a) $L(x_1 \dots x_m | \theta) = \prod_{i=1}^{m} f(x_i; 1, \theta)$

$= \prod_{i=1}^{m} \frac{1}{\theta} \cdot e^{-\frac{x_i}{\theta}} = \frac{1}{\theta^m} \cdot e^{-\frac{\sum_{i=1}^{m} x_i}{\theta}} \to \bar{x} \cdot m$

$= \frac{1}{\theta^m} \cdot e^{-\frac{m\bar{x}}{\theta}}$

$\ln L = -m \ln \theta - \frac{m\bar{x}}{\theta}$

$\frac{d \ln L}{d\theta} = -\frac{m}{\theta} + \frac{m\bar{x}}{\theta^2}$

$\frac{d \ln L}{d\theta} = 0 \Rightarrow \frac{m}{\theta} = \frac{m\bar{x}}{\theta^2} \Rightarrow \bar{\theta} = \bar{x}$

b) $\bar{\theta}$ este selv. corect dacă $M(\bar{\theta}) = \theta$ și $\lim_{m \to \infty} D(\bar{\theta}) = 0$

$M(\bar{\theta}) = M(\bar{x}) = M(x) = \theta$

$D(\bar{\theta}) = D(\bar{x}) = \frac{D(x)}{m} \xrightarrow{m \to \infty} 0$ $\Rightarrow$ este estimator selv. corect

c) $e(\bar{\theta}) = \frac{1}{I_m(\theta) \cdot D(\bar{\theta})}$

$I_n(\theta) = -M\left[\frac{d^2 \ln L(x_1|\theta)}{d\theta^2}\right]$

$L(x_1|\theta) = L(x|\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \Rightarrow \ln L = -\ln\theta - \frac{x}{\theta}$

$\frac{d \ln L}{d\theta} = -\frac{1}{\theta} + \frac{x}{\theta^2} \Rightarrow \frac{d^2 \ln L}{d\theta^2} = \frac{1}{\theta^2} - \frac{2x}{\theta^3}$

$M\left(\frac{1}{\theta^2} - \frac{2x}{\theta^3}\right) = \frac{1}{\theta^2} - \frac{2}{\theta^3} M(x) = \frac{1}{\theta^2} - \frac{2}{\theta^3} \cdot \theta = -\frac{1}{\theta^2}$

$I_1(\theta) = \frac{1}{\theta^2} \Rightarrow I_m(\theta) = m \cdot I_1(\theta) = \frac{m}{\theta^2}$

$e = \frac{1}{\frac{m}{\theta^2} \cdot \frac{\theta^2}{m}} = 1$

d) $H_0: \theta = 2$

$H_1: \theta = 1$