\lambda \\ y = -5-4\lambda \\ z = 2-\lambda \end{cases}$$

$y = \lambda$

$\frac{x+4}{3} = \lambda$

$4x+16 = 3\lambda$

$x = \frac{1}{4}(3\lambda-16) = \frac{3}{4}\lambda - 4$

$\frac{z+5}{-2} = \frac{\lambda}{4}$

$4z+20 = -2\lambda$

$z = -\frac{1}{2}\lambda - 5$

$$r: (-4,0,-5)+\lambda\left(\frac{3}{4},1,-\frac{1}{2}\right)$$

Continuação 2d

$$d(r,s) = \frac{\|\vec{AB} \cdot (\vec{r} \times \vec{s})\|}{\|\vec{r} \times \vec{s}\|}$$

$$(\vec{r} \times \vec{s}) = \begin{vmatrix} i & j & k \\ 3/4 & 1 & -1/2 \\ 6 & -4 & -1 \end{vmatrix} = \left(-3, -\frac{9}{4}, -9\right)$$

$$\begin{cases} \vec{AB} = (21,-5,2)-(-4,0,-5) = (25,-5,7) \\ \vec{r} = \left(\frac{3}{4}, 1, -\frac{1}{2}\right) \\ \vec{s} = (6,-4,-1) \end{cases}$$

$$\vec{AB} \cdot (\vec{r} \times \vec{s}) = \begin{vmatrix} 25 & -5 & 7 \\ 3/4 & 1 & -1/2 \\ 6 & -4 & -1 \end{vmatrix} = \frac{-507}{4}$$

$$\|\vec{r} \times \vec{s}\| = \sqrt{(-3)^2 + \left(-\frac{9}{4}\right)^2 + (-9)^2} = \frac{39}{4}$$

$$d(r,s) = \frac{\frac{507}{4}}{\frac{39}{4}} = \frac{507}{39} = 13$$

3) Ache os pontos de $r: \begin{cases} x + y = 2 \\ x = y + z \end{cases}$ que distam 3 do ponto $A = (0,2,1)$.

$\begin{cases} x = 2 - y \\ x = y + z \end{cases}$ subtraindo a segunda pela primeira

$\to 0 = 2 - 2y - z \to z = 2 - 2y$

$r = (2-y, y, 2-2y)$

$$d(P,Q) = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

$$3 = \sqrt{((2-y)-0)^2 + (y-2)^2 + ((2-2y)-1)^2}$$

$$3 = \sqrt{6y^2 - 12y + 9}$$

$$(3)^2 = \left(\sqrt{6y^2 - 12y + 9}\right)^2$$

$$\cancel{9} = 6y^2 - 12y + \cancel{9}$$

$6y^2 - 12y = 0 \to 6y(y-2) = 0$

$y = 0$ e $y = 2$

Se $y = 0 \to R = (2,0,2)$

Se $y = 2 \to R = (0,2,-2)$