① A multiple choice test consists of 10 questions, each having 4 possible answers (only one, of course, is correct). A student answers each question randomly.

a) (0.5 points) Find the probability that the student gives exactly 3 wrong answers.

b) (1 point) Find the probability that the student passes the test (i.e. answers at least 50% of the questions correctly)

c) (1 point) Let x denote the number of correct answers given by the student. Find the probability distribution function of x. What type of distribution is it?

d) (0.5 points) What is the expected number of correct answers given by the student.

e) (1.5 points) Show that $32\,P(1 < 2x < 9) \geq 17$

② Let $X_1, X_2 \ldots X_n$ be a random sample from a Gamma $(1,\theta)$ distribution, with $\theta > 0$ unknown. (for $x \in \text{Gamma}(a,b)$, the pdf is

$$f(x,a,b) = \frac{1}{b^a\,\Gamma(a)} \cdot x^{a-1} \cdot e^{-\frac{x}{b}} \qquad x > 0$$

$E(x) = ab$

$V(x) = ab^2$)

a) (1.5 points) Find the maximum likelihood estimator $\hat{\theta}$, for θ.

b) (0.5 points) Is it an absolutely correct estimator? Explain.

c) (1.5 points) Find the efficiency of $\hat{\theta}$, $\text{eff}(\hat{\theta})$

d) (1 point) At the significance level $\alpha \in (0,1)$, find a most powerful test for testing $H_0: \theta = 2$ against $H_1: \theta = 1$.