Chapter 1

Introduction

1.1 What do we mean by 'professional mathematics'?

We all know that a good mathematician uses the right method and gets the right answer! But this is not the full story. There's a lot more to being a proper mathematician, and in this module we talk about 'how to be a professional mathematician' or just 'professional maths' for short to encompass these ideas.

Each week, we'll talk in the lecture about one aspect of behaving like a professional mathematician. But here is a rough idea of the kind of things we're talking about:

- Presenting your work well, legibly, easily readable, neatly;
- Explaining what you are doing;
- Making conclusions from the 'sums' you do and summarising your findings;
- Using mathematical notation correctly;
- Stating any assumptions clearly, and understanding their implications;
- Building and understanding logical arguments;
- Understanding when something is proved beyond any doubt;
- Being prepared to experiment, have a go, try things out;
- Extracting the essential bits of a problem and then knowing what 'sums' you should do;
- Checking your answer (you won't have a teacher with you all your life!);
- Drawing graphs accurately and professionally (eg labelling the axes every time);
- Being able to apply ideas you learnt in one place in new contexts;

Why is this important? Here are a few reasons

- When you behave like a professional mathematician, you make fewer mistakes with the sums (you might have to trust me for now about this – but it's true).
- These skills are valued by employers – it's part of why maths degrees are highly valued.
- In the future, you will need to explain your work to other people – both mathematicians and non-mathematicians – and convince them that you are correct – and scribble algebraic scrawls don't make a convincing argument.
- Your tutors will be able to give you better feedback if they can read and understand your work.

In the exam, there is more emphasis on 'doing the sums', but remember that you'll do them correctly more often if you behave like a professional mathematician.

What should you have learnt by the end of this module?

By the end of this module you should be able to:

1. Use mathematical notation correctly and present mathematical work in a logical, clear and professional manner;
2. Apply algebraic manipulation, differentiation, integration and the solution of differential equations in a variety of ways;
3. Apply the techniques of calculus to solve problems.
4. Determine when to attempt an analytic solution and when to use a numerical one.
5. Use and graph a variety of different standard functions
6. Use and manipulate complex numbers
7. Derive and use Taylor and Maclaurin Series.

1.2 Writing as a professional mathematician

It is always the writer's responsibility to write neatly, clearly and legibly. Why?

So that you can

- Communicating ideas is important now, and in the future;

- you get better feedback;
- you keep your examiners happy;
- **you make fewer mistakes**

If tutors/examiners/your friends/your future colleagues/your future boss/your customers/clients/the project manager/the engineer/the designer/the government minister/the civil servant cannot read your work, or can misread your work, or can't follow your work.

- it's your responsibility;
- it doesn't matter whether the 'sums' are right or wrong;
- the work is worthless.

Some basics

- Put your name on everything you hand in.
- Show the title of the work clearly.
- Write from left to right, and top to bottom, like a book. Don't introduce columns. Don't right one bit further up the page than something which precedes it.
- Give yourself plenty of space.
- Write on one side of the paper only.
- Watch out for symbols, letters and numbers which can be confused.

1.2.1 Some specifics

Fractions
- Horizontal lines: $\frac{1}{2}x$ not $1/2x$
- Be very careful where you put the x.
- Fractions of fractions are rarely a good idea. Simplify

$$\frac{2}{\frac{x}{2}} = \frac{2 \times 2}{\frac{x}{2}} \times 2 = \frac{4}{x}$$

- Always reduce fractions: $\frac{1}{2}$ not $\frac{2}{4}$.

Simplifying Not always clear which is simpler, but try to simplify. Certainly...

$$3x + 6 + 4x + 10 = 7x + 16$$

$$\frac{x^2}{x^{-1}} = x^3.$$

Numbers and letters Numbers before letters in final answer

$$2x \quad \text{not} \quad x.2 \text{ or } x2.$$

Functions

1. $\sin x + 1$ means $(\sin x) + 1$ (but easily misread)
    (a) if we want $\sin(x + 1)$ you must include the brackets;
    (b) if we want $(\sin x) + 1$, write the 1 first: $1 + \sin x$.
2. Normally we need brackets, but by convention, $\sin 2x$ is usually taken to mean $\sin(2x)$ (but note, computers don't know this, so you always need the brackets when using computers)
3. Usual order for things multiplied together:

    $$\text{number} \times \text{algebraic} \times \text{other functions}$$

    eg $3(x + 2)\cos x$ not $3\cos x(x + 2)$

    eg xe^3x not e^3xx

    Why? Because we're less likely to misread it.

Multiplication dots Using a dot for multiplication is handy sometimes, but it's also potentially a problem.

$5! = 5.4.3.2.1$ is reasonably clear $2! = 2.1$ is not so clear. Do we mean $2! = 21/10$ or $2! = 2 \times 1$?

Multiplication dots are especially problematic which the second thing is negated. Writing $2 \times (-x)$ as $2. - x$ looks far too much like $2 - x$.

Change the order, or use brackets, eg $2(-x) = -2x$

And while we're talking about minus signs You can never turn a plus back into a minus by adding more ink.

Exercise 1.

Write these in the most sensible way:

1. The cosine of one-half of β, all multiplied by $2x$ plus 1
2. The cosine of Γ, over 5, multiplied by $2x$, plus 1
3. The cosine of ϕ, over 5, multiplied by $2x$ plus 1
4. The square root of $x+1$, times x squared times x cubed times 6 over 3
5. x squared times 6 times 4 times -1 divided by 4
6. $\ln(4x).5.e^x.x$

1.3 Solving equations and transposing formulae

What different operations can you perform when solving an equation or transposing a formula?

Things that **don't** change the value	Things that **do** change the value
gathering like-terms expanding brackets factorising simplifying	multiplying adding dividing (but not by zero or something that might be zero) squaring square-rooting taking the log integrating differentiating
Can be performed on an expression, ie on one side of the equation only	Can only be performed on an equation, and you must do exactly the same to both sides of the equation

Exercise 2. **Transposition & identifying the mathematical operations we use**

1. (a) Rearrange $i = \frac{2}{3+v} - 5a$ to make v the subject.
   (b) Identify every operation which you have performed to find your answer

(c) Now try the question again, but do it in a different way.

2. Solve this equation $\frac{x-1}{3} - \frac{3}{x} = \frac{x+5}{6}$. Again, identify every operation which you have performed.

Exercise 3.

1. Write these expressions in a sensible, concise, unmistakeable way.

   (a) $\cos(2x) \times (-5) \times x^2$

   (b) natural log of $(2 + x)$, multiplied by 3

   (c) 2 divided by $x + 1$

   (d) $x + 4$, divided by 2

   (e) 2, divided by $x + 4$.

2. Find the solution (or solutions) to the following equations. Be neat, clear, tidy. Summarise your result. Watch out for the letters and numbers which can easily be confused.

   (a) $\dfrac{3k-1}{2} = 4$

   (b) $2s + 5 = 3s + 1$

   (c) $\dfrac{8u^3 - 3}{3} = 8$

   (d) $\dfrac{3z}{2} - z = \dfrac{1}{z}$

   Check your answers by substituting the final value back into the equation.

3. For each of the following, make the letter in square brackets the subject. Be neat, clear, tidy. Summarise your result. Watch out for the letters and numbers which can easily be confused.

   (a) $\sqrt{\frac{a}{\alpha} - c} = e$ [α (not a)]

   (b) $\sqrt{(\rho^2 + P)} = 2\rho$ [ρ (not P)]

   (c) $M(a\gamma + by + c) = 0$ [y(not γ)]

   (d) $\frac{S}{X-x} = s$, [x]

4. Solve

   (a) $x^2 = 3$

   (b) $x(x + 4) = 0$

   (c) $(x^2 + 9)(x^2 - 4) = 0$

   Make sure you find all the possible answers.