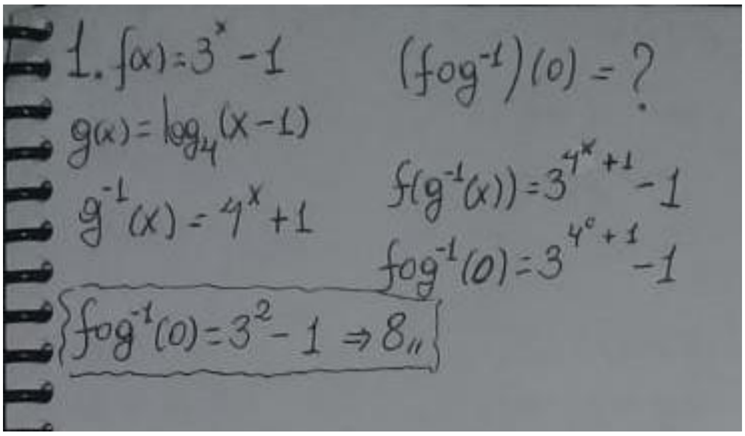
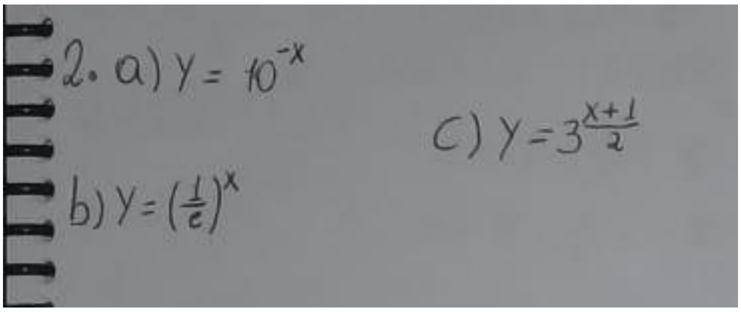
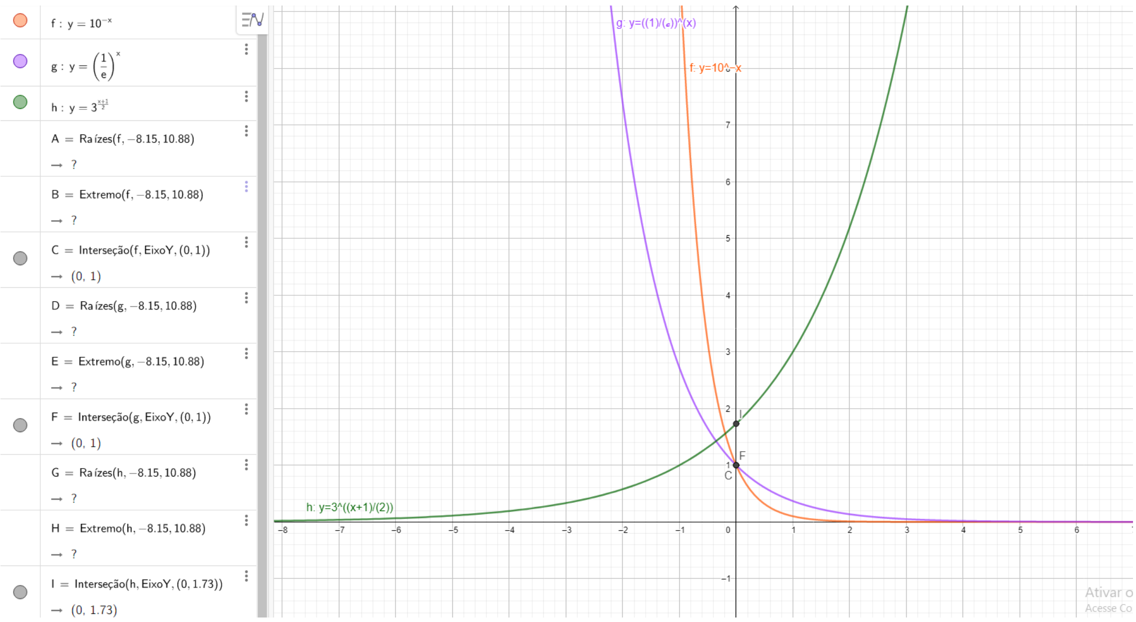
Nome: Eduardo Henrique de Almeida Izidorio
Matrícula: 2020000315
Disciplina: Matemática Básica
Semestre: 2020.2

LISTA DE EXERCÍCIOS 2







3. a) $4^{x^2+4x} = 4^{12}$

$x^2+4x = 12 \rightarrow x^2+4x-12=0$

$(x+6)(x-2)=0$

$x+6=0 \qquad x-2=0$

$x_1=-6 \qquad x_2=2$

$S=\{-6,2\}$

b) $\sqrt[x-1]{\sqrt[3]{2^{3x+1}}} - \sqrt[3x-7]{8^{x-3}} = 0$

$\sqrt[3x-3]{2^{3x+1}} = \sqrt[3x-7]{8^{x-3}}$

$\sqrt[3x-7]{(2^3)^{x-3}} = \sqrt[3x-7]{2^{3x-9}}$

$2^{\frac{3x+1}{3x-3}} = 2^{\frac{3x-9}{3x-7}} \Rightarrow \frac{3x+1}{3x-3} = \frac{3x-9}{3x-7}$

$(3x+1).(3x-7) = (3x-9).(3x-3)$

$\cancel{9x^2}-21x+3x-7 = \cancel{9x^2}-9x-27x+27$

$-21x+3x-7 = -9x-27x+27$

$-18x-7 = -36x+27$

$-18x+36x = 27+7$

$18x = 34$

$x = \frac{34 \div 2}{18 \div 2} = \boxed{\frac{17}{9}}$

c) $\frac{2^{5x+2}}{8^{2x+7}} = 4^{x-1}$

$\frac{2^{3x+2}}{2^{6x-21}} = 2^{2x-2}$

$2^{-3x+23} = 2^{2x-2}$

$-3x+23 = 2x-2$

$23+2 = 2x+3x$

$25 = 5x$

$x = \frac{25}{5} = 5$,,

4.a) $4^{x^2}+2\cdot 14^x = 3\cdot 49^x$

$4^x+2\cdot 14^x-3\cdot 49^x=0$

$2^{2x}+2\cdot(2\cdot 7)^x-3\cdot 7^{2x}=0$

$2^{2x}+2\cdot 2^x\cdot 7^x-3\cdot 7^{2x}=0$

$2^x\div(7^{2x})+2\cdot 2^x\cdot 7^x\div(7^{2x})-3\cdot 7^{2x}\div(7^{2x})=0\div(7^{2x})$

$\dfrac{2^{2x}}{7^{2x}}+2\cdot 2^x\cdot 7^{x-2x}-3=0$

$\left(\frac{2}{7}\right)^{2x}+2\cdot 2^x\cdot 7^{-x}-3=0$

$\left(\frac{2}{7}\right)^{2x}+2\cdot 2^x\cdot\frac{1}{7^x}-3=0$

$\left(\frac{2}{7}\right)^{2x}+2\cdot\left(\frac{2}{7}\right)^x-3=0$ $\qquad t=\left(\frac{2}{7}\right)^x$

$\left(\left(\frac{2}{7}\right)^x\right)^2+2\cdot\left(\frac{2}{7}\right)^x-3=0$

$t^2+2t-3=0$

$t^2+3t-t=0$

$t\cdot(t+3)-(t+3)=0$

$t+3=0 \rightarrow t=-3$

$t-1=0 \rightarrow t=1$

$\left(\frac{2}{7}\right)^x=-3 \rightarrow x\in\mathbb{R}$

$\left(\frac{2}{7}\right)^x=1 \rightarrow \left(\frac{2}{7}\right)^x=\left(\frac{2}{7}\right)^0 \quad x=0$

b) $2^{2x+2} - 6^x - 2.3^{2x+2} = 0$

$2^{2x}.2^2 - (2.3)^x - 2.3^{2x}.3^2 = 0$

$2^2.2^{2x} - 2^x.3^x - 2.3^2.3^{2x} = 0$

$4.2^{2x} - 2^x.3^x - 2.3^2.3^{2x} = 0$

$4.\left(\frac{2}{3}\right)^{2x} - \left(\frac{2}{3}\right)^x \cdot 18 = 0$

$t = \left(\frac{2}{3}\right)^x \to 4t^2 - t - 18 = 0$

$4t^2 - 8t - 9t - 18 = 0$

$4t.(t+2) - 9(t+2) = 0$

$(t+2)(4t-9) = 0$

$t+2=0 \quad 4t-9=0$

$t=-2 \quad t=\frac{9}{4}$

$\left(\frac{2}{3}\right)^x = -2 \; x \notin \mathbb{R}$

$\left(\frac{2}{3}\right)^x = \frac{9}{4} \to \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-2} \; x = -2$

5.a) $8 < 2^x < 32$

$2^3 < 2^x < 2^5$

$3 < x < 5$

$S = \{x \in \mathbb{R} / 3 < x < 5\}$

b) $0{,}0001 < 0{,}1^x < 0{,}01$

$10^{-4} < (10^x)^{-1} < 10^{-2}$

$\left(\frac{1}{10}\right)^4 < \left(\frac{1}{10^x}\right)^1 < \left(\frac{1}{10}\right)^2$

$\frac{1}{10^4} < \frac{1}{10^x} < \frac{1}{10^2}$

$4 < x < 2$

$S = \{x \in \mathbb{R} / 2 < x < 4\}$

c) $4 < 8^{|x|} < 32$

$8^{|x|} > 4$

$8^{|x|} < 32$

$2^{3x|x|} > 2^2$

$3x|x| > 2$

$3x > 2, x \geq 0$

$3x(-x) > 2, x < 0$

$x > \frac{2}{3}, x \geq 0$

$x < -\frac{2}{3}, x < 0$

$x \in \{\frac{2}{3}, +\infty\} \cup \{-\infty, -\frac{2}{3}\}$

$8^{|x|} < 32$

$2^{3x|x|} < 2^5$

$3x|x| < 5$

$3x < 5, x \geq 0$

$x\,x(-x) < 5, x < 0$

$x < \frac{5}{3}, x \geq 0$

$x > -\frac{5}{3}, x < 0$

$x \in \{-\frac{5}{3}, \frac{5}{3}\}$

$x \in \{\frac{5}{3}, -\frac{2}{3}\} \cup \{\frac{2}{3}, \frac{5}{3}\}$

6. a) $\log_2\left(\frac{a^2 \cdot \sqrt{b}}{\sqrt[3]{c}}\right)$

$\log_2(a^2\sqrt{b}) - \log_2(\sqrt[3]{c})$

$\log_2(a^2) + \log_2(\sqrt{b}) - \log_2(c^{\frac{1}{3}})$

$2\log_2(a) + \log_2(b^{\frac{1}{2}}) - \frac{1}{3} \cdot \log_2(c)$

$2\log_2(a) + \frac{1}{2} \cdot \log_2(b) - \frac{1}{3} \cdot \log_2(c)$

7. $\log_2(a-b)=m \to 2^8 = a+b$

$\log_2(a+b)=8 \to 2^m = a-b$

$\begin{cases} a+b=2^8 \\ a-b=2^m \end{cases}$

$2a = 2^8+2^m$

$a = \dfrac{2^8+2^m}{2}$

$a = \dfrac{2(2^7+2^{m-1})}{2}$

$a = 2^7+2^{m-1}$

$a+b=2^8$

$2^7+2^{m-1}+b=2^8$

$b=2^8-(2^7+2^{m-1})$

$b=2^8-2^7+2^{m-1}$

$b=128+2^{m-1}$

$b=2^7+2^{m-1}$

$\log_2(a^2-b^2)$

$\log_2((2^7+2^{m-1})^2-(2^7+2^{m-1})^2)$

Como $a=b \to \log_2 0 = 1$

8. $\left(\frac{a}{b}\right)^{\log c} \cdot \left(\frac{b}{c}\right)^{\log a} \cdot \left(\frac{c}{a}\right)^{\log b} = 1$

$\log\left(\left(\frac{a}{b}\right)^{\log c} \cdot \left(\frac{b}{c}\right)^{\log a} \cdot \left(\frac{c}{a}\right)^{\log b}\right) = \log 1$

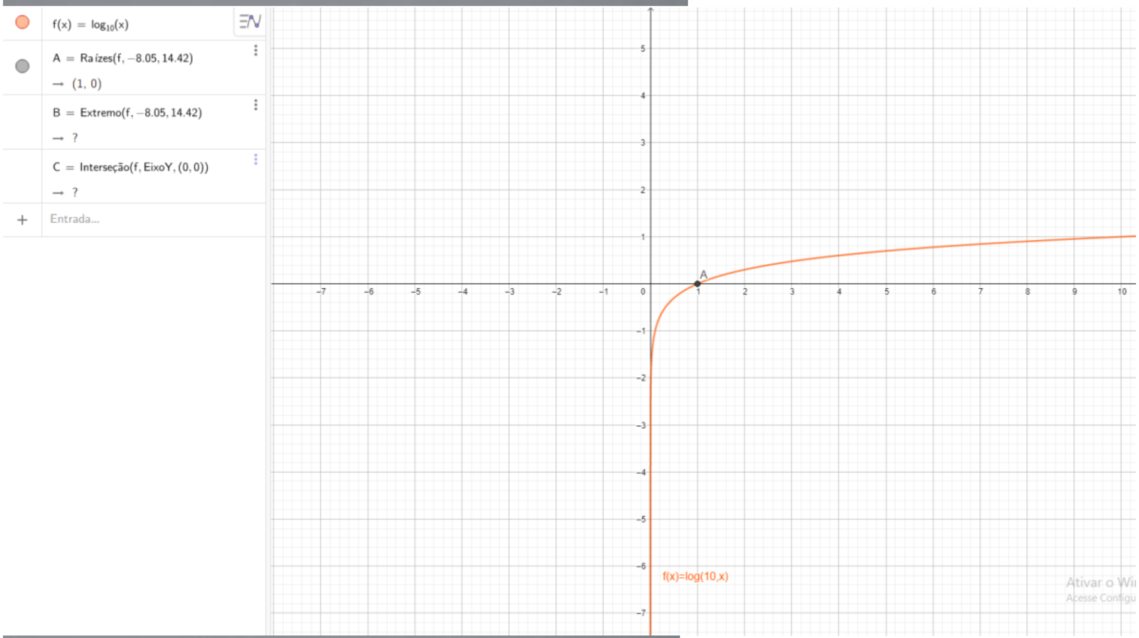
$\log c(\log a - \log b) + \log a(\log b - \log c) + \log b(\log c - \log a) = 0$

$\log a \cdot \log c - \log b \cdot \log c + \log a \cdot \log b - \log a \cdot \log c + \log b \cdot \log c - \log a \cdot \log b = 0$

$0=0$

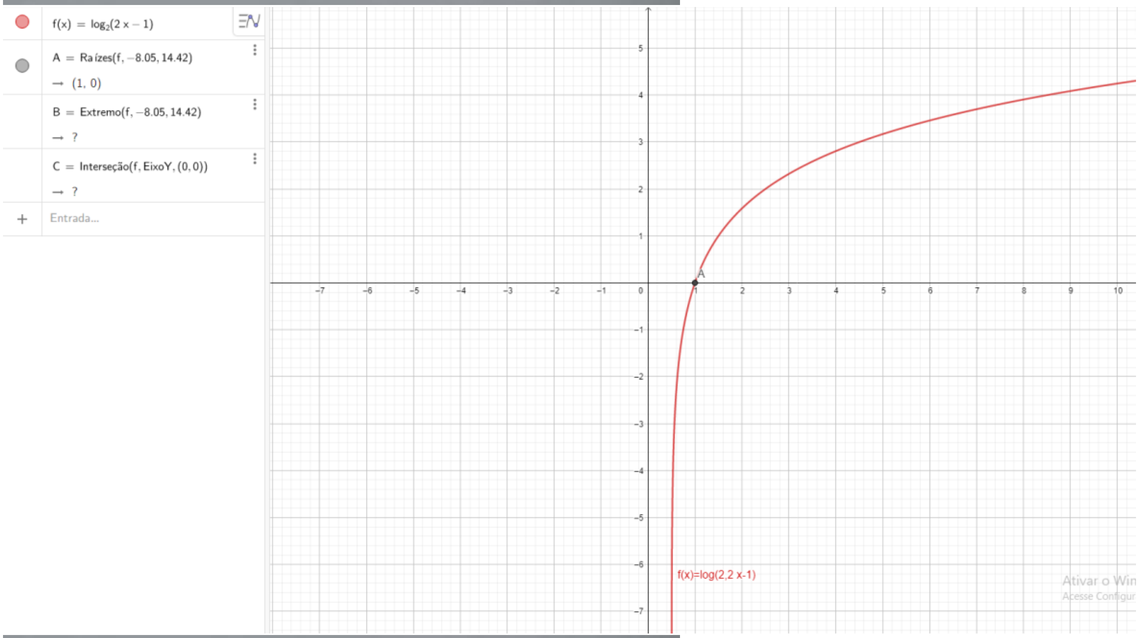
9. $Y = e^x \to Y^{-1} = \log_e(x)$

10. a) $f(x) = \log x$

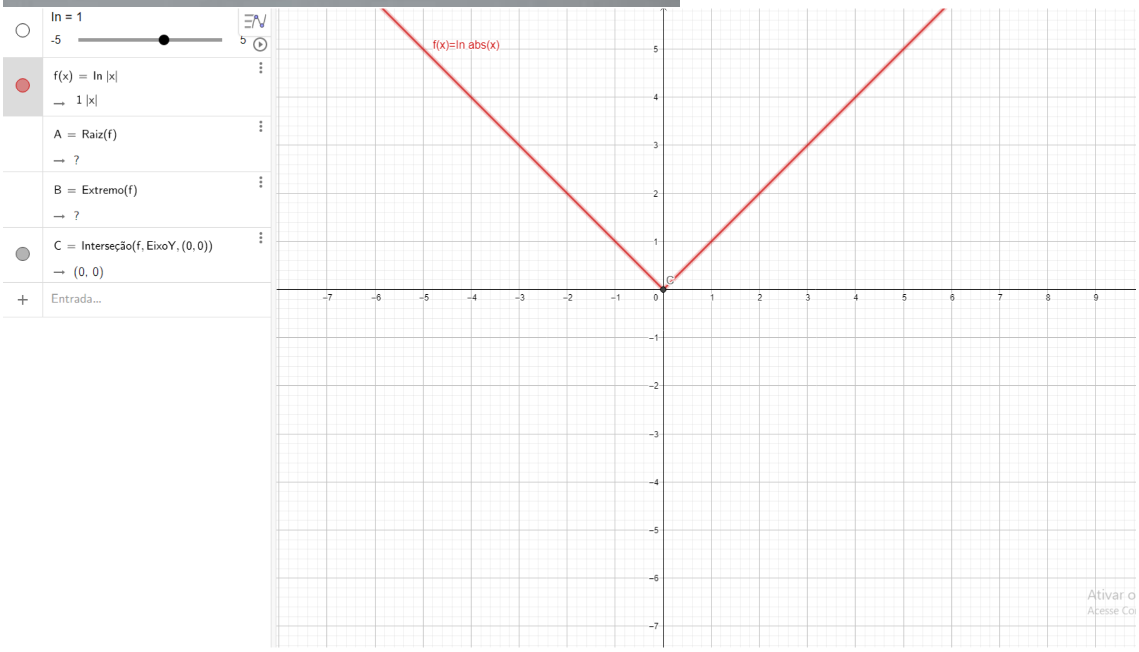


b) $f(x) = \log_{\frac{1}{3}} x$

c) $f(x) = \log_2(2X-1)$



d) $f(x) = \text{In}\,|x|$



11. a) $7^{\sqrt{x}} = 2$

$\log_7(7^{\sqrt{x}}) = \log_7(2)$

$\sqrt{x} = \log_7(2)$

$x = \log_7(2)^2$

$7^{\sqrt{\log_7(2)^2}} = 2$

$7^{\log_7(2)} = 2$

$2 = 2$ $\quad x = \log_7(2)^2$

b) $3^{2x+1} = 2$

$2x + 1 = \log_3(2)$

$2x = \log_3(2) - 1$

$2x \div 2 = (\log_3(2) - 1) \div 2$

$x = \log_3(2) \div 2 - 1 \div 2$

$x = \frac{1}{2} \cdot \log_3(2) - \frac{1}{2}$

c) $3^{x^2} = 5$

$x^2 = \log_3(5)$

$x = \pm\sqrt{\log_3(5)}$

$x_1 = -\sqrt{\log_3(5)}$

$x_2 = \sqrt{\log_3(5)}$

12. $T = 0$

$M(t) = Ce^{-kt}$

$M(0) = C \cdot e^{-k \cdot 0}$

$M(0) = C$

$\frac{C}{2} = C \cdot e^{-1600k}$

$\frac{1}{2} = e^{-1600k}$

$\frac{1}{2} = (e^{-100k})^{16}$

$e^{-100k} = \sqrt[16]{\frac{1}{2}}$

$e^{-100k} = (2^{-1})^{\frac{1}{16}}$ (1)

$M(100) = C \cdot e^{-100k}$ (2)

Substituindo (1) em (2) temos:

$M(100) = C \cdot 2^{-\frac{1}{16}}$

$M(0) - M(100) = C - C \cdot 2^{-\frac{1}{16}}$

$M(0) - M(100) = C \cdot (1 - 2^{-\frac{1}{16}})$

$(1 - 2^{-\frac{1}{16}})$ da quantidade Inicial.