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Item 6 – Forecasting demographic components: fertility

**A stochastic version of the United Nations World Population Prospects:
methodological improvements by using Bayesian fertility
and mortality projections¹**

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1. INTRODUCTION

The United Nations Population Division has prepared population estimates and projections for all countries of the world since the early 1950s. These World Population Prospects (WPP) are now used throughout the whole UN system and by many international organizations, as well as academic researchers. They have become a standard input for development planning, monitoring and global modeling.

Traditionally, the World Population Prospects have included high and low fertility variants, in which the total fertility was assumed to remain 0.5 children above or below the medium variant over most of the projection period. These assumptions are insensitive to the initial level of fertility and to the rate of fertility decline in various countries and may therefore oversimplify or even distort the future range of fertility variation.

Recent research in collaboration with the University of Washington has now enabled the Population Division to calculate probabilistic population projections for all countries of the world. Initial results indicate that the WPP projections might have considerably underestimated the uncertainty in population growth of developing countries- particularly in the Least Developed Countries. The 95% projection intervals of total population growth, as given by the probabilistic projections, are significantly wider in these countries than the high-low variant ranges in the current WPP projections.

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2. PREVIOUS STUDIES

Traditionally, population projections have used variants or scenarios in the projection of fertility, mortality and migration to indicate uncertainty or illustrate policy options. The variants are usually based on plausible alternatives, such as the “half-child” rule for high and low fertility projections in the United Nations *World Population Prospects*. This pragmatic approach has the advantage that results can be easily communicated to policy makers and planners. However, it has the disadvantage that variants do not indicate anything about their probability¹. They are just plausible alternatives. Demographers have therefore developed probabilistic population projections where the range of uncertainty is represented by projection intervals. Probabilistic projections come in various flavors²: Some authors have used historical forecast errors^{3,4}; others have based their projections on expert opinion^{5,6} or used time series analysis to project future population parameters⁷⁻⁹. Existing probabilistic population projections so far have been limited in their geographical scope either to macro-regions at the global scale¹⁰⁻¹², a specific region such as Europe¹³⁻¹⁵, or few countries for in-depth case studies¹⁶⁻²¹. So far, no attempt has been made to produce national-level probabilistic population projections for all countries of the world²².

3. OUR APPROACH TO PROBABILISTIC PROJECTIONS

We have used a Bayesian statistical approach to project total fertility and life expectancy at birth between 2005 and 2050, based on estimates between 1950 and 2005²³⁻²⁵. The fertility projection model consists of two components. The first component models the fertility decline from high or medium levels of fertility to a turnaround point close to replacement level fertility. The second component models changes in total fertility after the turnaround point. These changes are random fluctuations around and/or recovery towards replacement level fertility, which are modeled using a first order autoregressive time series model, an AR(1) model, with its mean fixed at approximate replacement-level fertility (2.1). In the projections of life expectancy at birth, expected gains in life expectancy are modeled as a function of its level, with random distortions added to it.

The projection methodology builds on the current methodology used in the World Population Prospects. Expected 5-year decrements in total fertility are modeled as a function of its level using a bi-logistic function²⁶. Similarly, expected 5-year gains in life expectancy at birth are modeled as a function of its level, using a bi-logistic function²⁶. In the Bayesian projection model, instead of using a given set of parameters to find the fertility decrements and gains in life expectancy in a country, a Bayesian hierarchical model^{27,28} is used to estimate these parameters for each country. Each of the unknown parameters of the bi-logistic functions is drawn from a probability distribution that represents the range of outcomes of that parameter across all countries. For a specific country, the posterior distribution of its parameters is determined by the world-level experience for all countries combined, as well as the observed declines in that country..

The Bayesian projection models yield a large number of trajectories of future total fertility and life expectancy, for all countries without generalized HIV/AIDS epidemic (i.e., less than 1% HIV prevalence in the general adult population age 15-49) and with a total population larger than 100,000 in 2009. We have used a male-dominant projection of life expectancy, and derived female life expectancies by assuming the same sex differentials as projected deterministically by the United Nations *World Population Prospects*. From the large number of trajectories (35,000), 1,000 trajectories were sampled to be used in standard cohort-component population projections^{29,30}. To convert the projections of total fertility into the age-specific fertility rates as required by the cohort component projection, percentages of age-specific fertility rates were taken from the 2008 Revision of the *World Population Prospects* and applied throughout the projection period. Projections of life expectancy at birth were converted into age- and sex-specific mortality rates and, subsequently, into survivor ratios which were then used in the cohort-component projections. For the historical periods, we used a standard Lee-Carter model⁷. For the future, we applied a modified Lee-Carter method, in which the time indicator of a Lee-Carter model was not projected by a random walk (as is the standard method), but was fitted to the trajectories of life expectancy at birth, as projected by the Bayesian projection model.

All other input data, including base population by age and sex as well as projections of net-migration, were identical to the 2008 Revision of the *World Population Prospects*³¹. Using the 1,000 sets of age-specific fertility rates and sex-specific survivor ratios, 1000 cohort component projections were calculated for the period 2010 to 2050 for all 196 countries with a population of more than 100,000 in 2009. From these trajectories the median of total population and a 95% projection interval was calculated for each country. These first results of our probabilistic population projections were compared to the high, low and medium variant of the 2008 Revision of the *World Population Prospects*³¹ (see figures with results in Appendix).

4. RESULTS

We have calculated probabilistic population projections for 196 countries of the world with a population of more than 100,000 (Atotal population projections, including 95% projection intervals and median) and compared these projections with the deterministic projections as provided by the *2008 Revision of the World Population Prospects*³¹. This work has yielded four important results:

- (1) While probabilistic projections are available for particular countries or regions this is the first time that probabilistic projections were calculated in a systematic and consistent way for almost all countries of the world – including 151 developing countries. We now can calculate projection intervals that are based on probability, rather than projection intervals that are based on high-low scenarios, as in the deterministic *World Population Prospects*. This is of particular relevance for developing countries, where large differences in the availability and quality of the empirical input data for the initial estimates are common. With the probabilistic projections these varying uncertainties in estimates are used to determine the probability of the projection intervals. The global coverage also allows us to analyze differences between probabilistic and deterministic projections for countries having very different initial levels of fertility and mortality.
- (2) We found that, in more than 80% of the countries, probabilistic medians in projected total population differ less than 10% from the medium variant of the *World Population Prospects*. Figure 1 illustrates the high correlation between the probabilistic median and the (deterministic) medium variant population projection for the year 2050. However, we also found that in less than 20% of the countries of the world probabilistic projections resulted in substantially higher or lower median projections than were previously projected in the (deterministic) *World Population Prospects*. This concerns primarily countries where the level of fertility is still rather high or where fertility declines have been rapid and recent.
- (3) We could confirm that in about 60% of the countries, the high-low scenarios of the (deterministic) *World Population Prospects* lie within the 95% confidence interval of the probabilistic population projections. This indicates, for the first time, how the United Nations projection scenarios are related to the uncertainty determined by a probabilistic projection.
- (4) We also found that the range of uncertainty as measured by a 95% projection interval in our probabilistic population projections for 2050 was much larger for some countries than the high-low variant range in the deterministic projections. Figure 2 plots the ratio of the 95% projection interval to the deterministic high-low range in population projections for 2050 against the level of total fertility in the five-year interval of 2000-2005. The figure reveals that in countries with high fertility in 2000-2005 the probabilistic projection resulted in a much wider 95% projection interval than the high-low variant range (i.e., +/- 0.5 child) in the *World Population Prospects*. As indicated by Map 1, these countries are mainly located in Africa and often belong to the group of Least Developed Countries (see appendix 1). These results seem to indicate that the range of future outcomes as given by the high-low variants in the deterministic *World Population Prospects* is narrower than the likely range of possible future scenarios in population projections for countries, which are still at the early stages of their fertility decline.

5. DISCUSSION

Uncertainty in population projections results from three major sources³²: the accuracy of the input data, the methods and assumptions for projecting future trends in fertility, mortality and migration, and the intrinsic unpredictability of all socio-economic events, which include demographic events such as migration.

The first source of uncertainty is identical between our probabilistic and deterministic projections. Both use historical *estimates* of population by age and sex, total fertility, life expectancy at birth and net-migration that had been prepared for the *2008 Revision of the World Population Prospects*. These estimates are based on a large number of empirical data sources, including censuses, population and vital registers as well as surveys, such as the Demographic and Health Surveys (DHS) the Multi-Cluster Indicator Surveys (MICS) and others. Typically, these empirical data sources do not provide unambiguous historical time series of demographic indicators, but widely ranging “data clouds” – particularly in developing countries. If the Bayesian projection model would be based on these raw, unadjusted data sets, the projection intervals would certainly be much wider. However, not all of the variation in these empirical data can be used in assessing uncertainty. Some of the empirical information is obviously wrong or deficient and can be corrected or adjusted by established demographic methods – such as the age-heaping in population counts or the underreporting of birth and children under the age of five. The Population Division is using various methods to check consistency between data sources, calculate intercensal growth balances, compare countries within a region and apply adjustments, if necessary³³. This estimation procedure results in relatively smooth historical time series of total fertility and life expectancy at birth in five-

year intervals, which certainly hide some of the actual annual fluctuations and uncertainties in demographic indicators, but also corrects unrealistic variation due to errors and data deficiencies. The projections based on the UN estimates can be viewed validly as probabilistic projections of future UN estimates.

In the Bayesian projection model, the projections of future trends in fertility and mortality are based on the historical trends of fertility and mortality declines in all countries in the world. The projected variance thus results from the variance in the pace of observed declines of fertility and mortality in all countries. This method of taking into account uncertainty seems more appropriate than assuming a single high or low variant of fertility decline for all countries, as in the (deterministic) *World Population Prospects*. Therefore, we likely improve the second source of uncertainty.

The third source of uncertainty is the fact that human behavior is intentional and ultimately unpredictable. While the past decline of fertility in most countries strongly indicates a general trend towards low fertility, it is possible that some populations might not follow these trends. Demographers have discussed the phenomenon of “stalled fertility decline”^{34,35}. The universal trends towards higher life expectancy might also slow down or even reverse due to massive increases in unhealthy and risky life styles (obesity, alcohol misuse), emergence of devastating pandemics, or widespread wars. Migration, in particular, will always include a component of inherent unpredictability, because it can be induced or stopped by actions of governments and by natural conditions. No one can exclude the possibility of massive displacement of people due to conflict, economic collapse, or natural disaster³⁶. We have partially accounted for this source of uncertainty by adding random distortion terms to future trajectories. The variance of these terms is estimated based on observed past distortions in all countries.

6. CONCLUSION

While population projections will always include several elements of uncertainty, our probabilistic projections allow us to quantify those aspects of uncertainty which can be derived from countries' past experience. The Bayesian Hierarchical Model is data-driven, since it uses the UN estimates which are based on empirical data. It does not depend on expert opinion to determine uncertainties and is fully reproducible.

Our probabilistic projections show that the uncertainty in future total population trends in countries with high fertility is probably larger than indicated by the range between high and low variants in the United Nations *World Population Prospects*.

Figure 1: Total Population in 2050: Probabilistic Median versus Medium Variant WPP

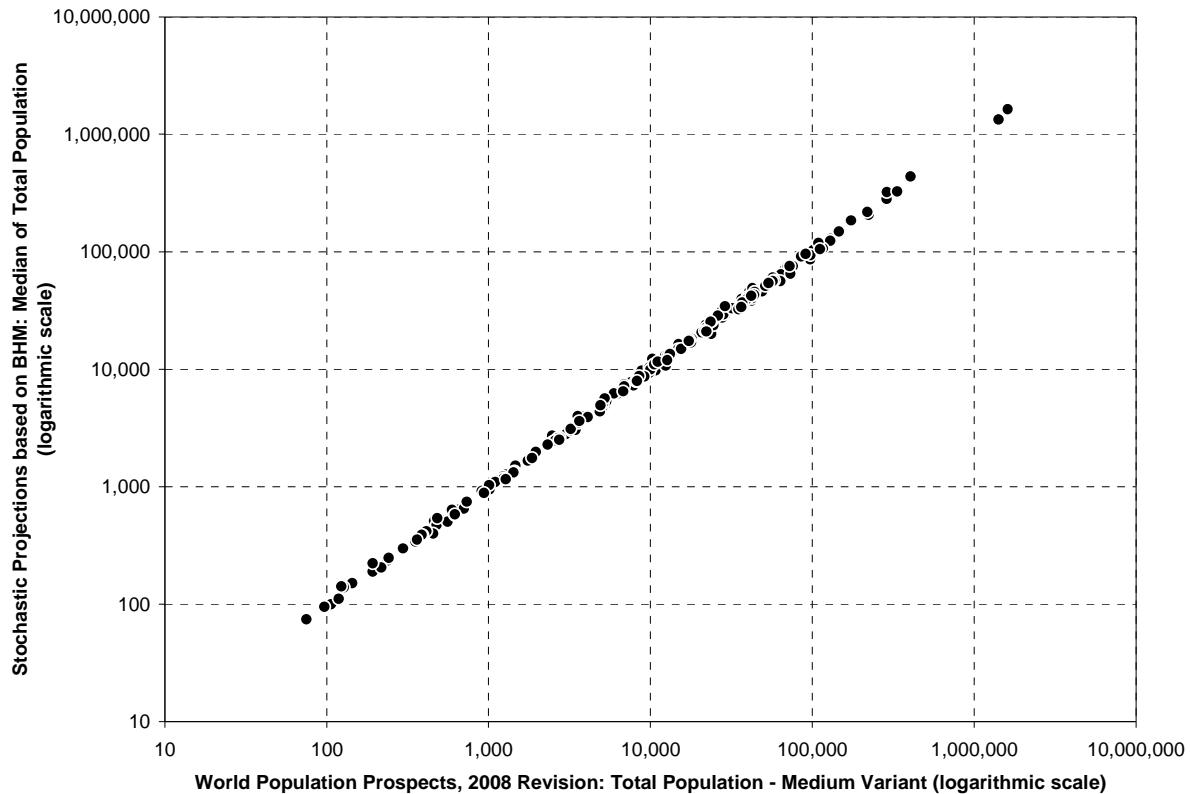
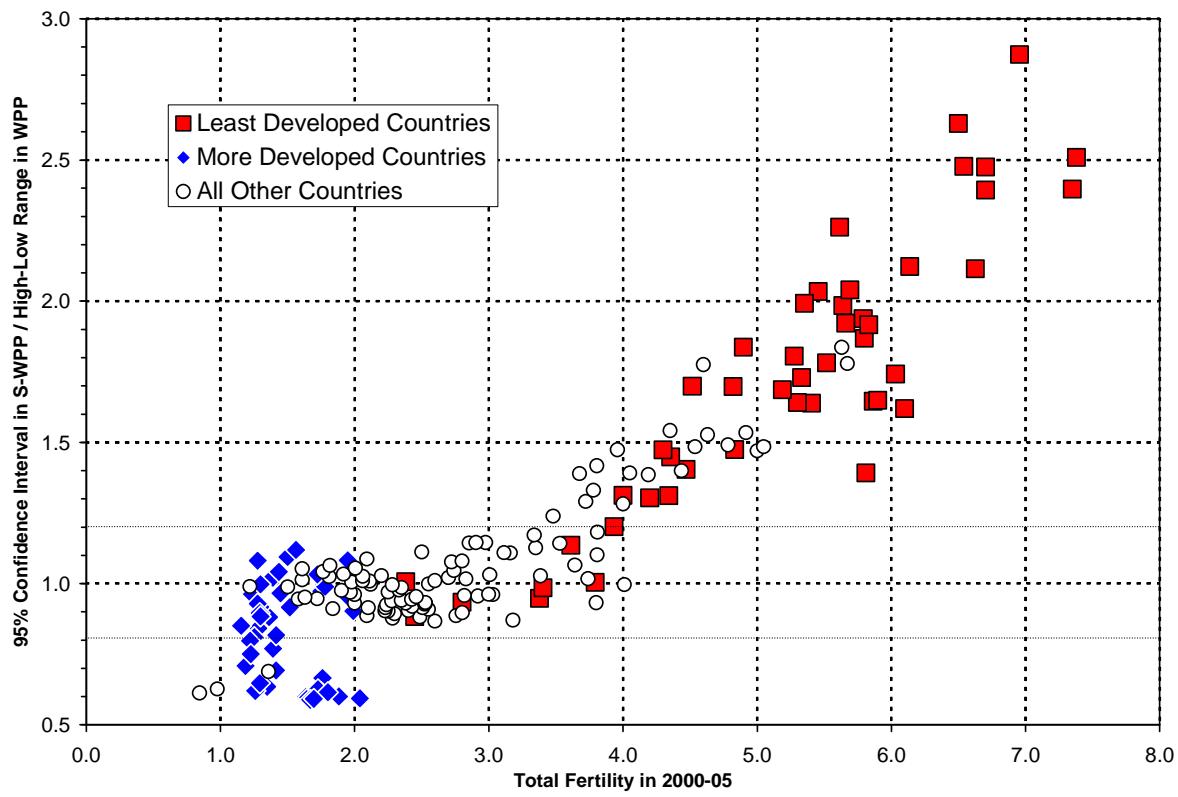
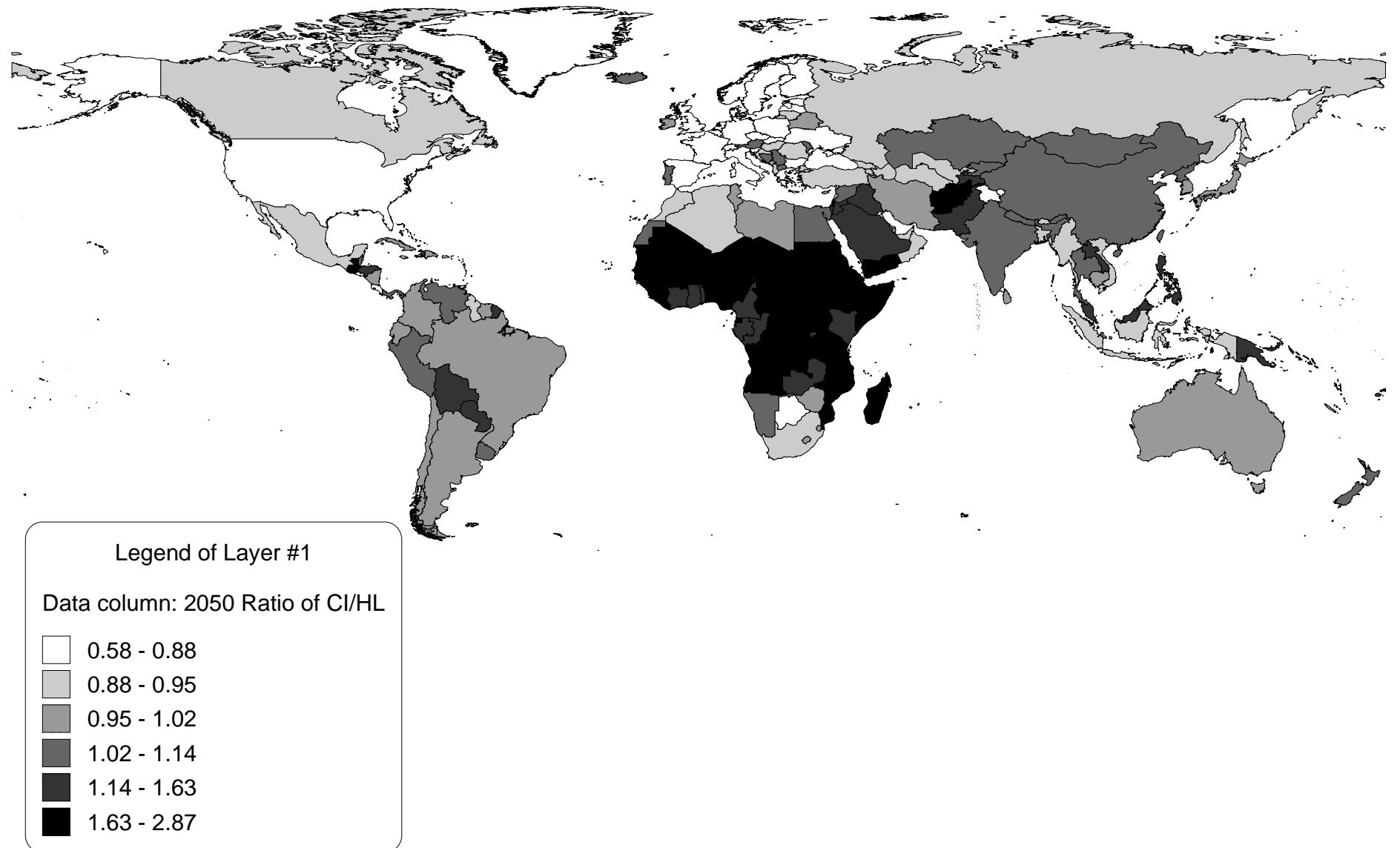


Figure 2: Ratio of probabilistic projection interval and high-low range in WPP versus total fertility in 2005-10



Map 1: Ratio of probabilistic projection interval and high-low range in WPP versus total fertility in 2005-10



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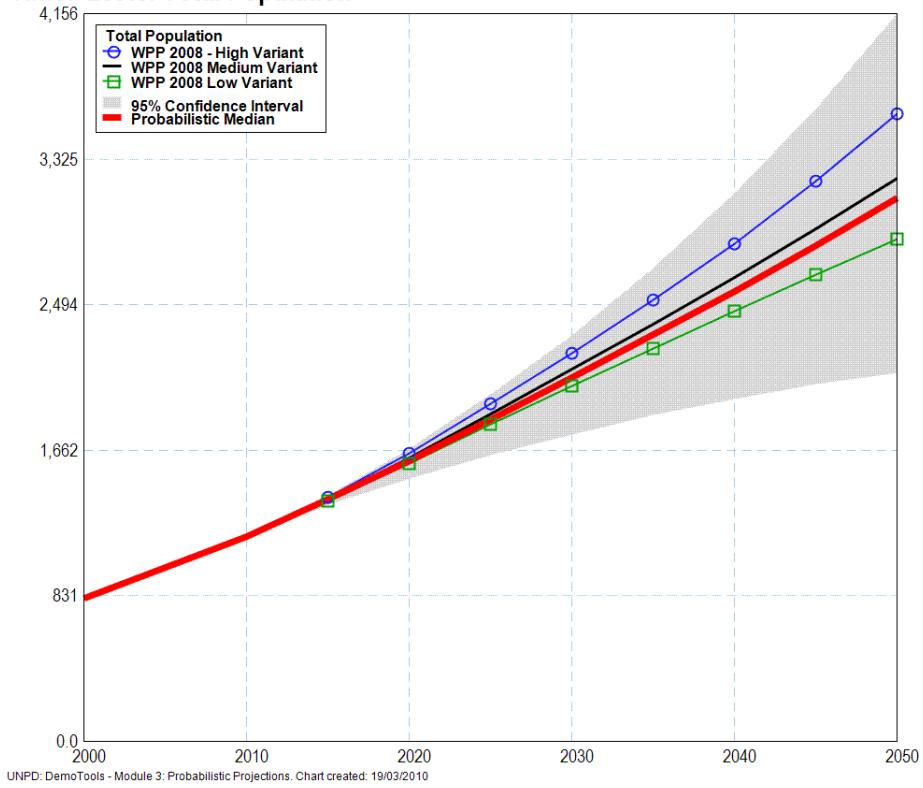
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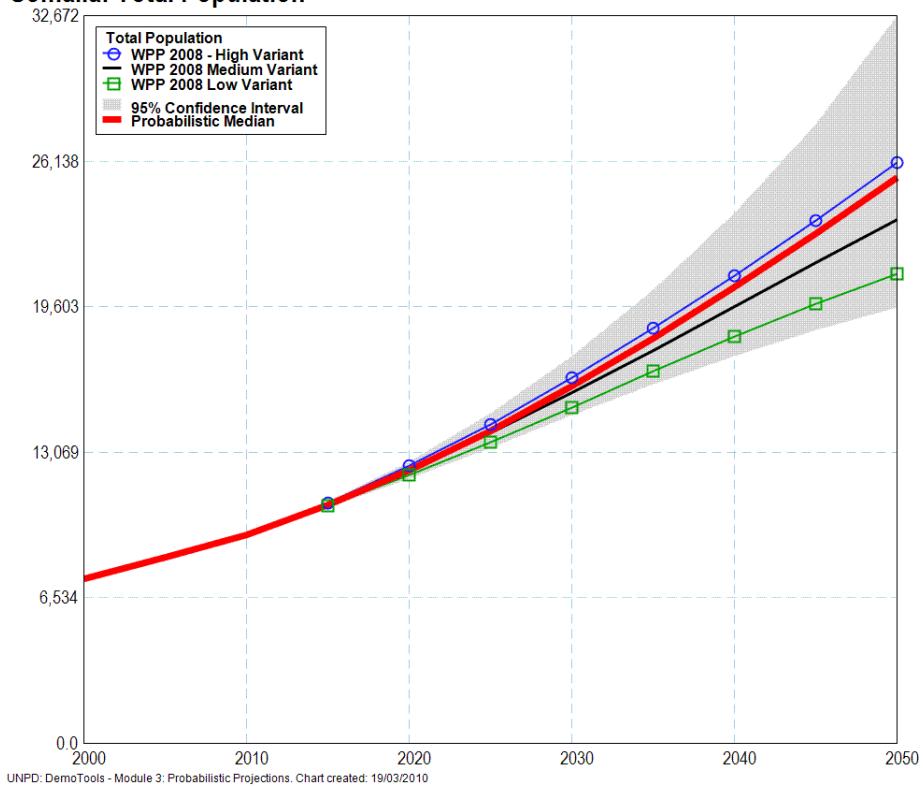
Appendix 1

Total population of selected Least Developed Countries, 2000-2050:
Probabilistic projections and World Population Prospects (2008 Revision)
(Projection Base Year: 2005)

Timor-Leste: Total Population



Somalia: Total Population



Niger: Total Population

69,372

Total Population
WPP 2008 - High Variant
WPP 2008 Medium Variant
WPP 2008 Low Variant
95% Confidence Interval
Probabilistic Median

55,498

41,623

27,749

13,874

0.0

2000 2010 2020 2030 2040 2050

UNPD: DemoTools - Module 3: Probabilistic Projections. Chart created: 19/03/2010

Chad: Total Population

35,689

Total Population
WPP 2008 - High Variant
WPP 2008 Medium Variant
WPP 2008 Low Variant
95% Confidence Interval
Probabilistic Median

28,551

21,413

14,276

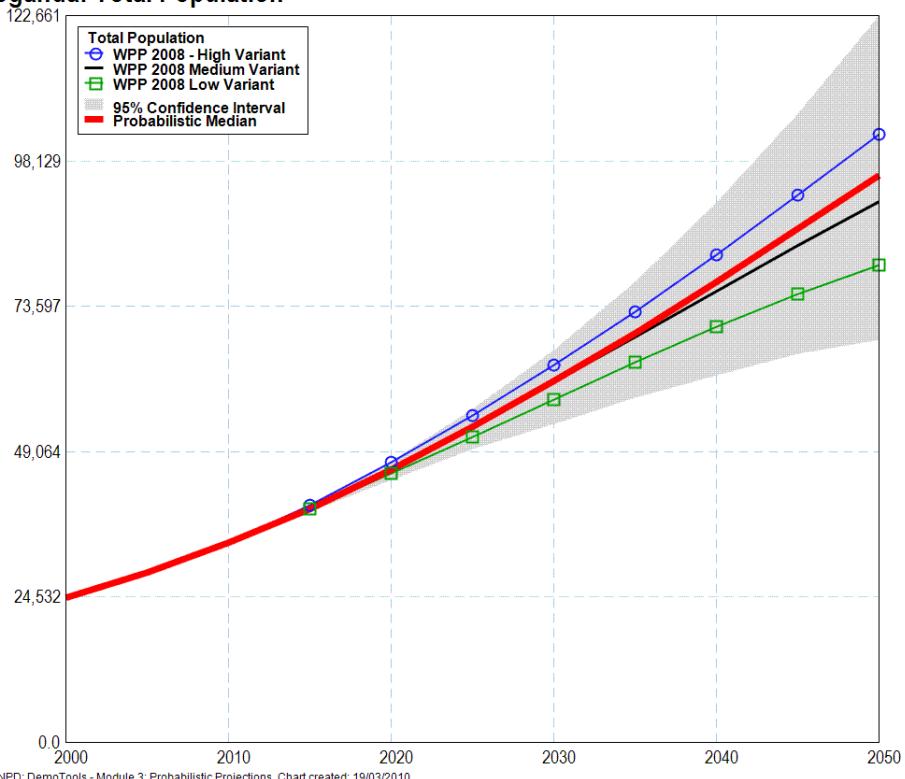
7,138

0.0

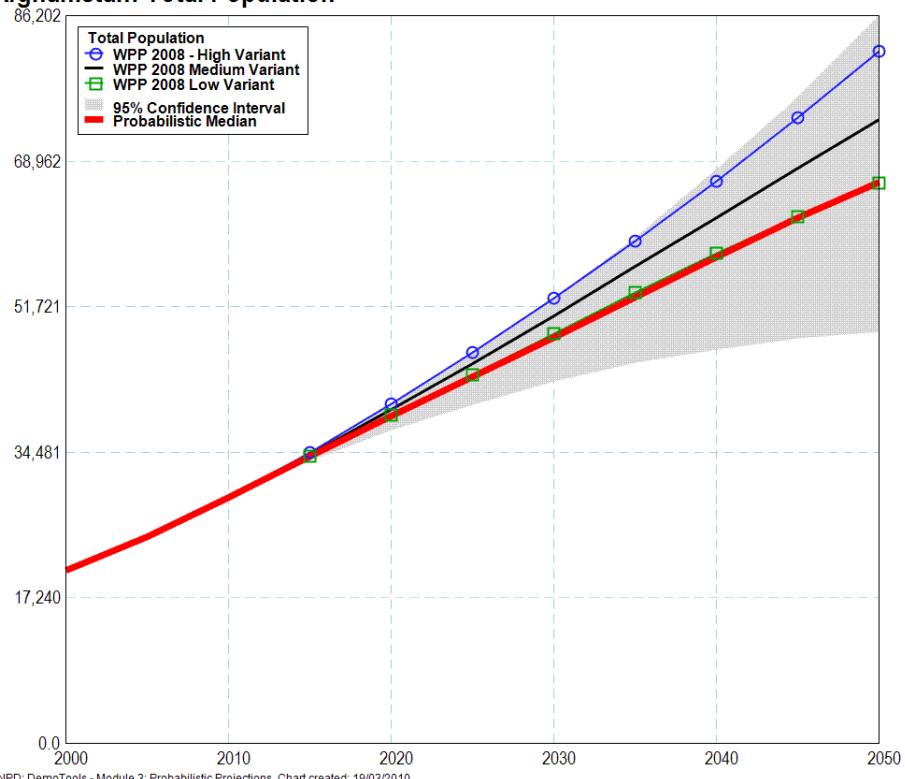
2000 2010 2020 2030 2040 2050

UNPD: DemoTools - Module 3: Probabilistic Projections. Chart created: 19/03/2010

Uganda: Total Population



Afghanistan: Total Population

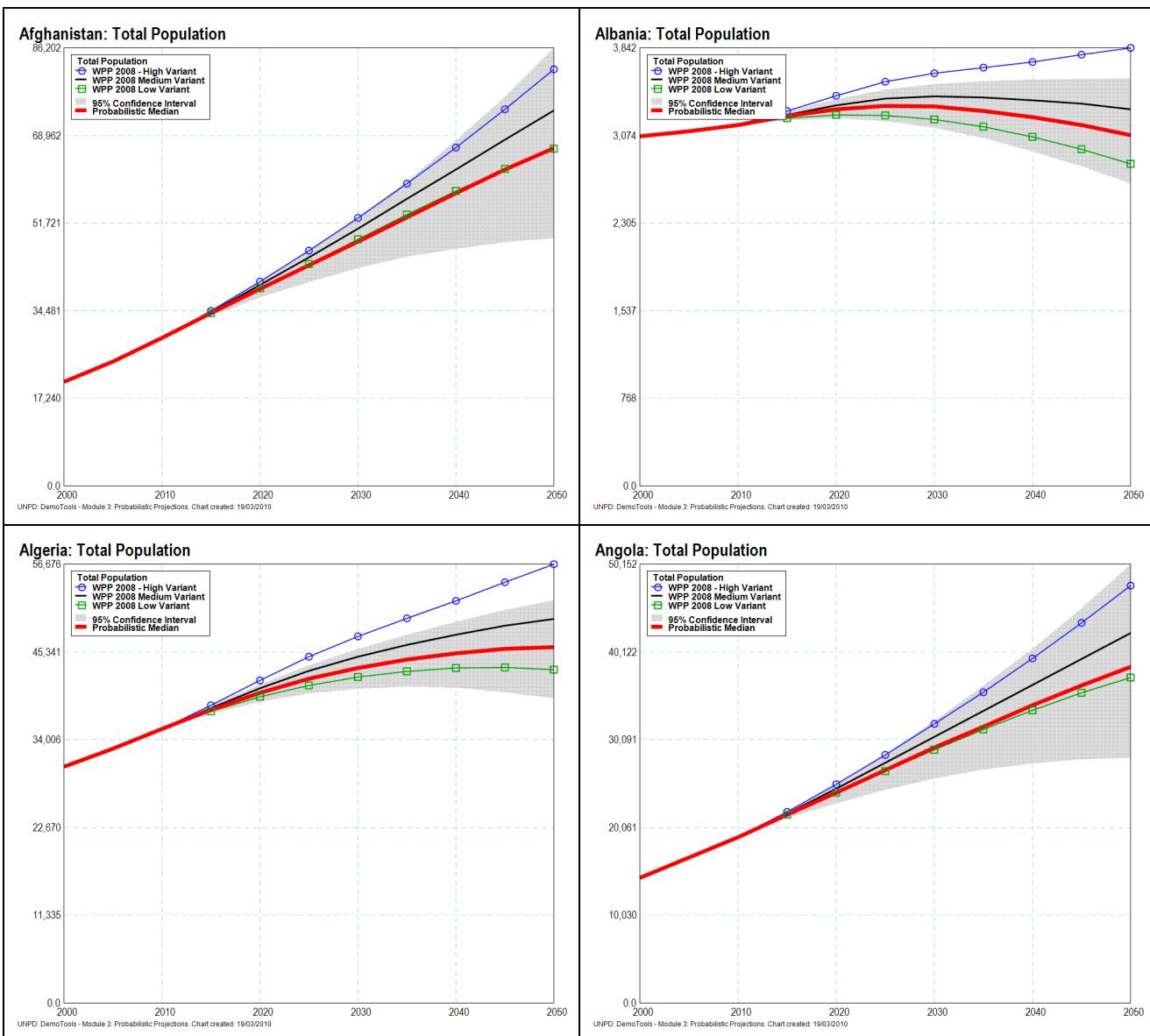


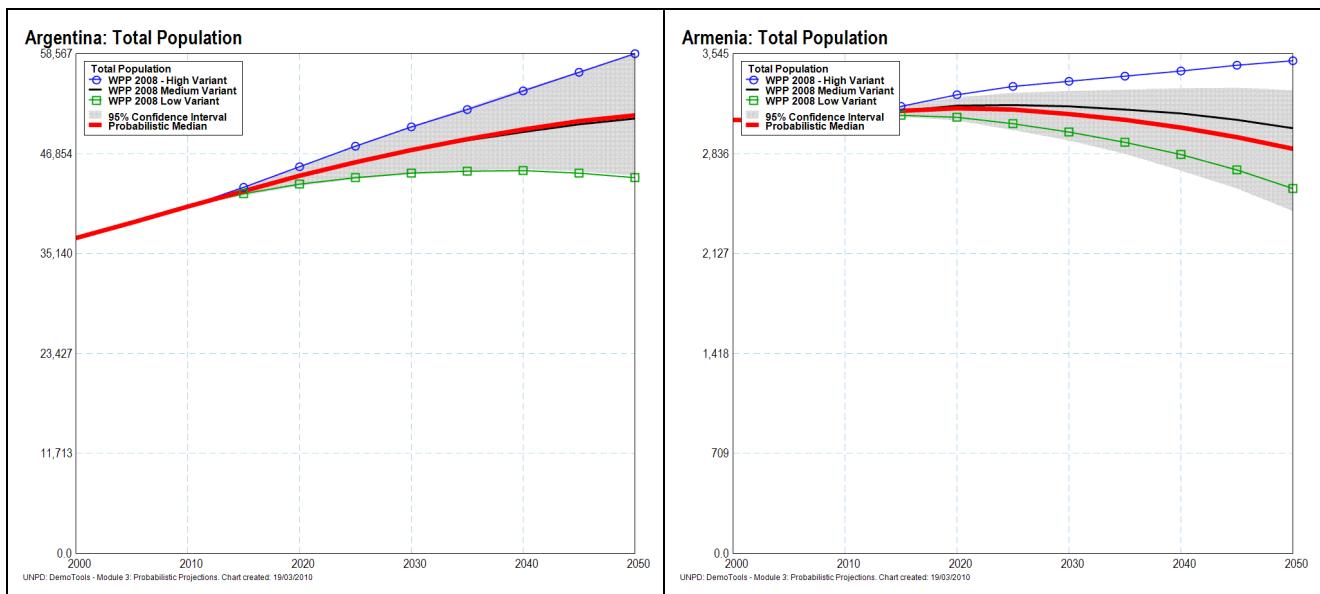
Appendix 2

Total population of selected countries, 2000-2050 (Projection Base Year: 2005)

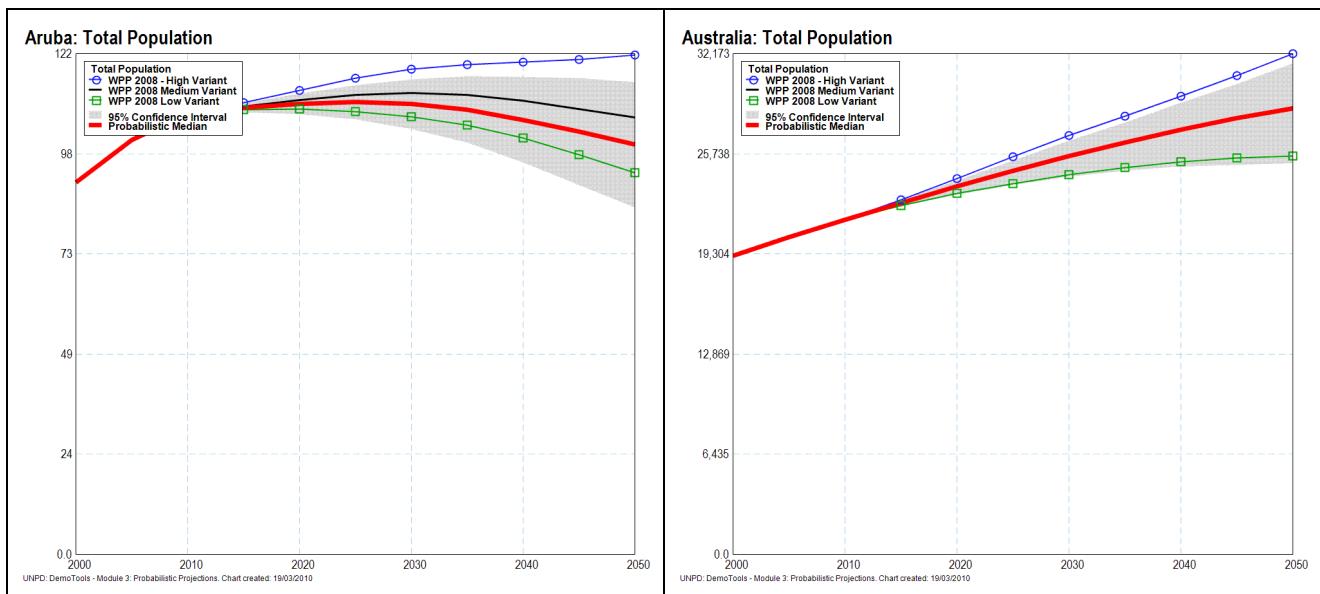
These probabilistic population projections have been computed using stochastic projections of fertility and mortality, generated by a Bayesian Hierarchical Model (Raftery / Alkema, et. al.).

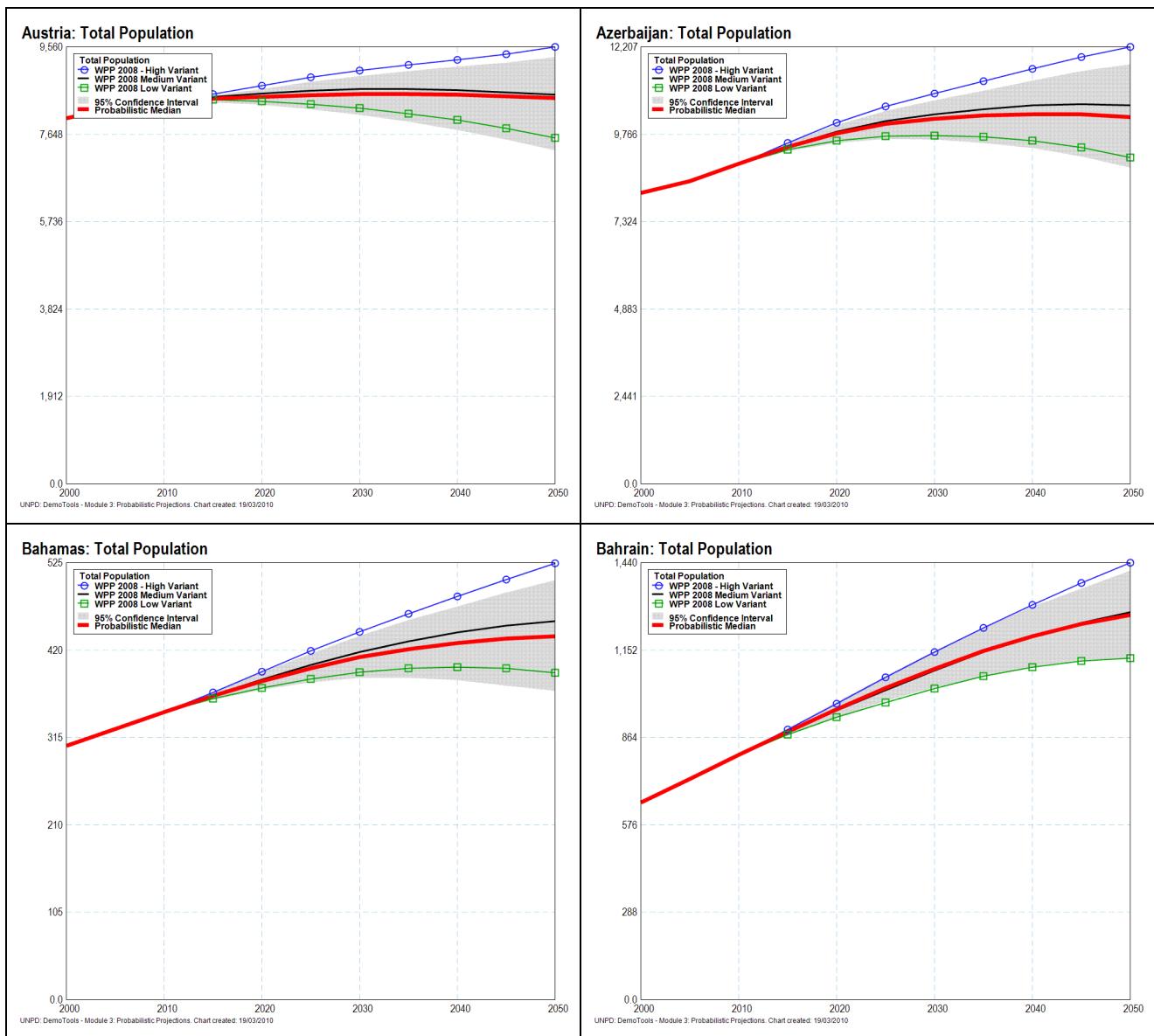
The projections are available for all countries of the world with a population of more than 100,000 in 2008. They have been plotted together with the high, medium and low variants of the *2008 revision of the World Population Prospects*.



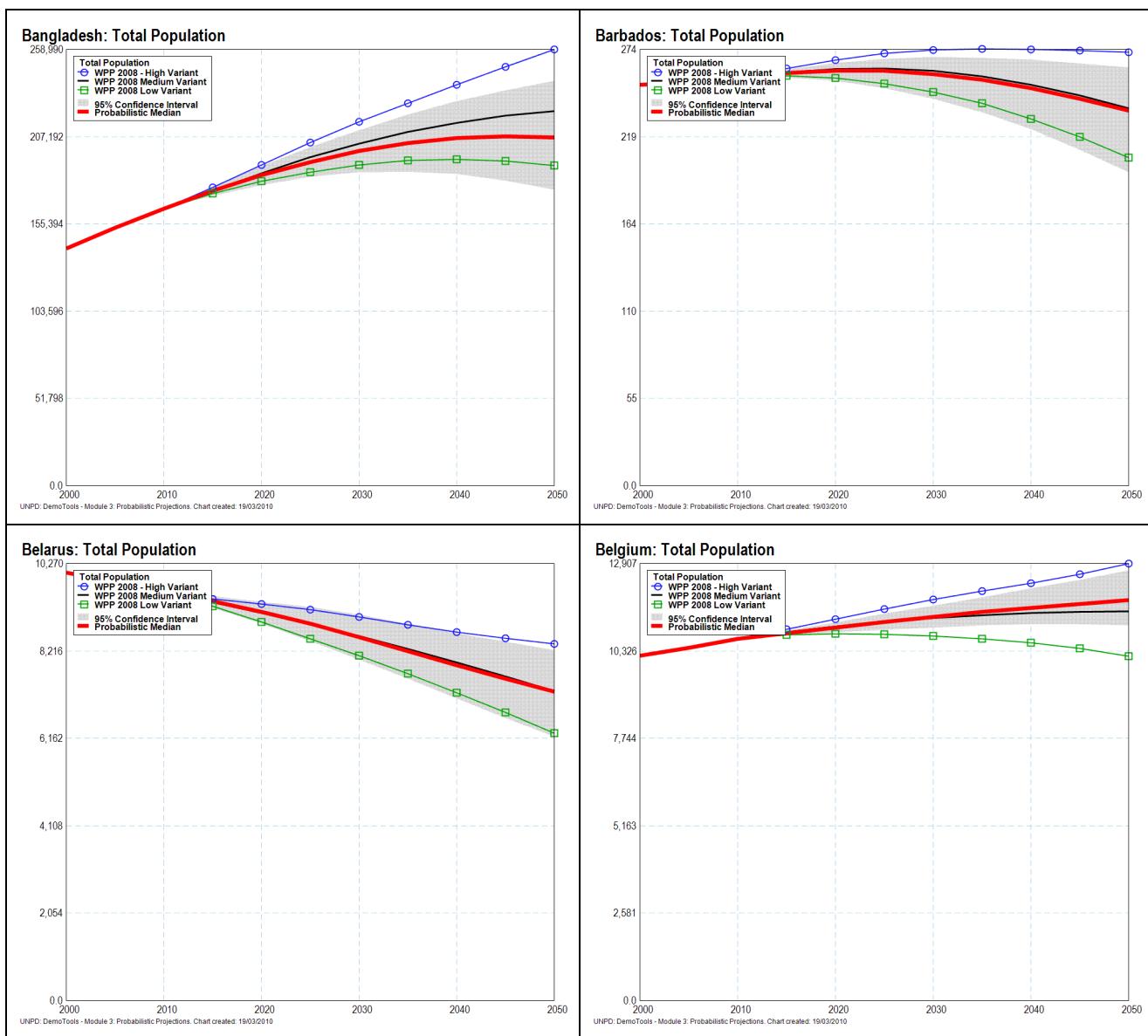


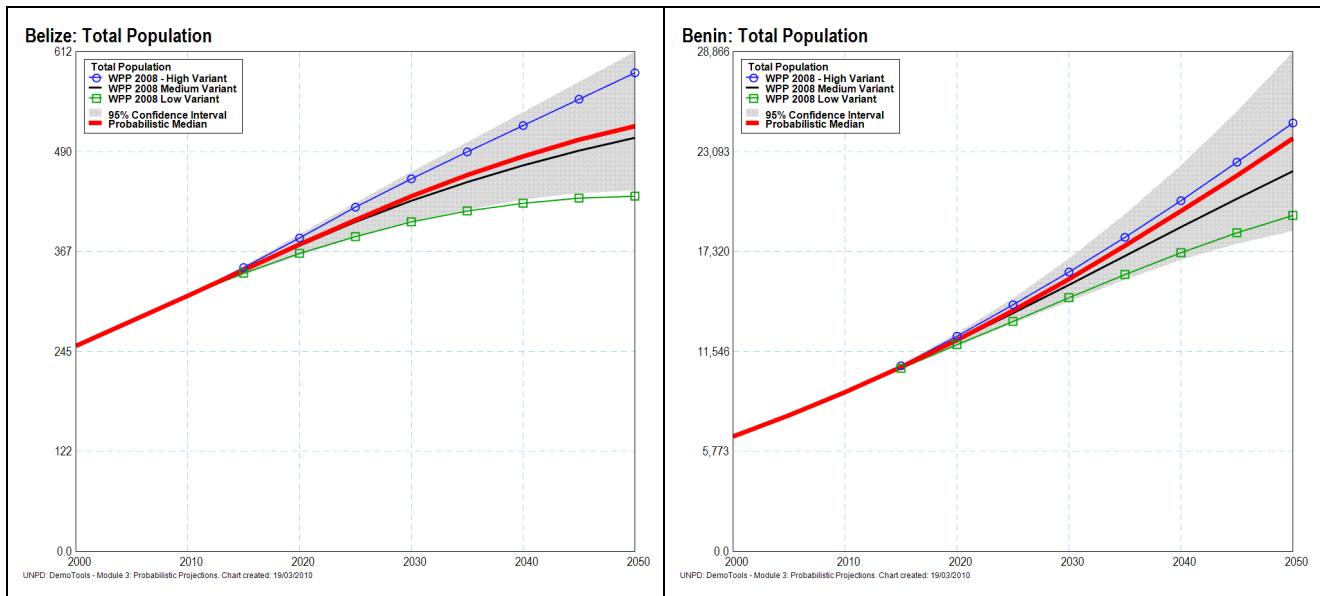
(All population numbers are in thousands)





(All population numbers are in thousands)





(All population numbers are in thousands)