

Considerăm selecție de volum 1, $m=1$
 $L(\theta) = \prod_{i=1}^m f(x_i; i, \theta) = f(y; 1, \theta)$.

$$L(\theta_0) = L(2) = \frac{1}{2} \cdot e^{-\frac{y}{2}}$$

$$L(\theta_1) = L(1) = e^{-y}$$

$$\frac{L(\theta_1)}{L(\theta_0)} = \frac{e^{-y}}{\frac{1}{2} \cdot e^{-\frac{y}{2}}} = \frac{2}{e^{\frac{y}{2}}} \geq K_\alpha \Rightarrow e^{\frac{y}{2}} \leq \frac{2}{K_\alpha} \Rightarrow$$

$$\frac{y}{2} \leq \ln 2 - \ln K_\alpha \Rightarrow y \leq \underbrace{2 \ln 2 - 2 \ln K_\alpha}_{k'_1}$$

$$RC: \{y \leq k'_1\}$$

$$\begin{aligned} \alpha &= P(\text{eroare de tip 1}) = P(\text{rejection } H_0 | H_0) = \\ &= P(y \leq k'_1 | \theta = 2) = \\ &= \int_0^{k'_1} \frac{1}{2} e^{-\frac{x}{2}} dx = -e^{-\frac{x}{2}} \Big|_0^{k'_1} = -e^{-\frac{k'_1}{2}} + 1 \Rightarrow \\ e^{-\frac{k'_1}{2}} &= 1 - \alpha \Rightarrow -\frac{k'_1}{2} = \ln(1 - \alpha) \Rightarrow k'_1 = -2 \ln(1 - \alpha) \end{aligned}$$

$$RC: \{y \leq -2 \ln(1 - \alpha)\}, \alpha \in (0, 1).$$