

① A software firm produces accounting programs. The probability that one of their programs is defective is 0.2. A chain store purchases 20 accounting programs from that firm.

- Find the probability that exactly 3 programs are defective.
- Find the probability that at least 20% of the programs are working properly.
- Let  $X$  denote the number of programs that are working properly. Find the probability distribution function of  $X$ . What type of distribution is it?
- What is the expected number of programs that are working properly.

②  $X_1, X_2, \dots, X_n$  be a random sample drawn from a Poisson distribution with parameter  $\lambda$ , unknown (for  $X \in P(\lambda)$ ), the pdf is  $p(x, \lambda) = \frac{\lambda^x}{x!} e^{-\lambda}$ ,  $x=0, 1, \dots$ ,  $h(x) = k(x) = \lambda$

- Find the maximum likelihood estimator,  $\bar{X}$  for  $\lambda$ .
- Is it an absolutely correct estimator? Explain.
- Find the efficiency of  $\bar{X}$ ,  $e(\bar{X})$ .
- At the significance level  $\alpha \in (0, 1)$ , find the most powerful test for testing:  $H_0: \lambda=1$ , against  $H_1: \lambda=2$

①

a)

$$k=3$$

$$n=20$$

$$p=0.2$$

binomial

$$C_{20}^3 (0.2)^3 (0.8)^{17}$$

b)

80%  $\rightarrow$  16 cel putim  $\Rightarrow$  1-4 cel mult

$$k < 4 \quad 1 - \sum_{i=0}^3 C_{20}^k (0.2)^k (0.8)^{20-k}$$

c)

$$X \left( \begin{matrix} k \\ C_{20}^k (0.2)^k (0.8)^{20-k} \end{matrix} \right) \quad \text{Set dist. binomial}$$

$k=0, 20$

d)

$0.2 \times 20 = 4$  se aşteptă să fie defecte

$20 - 4 = 16 \Rightarrow$  numărul de programe care funcționează bine