Probability and Statistics Exam, 10

1) The probability of a certain basketball player making a free throw is known to be 0.6. Find the probability of the following events:

   a) **(1 point)** A : the player makes his first free throw only on the 5th shot;

   b) **(2 points)** B : the player makes the first at least 7 consecutive free throws.

2) **(2 points)** Let $X \in \chi^2(2, 1/2)$. Find the pdf of $Y = \sqrt{X}$.

3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x; \theta) = \theta x^{\theta - 1}$, for $0 < x < 1$, with $\theta > 0$, unknown.

   a) **(2 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

   b) **(2 points)** Find the maximum likelihood estimator, $\bar{\theta}$, for θ.