Chapter 3

Equals means Equals

> ...to avoid the tedious repetition of these words: "is equal to", I will set (as I do often in work use) a pair of parallels (or Gemowe lines) of one length (thus =), because no two things can be more equal.

First recorded use of the equals sign, Robert Recorde (1557)

The equals sign, =, is one of the most common in mathematics, and one of the earliest learned by children. Despite this, or maybe because of it, it is still badly abused. (Houston, 2009)

You all know what the equals sign means. You've been using it for years. It means "is equal to". However, we need to take note of Kevin Houston's comment above, and make sure you don't abuse it.

Three key points

- An equals sign must always have something either side of it. "is equal to" means nothing unless we say this thing is equal to that thing.
- "This thing" and "that thing" should be symbolic expressions, not words. Don't write something like "The radius = 3". Instead, either: The radius $r = 3$, or "The radius is 3".
- Except for the exception given below, do not start a line with = because this means you don't have an expression on the left hand side of the equals.

The big exception

There is one exception to the rules given above, which also breaks our rule from last week about giving text explanations for our work before we do the algebraic manipulation. We make this exception because it's so useful. You've met this format for maths before, eg:

$$\begin{aligned}(x+3)^2 &= (x+3)(x+3) \\ &= x^2+3x+3x+3^2 && \text{(expanding the brackets)} \\ &= x^2+6x+9 && \text{(gathering like terms,simplifying } 3^2\text{).}\end{aligned}$$

Notice some important things about this:

- We still have something on the LHS of the equals sign, on the first line.
- The equals signs are aligned with each other vertically
- Now, we put any explanations on the RHS, on the same line where we did the manipulation.
- We can only do this when the algebraic operations we are doing are ones which don't change the value of the expression. As soon as we do anything which changes the value, then we would need to change the LHS as well as the RHS. For example, solve $2x^2+4=(x+3)(x-3)$.

  Solution

  $$\begin{aligned}2x^2+4 &= (x+3)(x-3) \\ &= x^2-9 && \text{(expanding the brackets)}\end{aligned}$$

  Subtracting x^2+4 from both sides gives

  $$\begin{aligned}x^2 &= x^2-9-(x^2+4) \\ &= x^2-9-x^2-4 && \text{(removing the brackets)} \\ &= -13 && \text{(simplifying)}\end{aligned}$$

  Taking the square root of both sides gives

  $$\begin{aligned}x &= \pm\sqrt{13} \\ &= \pm i\sqrt{13} && \text{(since } \sqrt{-13}=i\sqrt{13}\text{)}\end{aligned}$$

For any calculation done using the exception, it's a good idea to

- repeat the LHS on the last line, even if you don't really need to, so that the final answer is clearly stated.
- repeat the LHS if you've turned the page.

Exercise 4. **Equals**

The image on the right shows a very poorly written answer to the question “Solve $3x(x-3)=5$”. Rewrite it properly.