

Probability and Statistics Exam, 2

A software firm produces accounting programs. The probability that one of their programs is defective is 0.2. A chain store purchases 20 accounting programs from that firm.

- a) (1 point) Find the probability that exactly 3 programs are defective.
- b) (1.5 point) Find the probability that at least 80% of the programs are working properly.
- c) (1.5 points) Let X denote the number of programs that are working properly. Find the probability distribution function of X . What type of distribution is it?
- d) (0.5 points) What is the expected number of programs that are working properly?

Let X_1, X_2, \dots, X_n be a random sample drawn from a *Poisson* distribution with parameter λ , unknown.

(for $X \in \mathcal{P}(\lambda)$, the pdf is $p(x; \lambda) = \frac{\lambda^x}{x!} e^{-\lambda}$, $x = 0, 1, \dots$, $E(X) = V(X) = \lambda$).

- a) (1.5 points) Find the maximum likelihood estimator, $\bar{\lambda}$, for λ .
- b) (0.5 points) Is it an absolutely correct estimator? Explain.
- c) (1.5 points) Find the efficiency of $\bar{\lambda}$, $e(\bar{\lambda})$.
- d) (1 point) At the significance level $\alpha \in (0, 1)$, find a most powerful test for testing $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$.