Exam 5

1) There are 30 computers in a classroom, 4 of which are very slow. 20 students come to class and are seated randomly each in front of a computer

a) (0,5P) Find the P that none of the slow computers are used

b) (1P) Find the P that at most half of the slow computers are used

c) (1P) Let X denote the no. of slow comp. that are being used. Find the pdf of X. What type of distribution is it?

d) (0,5P) What is the expected no. of slow comp. being used?

e) (1,5P) Show that $P(-3<X<3) \geq 1$

2) Let $X_1, X_2, \ldots, X_n$ be a random sample drawn from an exponential distribution with parameter $\lambda > 0$, unknown. (for $X \in Exp(\lambda)$, the pdf is $f(x;\lambda) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}$, $x>0, \lambda>0, E(X)=\lambda$, $V(X) = \lambda^2$).

a) (1,5P) Find the maximum likelihood estimator, $\bar{\lambda}$, for λ.

b) (0,5P) Is it an absolutely correct estimator? Explain.

c) (1,5P) Find the efficiency of $\bar{\lambda}$, $e(\bar{\lambda})$.

d) (1P) At the significance level $\alpha \in (0,1)$, find a most powerful test for testing $H_0: \lambda = 1$ against $H_1: \lambda = 2$