15210: Parallel and Sequential Data Structures and Algorithms

BigNumLab

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5.1

For Union, we have

$$W(|S_1|,|S_2|) = \Theta(mlog\frac{m+n}{m})$$

where $m = min\{|S_1|,|S_2|\}, n = max\{|S_1|,|S_2|\}$

We can use Θ here because we have a specific number of members of the set, and we don't have to enlarge the work to get this result.

Since we are applying union operation on (k, k), (2k, k), (3k, k)…((n-1)k, k), for the ith iteration,

$$W_i = \Theta(klog\frac{(i+1)k}{k})$$

Therefore, total work

$$W(n,k) = \sum_{i=1}^{n\text{-}1} \Theta(klog\frac{(i+1)k}{k}) = klogn! = knlogn$$

Since union is using tree, the span for ith union is $logik$. And the span for iter is the sum of all the iterations, therefore

$$S = max\{S_{iter}, S_{union}\} = S_{iter} = \sum_{i=1}^{n\text{-}1} logik = logk^{n\text{-}1}(n\text{-}1)! = \Theta(\text{nlogkn})$$

5.2

The recurrence of the work and span are:

$$W(n,k) = 2W\left(\frac{n}{2},2k\right) + W_{combine}$$

Use block method. In each level, union two sets so the number of elements doubles; and the number of sets decreased to half. This tree is balanced. As shown below:

$$\frac{n}{2^i}\times k\times 2^i = nk$$

That is,

$$W_{combine} = \Theta(nk)$$

The depth of the recurrence is $\log n$. Hence if we sum up each level, the total work should be:

$$W(n, k) = \Theta(nk \log n)$$

For reduce, span is

$$S_{reduce} = \log n \ max\{S_{union}\}$$

For union, span is

$$S_{union}(ik, ik) = logik$$

Therefore,

$$max\{S_{union}\} = log\frac{n}{2}k$$

$$S_{reduce} = \Theta(logn \ lognk)$$