15210: Parallel and Sequential Data Structures and Algorithms

BabbleLab

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7.1

For union of two sets , define

The work and span are

For integer, , apply to the above equation, we have

7.2

It is not necessary to take time to generate a whole new set. Since original set won’t be changed, we can simply pass a new reference of that set and do not need to worry about any change by this operation. So we can use something like Seq.subseq to perform faster copy operation.

7.3

Define the 3 elements as a, b, c, (as the same order) respectively.

If we pick b as the pivot, a will be compared with b but never be compared with c.

If we pick a as the pivot, it will be compared with both b and c.

If we pick c as the pivot, a will be compared to c, and then a and b would enter next recursive call of quicksort. In next round, the probability of a and b are compared are as shown in class.

Therefore,

7.4

Number of calls to lt is the same as number of comparisons

We have a fixed input b at the beginning. To make this function reasonable, b should be less than any number in S.

The first 3 numbers from S are special:

1st 🡪 has to be 3 comparisons

2nd 🡪 has to be 3 comparisons

3rd 🡪 2 or 3 comparisons, with probability 1/2 vs. 1/2

Define

Then we have

Consider

Therefore,