a)

$M(x) = D(x) = \lambda$

$$L(x_1 \dots x_m | \lambda) = \prod_{i=1}^{m} f(x_i | \lambda) =$$

$$= \prod_{i=1}^{m} \frac{\lambda^{x_i}}{x_i!} \cdot e^{-\lambda}$$

$$= e^{-\lambda m} \prod_{i=1}^{m} \frac{\lambda^{x_i}}{x_i!}$$

$$= e^{-\lambda m} \underbrace{\prod_{i=1}^{m} \frac{\lambda}{x_i!}}_{=k} \cdot \lambda^{\sum_{i=1}^{m} x_i} \qquad \sum_{i=1}^{m} x_i = m\overline{x}$$

$$= e^{-\lambda m} \cdot k \cdot \lambda^{m\overline{x}}$$

$$\ln L = \ln k + m\overline{x} \ln \lambda - \lambda m$$

$$\frac{\partial \ln L}{\partial \lambda} = \frac{m\overline{x}}{\lambda} - m$$

$$\frac{\partial \ln L}{\partial \lambda} = 0$$

$$\frac{m\overline{x}}{\lambda} = m$$

$$\overline{x} = \lambda$$

$$\lambda = \overline{x}$$

b) $M(\overline{x}) = M(x) = \lambda$

$$D(\overline{x}) = \frac{D(x)}{m} = \frac{\lambda}{m}$$

$$\lim_{m \to \infty} \frac{\lambda}{m} \to 0$$

c) $I_1(\lambda) = -M\left[\frac{\partial^2 \ln L(x_1|\lambda)}{\partial \lambda^2}\right]$

$$L(x_1|\lambda) = L(x|\lambda) = \frac{\lambda^x}{x!} \cdot e^{-\lambda}$$

$$\ln L = -\lambda + \ln \lambda^x - \ln x!$$

$$\frac{\partial \ln L}{\partial \lambda} = \cancel{} - 1 + x \cdot \frac{1}{\lambda}$$