Model curs

(1) (U, V) – vector aleator discret

$U \backslash V$	-1	1	b
1	$0,25$	$0,05$	a
3	$0,3$	a	$0,1$

a) $a, b = ?$, $E(V) = 0,15$

$E(X) = \sum_{i=1}^{m} x_i \cdot p_i$

V	-1	1	b
	$0,55$	$0,05+a$	$0,1+a$

U	1	3
	$0,30+a$ $\overset{\shortparallel}{0,45}$	$0,4+a$

$\sum_{i=1}^{m} P(U=i) = 1 \Leftrightarrow 2a + 0,70 = \underline{1} \Rightarrow 2a = 0,30 \Rightarrow a = 0,15$

$$\left.\begin{array}{l} P(V=b) = 0,25 \\ P(V=1) = 0,20 \\ P(V=-1) = 0,55 \\ E(V) = 0,15 \end{array}\right\} \Rightarrow 0,25 \cdot b + 0,20 \cdot 1 + (-1) \cdot 0,55 = 0,15$$

$0,25 \cdot b - 0,35 = 0,15 \Rightarrow 0,25b = 0,50 \,|_{\cdot 100}$

$\Rightarrow b = 2$

b) U și V – indepen. ?

$P(U=u, V=v) = P(U=u) \cdot P(V=v) \Rightarrow$ indepen.

Căutăm un contraexemplu

$P(U=1, V=-1) = P(U=1) \cdot P(V=-1) \Leftrightarrow 0,25 = 0,45 \cdot 0,55$

$0,25 = 0,25$

$P(U=1, V=1) = P(U=1) \cdot P(V=1) \Leftrightarrow 0,05 = 0,45 \cdot 0,2$

$0,05 = 0,09$ (F) $\Rightarrow$ U și V nu sunt indepen

c) $E((U-3)^2)$? ?

(2) $X \sim \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{5^3}{6^3} & \frac{5^2 \cdot 3}{6^3} & \frac{5 \cdot 3}{6^3} & \frac{1}{6^3} \end{pmatrix}$ 6^3 posib.

a) $E(X) = \sum_{i=1}^{n} x_i \cdot P(X = x_i)$

$$E(X) = \frac{5^3}{6^3} \cdot 0 + \frac{5^2 \cdot 3}{6^3} \cdot 1 + 2 \cdot \frac{5 \cdot 3}{6^3} + 3 \cdot \frac{1}{6^3}$$

$$= \frac{3 \cdot 5^2 + 30 + 3}{6^3} = \frac{108}{216} = \frac{1}{2}$$

b) $X \sim Bino\left(432, \frac{1}{6^3}\right) \Rightarrow E(Y) = 432 \cdot \frac{1}{6^3} = 2.$

(3) $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} \frac{3a^3}{x^4}, & x > a \\ 0, & x \le a \end{cases}, a > 0.$$

$P(X \le 2a)$

$E(X)$

$$P(X \le 2a) = \int_a^{2a} \frac{3a^3}{x^4} dx = 3a^3 \int_a^{2a} x^{-4} dx = \cancel{3}a^3 \cdot \left(-\frac{1}{\cancel{3} x^3}\right)$$

$$= -\frac{a^3}{x^3}\Big|_a^{2a}$$

$$= -\frac{a^3}{(2a^3)} + \frac{\cancel{a^3}}{\cancel{a^3}}$$

$$= -\frac{1}{8} + \overset{8}{1} = \frac{7}{8}$$

$$E(X) = \int_a^{\infty} \cancel{x} \cdot \frac{3a^3}{x^{\cancel{4}\,3}} dx = 3a^3 \int_a^{\infty} \frac{1}{x^3} dx$$

$$= 3a^3 \left(-\frac{1}{2x^2}\right)\Big|_a^{\infty} = 3a^{\cancel{3}} \cdot \frac{1}{2\cancel{a^2}}$$

$$= \frac{3a}{2}$$

(4) X, Y val binare $(0,1)$ a) $P(X=Y)$

$P(X=1) = 0,4.$

$P(Y=1 \mid X=1) = 0,2$

$P(Y=0 \mid X=0) = 0,3$

$X \sim \begin{pmatrix} 0 & 1 \\ 0,6 & 0,4 \end{pmatrix}$

$P(X=1) = 0,4 \Rightarrow P(X=0) = 0,6$

$P(X=Y=1) + P(X=Y=0) =$

$= P(X=1)\, P(Y=1 \mid X=1) + P(Y=0 \mid X=0) \cdot P(X=0)$

$= 0,4 \cdot 0,2 + 0,3 \cdot 0,6$

$= 0,08 + 0,9 = 0,98$

b) $E(Y) = P(Y=0) \cdot 0 + P(Y=1) \cdot 1 = P(Y=1)$

$P(Y=1) = P(Y=1 \mid X=1) \cdot P(X=1) + P(Y=1 \mid X=0) \cdot P(X=0)$

$= 0,2 \cdot 0,4 + (1 - P(Y=0 \mid X=0)) \cdot P(X=0)$

$= 0,08 + 0,7 \cdot 0,6 = 0,50$

c) $E(X \cdot Y) = P(X=Y=1) \cdot 1 + P(X=Y=0) \cdot 0 = P(X=Y=1).$

$P(X=Y=1) = P(Y=1 \mid X=1) \cdot P(X=1) = 0,2 \cdot 0,4 = 0,08$

d) $E(\underbrace{2^1 Y + 2^0 X}_{\text{binar} \rightarrow B_{10}}) = 2E(Y) + E(X) = 2 \cdot 0,50 + 0,4 = 1,4.$

(5) $f_X : \mathbb{R} \rightarrow \mathbb{R}$

$f_X(t) = \begin{cases} c(3-t)^2, & 0 < t < 3 \\ 0, & \text{altfel} \end{cases}$

a) $c = ?$ b) $F_X - ?$ c) $P(1 < X < 2)$ $P(X < 2 \mid X > 1)$

a) $\int_0^3 c(3-t)^2 = 1 \Rightarrow -c \left. \frac{(3-t)^3}{3} \right|_0^3 \Leftrightarrow c \frac{3^{\cancel{3}2}}{\cancel{3}} = 1$

$9c = 1 \Rightarrow c = \frac{1}{9}$

b) $F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt = \begin{cases} 0, & x \leq 0 \\ \int_0^x \frac{1}{9} \cdot (3-t)^3\,dt \\ 1, & x \geq 3 \end{cases} = \begin{cases} 0, & x \leq 0 \\ \left. \frac{(3-t)^3}{27} \right|_0^x, & 0 < x < 3 \\ 1, & x \geq 3 \end{cases}$

$$= \begin{cases} 0, & x \le 0 \\ \frac{x^3}{27} - \frac{x^2}{3} + x, & 0 < x < 3 \\ 1, & x \ge 3 \end{cases}$$

c) $P(1 < X < 2) = F_X(2) - F_X(1) = \frac{2^3}{27} - \frac{2^2}{3} + 2 - \frac{1}{27} + \frac{1}{3} - 1$

$$= \frac{8-1}{27} + \frac{1-4}{3} \cancel{+1}$$

$$= \frac{7}{27}$$

$$P(X < 2 \mid X > 1) = \frac{P(1 \le X < 2)}{P(X > 1)}$$

$$= \frac{F_X(2) - F_X(1)}{1 - F_X(1)}$$

(6) 4 bile verzi
5 bile albastre
6 bile roșii

$i \in \{1, 2, 3\}$

Formula înmulțirii totale

$P(A_1 \cap \ldots \cap A_m)$

$= P(A_1) \cdot P(A_2 \mid A_1) \cdots$

$\cdots P(A_m \mid A_1 \cap \ldots \cap A_{m-1})$

A_i: „se obț. o bilă albastră

$$P(V_2 \cap R_3) = P(V_1 \cap V_2 \cap R_3) + P(A_1 \cap V_2 \cap R_3) + P(R_1 \cap V_2 \cap R_3)$$

$$= P(V_1) \cdot P(V_2 \mid V_1) \cdot P(R_3 \mid V_1 \cap V_2) + P(A_1) P(V_2 \mid A_1) P(R_3 \mid A_1 \cap V_2)$$

$$+ P(R_1) P(V_2 \mid R_1) P(R_3 \mid R_1 \cap V_2)$$

(7) ① ② ③ ④ ⑤

a) $\frac{2!\cdot 3!}{5!}$ b) $\frac{A_3^2 \cdot 3!}{5!}$ c) $\frac{2\cdot 4\cdot 3!}{5!}$ → permutare bile

d) $\frac{2!\cdot 3!}{5!}$

(8) cod 5 cifre

a) $\frac{A_{10}^5}{10^5}$ b) $\frac{5!}{10^5}$ c) $\frac{C_5^3\cdot 10\cdot 9}{10^5}$ d) $\frac{\frac{5!}{3!\cdot 1!\cdot 1!}\cdot 3+\frac{5!}{2!\cdot 2!\cdot 1!}\cdot 3}{10^5}$

(9) 2 urne   I – 2 bile negre, 3 roșii
II – 3 bile negre, 2 roșii

1 zar → nr par → 2 bile I cu ret.
↓
2 bile II fără ret.

a) distribuție
Bino → cu ret.
hipergeom → fără ret.

$$P(X=k)=C_2^k\cdot\left(\frac{3}{5}\right)^k\cdot\left(\frac{2}{5}\right)^{2-k}+\frac{C_2^k\cdot C_3^{2-k}}{C_5^2}$$

(10) $F_T(t) = \begin{cases} 0, & t<0 \\ t/10, & t\in[0,10] \\ 1, & t>10 \end{cases}$ 16:30 → 16:39

a) $P(T\geq 9) = 1-P(T\leq 9) = 1-F_T(9) = 1-\frac{9}{10} = \frac{1}{10}$.

b) $E(X) = \int_{-\infty}^{\infty} x\, f(x)\,dx$. $f(x) = \int_{-\infty}^{+\infty} F_T(x)\,dx$

$f(x) = \begin{cases} \frac{1}{10}, & t\in[0,10] \\ 0, & t\notin[0,10] \end{cases}$ $E(X) = \int_0^{10} \frac{1}{10}\cdot x\,dx = \frac{1}{10}\cdot \frac{x^2}{2}\Big|_0^{10}$

$= \frac{10}{2} = 5$

c) 5 zile să piardă de 2 ori

Z = nr de zile din 5 când pierde buswl

$Z \sim Bino(5, p)$

$P(Z\geq 2) = 1-P(Z=0) - P(Z=1) = 1 - C_5^0\, p^0(1-p)^5 - C_5^1\, p^1(1-p)^4$

$= 1-\left(\frac{9}{10}\right)^5 - \frac{5!}{4!}\cdot\frac{1}{10}\cdot\left(\frac{9}{10}\right)^4$

$= 1-\left(\frac{9}{10}\right)^5 - \frac{1}{2}\cdot\left(\frac{9}{10}\right)^4$

d) $Y \sim Geom(p) = p(1-p)^k$ $p = P(T\geq 9) = \frac{1}{10}$.

(11) $[1,2]\times[2,4] \subset \mathbb{R}^2$ se alege uniform un pct.

(X,Y) coord. $(X,Y) \sim Uniform([1,2]\times[2,4])$

$f_{(X,Y)} : \mathbb{R}^2 \to \mathbb{R}$

$f_{(X,Y)}(x,y) = \begin{cases} 1/2, & (x,y)\in[1,2]\times[2,4] \\ 0, & \text{altfel} \end{cases}$

(11) $f_X(x) = \begin{cases} \frac{1}{2-1}, & x \in [1,2] \\ 0, & \text{altfel} \end{cases}$ $f_Y(y) = \begin{cases} \frac{1}{2}, & y \in [2,4] \\ 0, & \text{altfel} \end{cases}$

$X \sim \text{Unif}(1,2)$ $Y \sim \text{Unif}(2,4)$

a) $P\left((X,Y) \in \left[\frac{4}{3}, \frac{5}{3}\right] \times \left[\frac{5}{2}, \frac{7}{2}\right]\right)$:

$$P\left(\frac{4}{3} \le X \le \frac{5}{3}, \frac{5}{2} \le Y \le \frac{7}{2}\right) = \int_{\frac{4}{3}}^{\frac{5}{3}} 1\,dx \cdot \int_{\frac{5}{2}}^{\frac{7}{2}} \frac{1}{2}\,dy$$

$$= \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

b) $$E(X^2+Y^2) = E(X^2) + E(Y^2) = \int_1^2 x^2 \frac{1}{2-1}\,dx + \int_2^4 y^2 \frac{1}{4-2}\,dy$$

$$= \frac{x^3}{3}\Big|_1^2 + \frac{1}{2} \frac{y^3}{3}\Big|_2^4$$

$$= \frac{8-1}{3} + \frac{1}{2} \cdot \frac{64-8}{3}$$

$$= \frac{7}{3} + \frac{28}{3} = \frac{35}{3}$$

(14) $X_1 \dots X_n \dots$ v.a. indep. $Y_i = \max\{X_i, X_{i+1}\}$

$P(X_i = -1) = P(X_i = 1) = 0{,}5$ $1 \le i \le n-1$

a) distr. Y_i, $1 \le i \le n-1$

X_i, X_{i+1} v.a. indepen.

$$P(Y_i = -1) = P(\max\{X_i, X_{i+1}\} = -1) = P(X_i = -1, X_{i+1} = -1)$$

$P(X_i=-1, X_{i+1}=-1) = P(X_i=-1) \cdot P(X_{i+1}=-1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

$\Rightarrow P(Y_i=1) = \frac{3}{4}$

$Y_i \sim \begin{pmatrix} -1 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

b) $E(Y_i) = \frac{1}{4} \cdot (-1) + \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$ $\quad V(Y) = 1 - \left(\frac{1}{2}\right)^2$

$V(X) = E(X^2) - E^2(X)$ $\quad = \frac{3}{4}$

c) $Z_n = \frac{1}{n}(X_1^5 + \dots + X_n^5)$ $\quad$ LTNM $\Rightarrow E(X_1^5) = 0$

$(Z_n)_n \xrightarrow{a.s.} E(X_1^5) = 0$

$\frac{1}{2} \cdot (-1)^5 + \frac{1}{2} = 0$

(15) 10 min $\quad X \sim Exp(\lambda)$

a) $P(T \le 15) = ?$

b) $E(X) = ?$ $\quad P(T > 15) = 0,1$.

$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \le 0 \end{cases}$

$E(X) = \frac{1}{\lambda}$

$f_X(x) = \begin{cases} \frac{1}{10} \cdot e^{-\frac{1}{10}x}, & x > 0 \\ 0, & x \le 0 \end{cases}$

$E(X) = 10 \Rightarrow \lambda = \frac{1}{10}$

$P(T \le 15) = \int_0^{15} \lambda e^{-\lambda t} dt = \frac{1}{10} \int_0^{15} e^{-\frac{t}{10}} dt$

$\cancel{\frac{1}{10}} \cdot \left(-\frac{\cancel{10}}{e^{t/10}}\right) \Big|_0^{15}$

$-\frac{1}{e^{15/10}} + 1 = 1 - e^{-1,5}$

b) $P(T>15) = 1 - P(T \le 15) = e^{-15\lambda} \to e^{-\lambda \cdot 15} = 0{,}1$

$\hookrightarrow 0{,}1$

$-\lambda \cdot 15 = \ln \frac{1}{10}$

$E(X) = \frac{1}{\lambda} = \frac{15}{\ln 10}$

Curs final

(1) $U = [1,2,3,1,2,4,1,2,2,4]$

vector de date alese $\longrightarrow$ cu ret. U: $X = [U_{i_1}, \dots, U_{i_5}]$

$\searrow$ fără ret. U: $Y = [U_{j_1}, U_{j_2}, U_{j_3}]$

2. v.a. indică de câte ori apare 1 în X

a) 1 : 3

2 : 4

3 : 1

4 : 2

(3) $f: \mathbb{R} \to [0, \infty)$

$$f(x) = \begin{cases} \frac{2x}{a^2}, & x \in [0, a] \\ 0, & \text{altfel} \end{cases}$$

a) $F_X(X) = ?$ $P(X < \frac{a}{2})$

$P(|X - \frac{a}{2}| < \frac{a}{4})$

a) $$F_X(x) = \int_{-\infty}^{x} f(t)\,dt = \begin{cases} \int_0^a \frac{2x}{a^2}\,dx \\ 0, \text{altfel} \end{cases} = \begin{cases} 1/a, & x \in [0, a] \\ 0, & \text{altfel} \end{cases}$$

$$\int_0^a \frac{2x}{a^2}\,dx = \frac{2}{a^2} \cdot \int_0^a x\,dx = \frac{\cancel{2}}{a^2} \cdot \frac{x^2}{\cancel{2}}\Big|_0^a = \frac{x^2}{a^2}\Big|_0^a$$

$$= \frac{1}{a}$$

$$P\left(X < \frac{a}{2}\right) = \int_0^{\frac{a}{2}} \frac{1}{a}\,dx = \frac{x}{a}\Big|_0^{\frac{a}{2}} = \frac{\cancel{a}}{2} \cdot \frac{1}{\cancel{a}} = \frac{1}{2}$$

$$P\left(|X - \frac{a}{2}| < \frac{a}{4}\right) = P\left(X < \frac{3a}{4}, X > \frac{a}{4}\right) = P\left(X < \frac{3a}{4}\right) \cdot P\left(X > \frac{a}{4}\right)$$

$$= \frac{9}{16}$$

$|X - \frac{a}{2}| < \frac{a}{4}$ $\longrightarrow$ $X - \frac{a}{2} < \frac{a}{4} \Rightarrow X < \frac{3a}{4}$

$\downarrow$

$\frac{a}{2} - X < \frac{a}{4} \Rightarrow \frac{a}{4} < X$

$$P\left(X < \frac{3a}{4}\right) = \int_0^{\frac{3a}{4}} \frac{1}{a}\,dx = \frac{1}{a} \cdot x\Big|_0^{\frac{3a}{4}} = \frac{3\cancel{a}}{4} \cdot \frac{1}{\cancel{a}} = \frac{3}{4}$$

$$P\left(X > \frac{a}{4}\right) = 1 - P\left(X \le \frac{a}{4}\right) = 1 - \int_0^{\frac{a}{4}} \frac{1}{a}\,dx = 1 - \frac{x}{a}\Big|_0^{\frac{a}{4}}$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

(4) $f: \mathbb{R} \to \mathbb{R}$

$$f(t) = \begin{cases} \frac{1}{\delta}, & t \in [a, a+\delta] \\ 0, & t \notin [a, a+\delta] \end{cases} \qquad V(T) = \frac{3}{4}$$

a) $E(T) = ?$, $E(T^2) = ?$

b) $\delta = ?$

$$F_T(t) = \begin{cases} \int_a^{a+\delta} \frac{1}{\delta}\, dt = \frac{a+\delta - a}{\delta} = 1 \\ 0, \text{ altfel} \end{cases}$$

$V(T) = \frac{3}{4}$

$V(T) = E(T^2) + E^2(T)$

$$E(T) = \int_a^{a+\delta} t \cdot \frac{1}{\delta}\, dt = \frac{t^2}{2\delta}\Big|_a^{a+\delta} = \frac{(a+\delta)^2 - a^2}{2\delta}$$

$$= \frac{2a\delta + \delta^2}{2\delta} = \frac{2a+\delta}{2}$$

$$E(T^2) = \int_a^{a+\delta} t^2 \cdot \frac{1}{\delta}\, dt = \frac{t^3}{3\delta}\Big|_a^{a+\delta}$$

$$(a+\delta)^3 = (a+\delta)^2 (a+\delta) = (a^2 + 2a\delta + \delta^2)(a+\delta)$$

$$= a^3 + a^2\delta + 2a^2\delta + 2a\delta^2 + a\delta^2 + \delta^3$$

$$= a^3 + 3a^2\delta + 3a\delta^2 + \delta^3$$

$$E(T^2) = \frac{a^3}{3\delta} + \frac{\cancel{3}a^2\cancel{\delta}}{\cancel{3\delta}} + \frac{\cancel{3}a\,\delta^{\cancel{2}}}{\cancel{3\delta}} + \frac{\delta^{\cancel{3}\,2}}{3\cancel{\delta}} - \frac{a}{3\delta}$$

$$= \frac{a^3}{3\delta} + \frac{\delta^2}{3} - \frac{a}{3\delta} + a^2 + a\delta = a^2 + a\delta + \frac{\delta^2}{3}$$

b) $\frac{3}{4} = a^2 + a\alpha + \frac{\alpha^2}{3} - \left(a + \frac{\alpha}{2}\right)^2$

$\frac{3}{4} = \cancel{a^2} + \cancel{a\alpha} + \frac{\alpha^2}{3} - \cancel{a^2} - \cancel{a\alpha} - \frac{\alpha^2}{4} \Leftrightarrow 9 = \alpha^2 \Rightarrow \alpha = 3$

(5) $X_i \sim$ Bernoulli (p_i), $i \in \{1,2,3\}$ v.a. indepen.

$X_i = 1$ R_i fumat $\qquad P = ?$

$X_i = 0$: R_i nefumat. $\qquad X \sim \begin{pmatrix} 1 & 2 & 3 \\ 0{,}8 & 0{,}7 & 0{,}9 \end{pmatrix}$

$P((R_1 \cap R_3) \cup R_2)$

$= P(R_1 \cap R_3) + P(R_2) - P((R_1 \cap R_3) \cap R_2)$

$= P(R_1) \cdot P(R_3) + P(R_2) - P(R_1) P(R_2) P(R_3)$

$= 0{,}8 \cdot 0{,}9 + 0{,}7 - 0{,}8 \cdot 0{,}7 \cdot 0{,}9$

$= 1{,}42 - 0{,}50 = 0{,}92$

(6) $P(T > 2)$ $\qquad P(T<2) = p_1 \cdot P(T_1 < 2) + p_2 \cdot P(T_2 < 2) + p_3 \cdot P(T_3 < 2)$

$P(T = 2)$

$P(T < 2)$

$f_{T_1}(T_1) = \begin{cases} \frac{1}{4}, & x \in [1,5] \\ 0, & \text{altfel} \end{cases} \qquad f_{T_2}(T_2) = \begin{cases} \frac{1}{2}, & T_2 \in [1,3] \\ 0, & \text{altfel} \end{cases}$

$f_{T_3}(T_3) = \begin{cases} \frac{1}{3}, & x \in [1,4] \\ 0, & \text{altfel} \end{cases}$

$T_1: \int_1^2 \frac{1}{4}\, dt = \frac{t}{4}\Big|_1^2 = \frac{1}{4}.$

$T_2: \int_1^2 \frac{1}{2}\, dt = \frac{t}{2}\Big|_1^2 = \frac{1}{2}$

$T_3: \int_1^2 \frac{1}{3}\, dt = \frac{t}{3}\Big|_1^2 = \frac{1}{3}$

$P(T<2) = 0{,}4 \cdot \frac{1}{4} + 0{,}4 \cdot \frac{1}{2} + 0{,}2 \cdot \frac{1}{3} = 0{,}1 + 0{,}2 + 0{,}06$

$= 0{,}36$

(7) 4 bilete câștig
8 bilete necâștig.

3 bilete f. returnare

a) p ("nu s-a extras bilet c")

$$X \sim \begin{pmatrix} 1 & 2 & 3 \\ \frac{8}{12} & \frac{7}{11} & \frac{6}{10} \end{pmatrix}$$

$$\frac{\cancel{8}^{2} \cdot 7 \cdot \cancel{6}^{3}}{\cancel{12}_{4} \cdot 11 \cdot \cancel{10}_{5}} = \frac{14}{55}$$

b) X v.a. nr. bilete câșt. extrase

$$E(x) = \sum_{i=1}^{n} x_i \cdot p_i$$

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(1) mulțimea $\{0, 1, 2, 3, 4, 5, 6\}$ codul x - 4 cifre distincte

a) cazuri posibile: $7 \cdot 6 \cdot 5 \cdot 4$

b) $X = 1234$ sau $X = 0246$ $\rightarrow p = \frac{2}{7 \cdot 6 \cdot 5 \cdot 4}$

c) $X = 0123, 1234, 2345,$
3456

$p = \frac{4}{7 \cdot 6 \cdot 5 \cdot 4}$

d) x -> doar cifre pare $p = \frac{4!}{7 \cdot 6 \cdot 5 \cdot 4}$

e) $X = \ldots 4$ $p = \frac{A_6^3 \cdot 3!}{4 \cdot 5 \cdot 6 \cdot 7}$

f) $1 - P(\overline{F}) = 1 - \frac{4!}{4 \cdot 5 \cdot 6 \cdot 7}$

(3) $f(x) = \begin{cases} \frac{1}{2}, & x \in [a, a+2] \\ 0, & \text{altfel} \end{cases}$

a) $P(X \leq a) \quad P(X \leq a + \frac{1}{2})$

$$P(X \leq a) = \int_a^a \frac{1}{2} dx - \int_{-\infty}^a \frac{1}{2} dx = 0.$$

$$P(X \leq a + \frac{1}{2}) = \int_a^{a+\frac{1}{2}} \frac{1}{2} dx - \int_a^a \frac{1}{2} dx - \int_{-\infty}^a \frac{1}{2} dx$$

$$= \frac{x}{2} \Big|_a^{a+\frac{1}{2}} = \frac{a+\frac{1}{2}}{2} - \frac{a}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

$$F(x) = \int_{-\infty}^{a+2} f(x)\, dx = \begin{cases} 1, & x \geq a \\ 0, & x < a \end{cases}$$

$$\lim_{x \to a} F(x) = \lim_{x \to a} 1 = 1.$$

$$F(a + \tfrac{1}{2}) = \tfrac{1}{4}.$$