15210: Parallel and Sequential Data Structures and Algorithms

SegmentLab

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7.1

$$x \cdot (1 - x^d)$$

7.2

Let

$$\frac{d(x(1-x^d))}{dx} = 1 - x^d + x \cdot (-dx^{d-1}) = 1 - (d+1)x^d = 0$$

Then,

$$x = (\frac{1}{d+1})^{\frac{1}{d}}$$

7.3

k = 3

Denote number of vertices with degree i as n_i. We have

$$n_1 + n_2 + n_3 = n$$

$$n_1 + 2n_2 + 3n_3 = 2(n-1)$$

Assume that $n_1 + n_2 < \frac{n}{3}$, then $n_3 > \frac{2n}{3}$, therefore

$$n_1 + 2n_2 + 3n_3 > n_1 + 2n_2 + 2n > 2n - 2$$

Contradict with $n_1 + 2n_2 + 3n_3 = 2(n-1)$, therefore, the assumption is wrong, that is, $n_1 + n_2 \geq \frac{n}{3}$

7.4

Assign each edge with a unique random number.

For each edge e, if the two vertices has degrees of:

1,1 => contract and finish the algorithm

1,2 => contract if e is of larger weight

2,2 => contract if weight of e is of larger than any adjacent edges

either of the two is 3 => do nothing

We have proved that $n_1 + n_2 \geq \frac{n}{3}$

The probability that an edge will get contracted is at least

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

That is, in each round a fixed fraction of edges $(\frac{1}{9})$ are expected to be removed.

7.5

Since we would finally get one vertex with no edges, we would do n-1 contractions, making the total work $O(n)$.

Since we contract a fixed fraction, we expect to have $logn$ rounds, and each round has span of $O(1)$ since it is parallel. That is,

$$S(n) = S(\frac{8n}{9}) + O(1) = O(logn)$$