here are 10 hot lunches left at a cafeteria, 6 of which are vegetarian. Four students come to lun
ate and randomly pick up a plate.

a) **(0.5 points)** Find the probability that one student got a vegetarian lunch.

b) **(1 point)** Find the probability that at most half of the students got a vegetarian lunch.

c) **(1 point)** Let X denote the number of students who got a vegetarian lunch. Find the pr
bility distribution function of X. What type of distribution is it?

d) **(0.5 points)** What is the expected number of students getting a vegetarian lunch?

e) **(1.5 points)** Prove that $5P(|X| \geq 3) \leq 4$.

t $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;p) = p^x(1 -$
$= 0, 1$, $E(X) = p$, $V(X) = p(1-p)$, where $p \in (0,1)$ is unknown.

a) **(1.5 points)** Find the maximum likelihood estimator, $\overline{p}$, for p.

b) **(0.5 points)** Is it an absolutely correct estimator? Explain.

c) **(1.5 points)** Find the efficiency of $\overline{p}$, $e(\overline{p})$.

d) **(1 point)** At the significance level $\alpha \in (0,1)$, find a most powerful test for testing H
against $H_1 : p = 1/4$.