Nome: Eduardo Henrique de A. Izidorio
Matrícula: 2020000315
Disciplina: Geometria Analítica

Avaliação I

1) Faça o que se pede:

a) Ache m para que os vetores $\vec{u} = (m, 1, m+1)$, $\vec{v} = (0, 1, m)$ e $\vec{w} = (0, m, 2m)$ sejam L. D.

$$\begin{vmatrix} m & 1 & m+1 \\ 0 & 1 & m \\ 0 & m & 2m \end{vmatrix} \begin{matrix} m & 1 \\ 0 & 1 \\ 0 & m \end{matrix}$$

$0 \quad m^3 \quad 0$

$2m \quad 0 \quad 0$

$-m^3 + 2m = 0 \quad \cdot(-1)$

$m^3 - 2m = 0$

$m(m^2 - 2) = 0 \to m = 0$

$\to m = \pm\sqrt{2}$

m só pode ser zero.

$m^3 - 2m = 0$

$0^3 - 2 \cdot 0 = 0$

$0 - 0 = 0$

$0 = 0$

b) Se $E = (\vec{e_1}, \vec{e_2}, \vec{e_3})$ é base, prove que $F = (\alpha\vec{e_1}, \beta\vec{e_2}, \gamma\vec{e_3})$ é base desde que α, β, γ não sejam nulos.

$\alpha, \beta, \gamma > 0$

Se E é base, significa que $\vec{e_1}, \vec{e_2}$ e $\vec{e_3}$ são L.I

$\vec{e_1} = 1$

$\vec{e_2} = 1$

$\vec{e_3} = 1$

$\vec{e_1} \cdot \vec{e_2} \cdot \vec{e_3} = 1$

$\alpha \cdot \beta \cdot \gamma = 1$

$\alpha \cdot \vec{e_1} = 1$

$\beta \cdot \vec{e_2} = 1$

$\gamma \cdot \vec{e_3} = 1$

$\alpha\vec{e_1} \cdot \beta\vec{e_2} \cdot \gamma\vec{e_3} = 1$

$\vec{e_1} = |1|, \vec{e_2} = |1|, \vec{e_3} = |1|$

$\alpha\vec{e_1} = |1|, \beta\vec{e_2} = |1|, \gamma\vec{e_3} = |1|$

$$\det = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1$$

$\det = 1 > 0$

2) Sejam $\vec{u}=(2,0,-3)$ e $\vec{v}=(1,1,1)$, Calcule:

a) A medida do ângulo entre $\vec{u}$ e $\vec{v}$.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|} \rightarrow$$

$$\cos\theta = \frac{(2,0,-3)\cdot(1,1,1)}{\sqrt{2^2+0^2+(-3)^2}\cdot\sqrt{1^2+1^2+1^2}}$$

$$\cos\theta = \frac{2+0+(-3)}{\sqrt{13}\cdot\sqrt{3}} \rightarrow \cos\theta = \frac{-1}{\sqrt{39}}$$

$$\cos\theta = -\frac{1}{\sqrt{39}} \,_{\shortparallel}$$

b) $Proj_{\vec{v}}\vec{u}$

$$Proj_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\cdot\vec{v}$$

$$= \left(\frac{(2,0,-3)\cdot(1,1,1)}{(1,1,1)\cdot(1,1,1)}\right)\cdot(1,1,1)$$

$$= \frac{-1}{3}\cdot(1,1,1)$$

$$Proj_{\vec{v}}\vec{u} = \left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right)_{\shortparallel}$$

c) $\vec{u}\cdot\vec{v}$ (produto escalar)

$$\vec{u}\cdot\vec{v} = (2,0,-3)\cdot(1,1,1)$$

$$= 2+0+(-3)$$

$$\vec{u}\cdot\vec{v} = -1_{\shortparallel}$$

d) $\vec{u}\times\vec{v}$ (produto vetorial)

0   $-3i$ $2j$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 2 & 0 & -3 \\ 1 & 1 & 1 \end{vmatrix}\begin{matrix} i & j \\ 2 & 0 \\ 1 & 1 \end{matrix}$$

0   $-3j$ $2K$

$$+3i - 2j - 3j + 2K$$

$$3i - 5j + 2K$$

$$\vec{u}\times\vec{v} = (3,-5,2) \;\#$$

3) A medida em radianos do ângulo entre $\vec{u}$ e $\vec{v}$ é $\frac{\pi}{6}$, e $\vec{w}$ é ortogonal a $\vec{u}$ e a $\vec{v}$. $\|\vec{u}\|=1$, $\|\vec{v}\|=1$ e $\|\vec{w}\|=4$, e $(\vec{u},\vec{v},\vec{w})$ base positiva, ache $[\vec{u},\vec{v},\vec{w}]$.

$$\|\vec{u}\times\vec{v}\| = \|\vec{u}\|\cdot\|\vec{v}\|\cdot \operatorname{sen}(\vec{u},\vec{v})$$

$$\|\vec{u}\times\vec{v}\| = \operatorname{sen}\left(\frac{\pi}{6}\right)$$

$$\|\vec{u}\times\vec{v}\| = \frac{1}{2}$$

$\vec{w}$ ortogonal a $\vec{u}$ e a $\vec{v}$. Logo, será $/\!/$ a $\vec{u}\times\vec{v}$

$$(\vec{u}\times\vec{v})\cdot\vec{w} = \|\vec{u}\times\vec{v}\|\cdot\|\vec{w}\|\cos(\vec{u}\times\vec{v},\vec{w})$$

$$[\vec{u},\vec{v},\vec{w}] = \left(\frac{1}{2}\right)\cdot 4\cos(0)$$

$$[\vec{u},\vec{v},\vec{w}] = 2_{\shortparallel}$$

4) Sejam os pontos $A=(0,0,0)$, $B=(1,1,0)$, $C=(0,1,1)$ e $D=(-4,0,0)$,
Calcule:

a) O Volume do Tetraedro ABCD;

$\vec{u} = \vec{AB} = (0,0,0)-(1,1,0) = (-1,-1,0)$

$\vec{v} = \vec{AC} = (0,0,0)-(0,1,1) = (0,-1,-1)$

$\vec{w} = \vec{AD} = (0,0,0)-(-4,0,0) = (4,0,0)$

Então

$$[\vec{u},\vec{v},\vec{w}] = \left|\begin{array}{ccc|cc} -1 & -1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 0 & -1 \\ 4 & 0 & 0 & 4 & 0 \end{array}\right| \rightarrow V_p = 4 \,//$$

$V_T = \frac{1}{6} \cdot V_p$

$V_T = \frac{4}{6}$

$V_T = \frac{2}{3} \,//$

b) A área do Triângulo ABC.

$\vec{AB} = (-1,-1,0)$, $\vec{AC} = (0,-1,-1)$

$$\vec{AB} \cdot \vec{AC} = \left[\begin{array}{ccc|cc} i & j & k & i & j \\ -1 & -1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 0 & -1 \end{array}\right] = 1i + 1K - 1j \rightarrow (1,1,-1)$$

$$\|\vec{AB} \cdot \vec{AC}\| = \sqrt{1^2+1^2+(-1)^2}$$
$$= \sqrt{1+1+1}$$
$$= \sqrt{3} \,//$$

$$A = \frac{1}{2} \cdot \|\vec{AB} \cdot \vec{AC}\| \rightarrow \boxed{A = \frac{\sqrt{3}}{2}} \,//$$