Probability and Statistics Exam, 13

1) A contestant participates in a game show where three important prizes are offered. His chances of winning the three prizes are $\frac{1}{6}$, $\frac{1}{3}$ and $\frac{1}{2}$, respectively.

- **a) (1 point)** Find the probability that the contestant wins exactly one prize.
- **b) (1.5 points)** Find the probability that the contestant loses at least two prizes.
- **c) (1.5 points)** Let X denote the number of prizes won by the contestant. Find the probability distribution function of X.
- **d) (1 point)** How many prizes can the contestant expect to win?

2) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a $Gamma(2, 3\theta)$ distribution, with $\theta > 0$ unknown. (for $X \in Gamma(a, b)$, the pdf is $f(x; a, b) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-x/b}$, $x > 0$, $E(X) = ab$, $V(X) = ab^2$)

- **a) (1.5 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.
- **b) (0.5 points)** Is it an absolutely correct estimator? Explain.
- **c) (2 points)** Find the efficiency of $\overline{\theta}$, $e(\overline{\theta})$.