Chapter 5

Functions: Exponential and Trigonometric

5.1 Symmetry and Reflections

5.1.1 Odd and Even Functions

Graphs of a function often have certain kinds of symmetry. The simplest kind of symmetry relate the values of a function at x and $-x$.

Definition 13. Even and Odd Functions

Suppose that $-x$ belongs to the domain of f whenever x does.
f is an

even function if $f(x) = f(-x)$ for all x in the domain of f. The graph of an even function is symmetric about the y-axis.

odd function if $f(-x) = -f(x)$ for all x in the domain of f. The graph of an odd function is symmetric about the origin.

Other kinds of symmetry exist - for example functions that have shifted to the left or right or up or down. These functions would then be symmetric around different axis or points.

5.1.2 Reflections in Straight Lines

Functions can also be reflected in lines. Some reflections of graphs are easily described in terms of the equations of the graphs:

1. Substituting $-x$ in place of x in an equation in x and y corresponds to reflecting the graph of the equation in the y-axis.
2. Substituting $-y$ in place of y in an equation in x and y corresponds to reflecting the graph of the equation in the x-axis.
3. Substituting $a-x$ in place of x in an equation in x and y corresponds to reflecting the graph of the equation in the line $x = a/2$.
4. Substituting $b-y$ in place of y in an equation in x and y corresponds to reflecting the graph of the equation in the line $y = b/2$.
5. Interchanging x and y in an equation in x and y correspnds to reflecting the graph of the equation in the line $y = x$.

Exercise 13.

1. For the following functions what (if any) symmetry does the graph of f possess? In particular, is f odd or even?

   (a) $f(x) = x^2 + 1$

   (b) $f(x) = x^3 - x$

   (c) $f(x) = \dfrac{x}{x^2 - 1}$

   (d) $f(x) = \dfrac{1}{x^2 - 1}$.