15210: Parallel and Sequential Data Structures and Algorithms

ParenLab

Zikang Wang (zikangw)

5.1

For a sequence, function subDist S would go along S to find the longest distance starting from its first element.

Function max S is a recursive function, it would compare the value of subDist S and the subsequence of S that has the largest parenthesis distance, then return the larger one of the two. The base case of this case is the last two elements of S, which would return 0 or 2.

Therefore, the work of this algorithm can be represented as

That is,

Since we used recursion and computed the distance in each turn of the recursion,

One improvement is saving the largest distances starting from each element in a sequence first, then compare, so it can be parallel and S would reduce to n.

But I’m busy and lazy…

5.2

1.

Guess:

So we need to find A, B and C

Base Case:

Inductive Case:

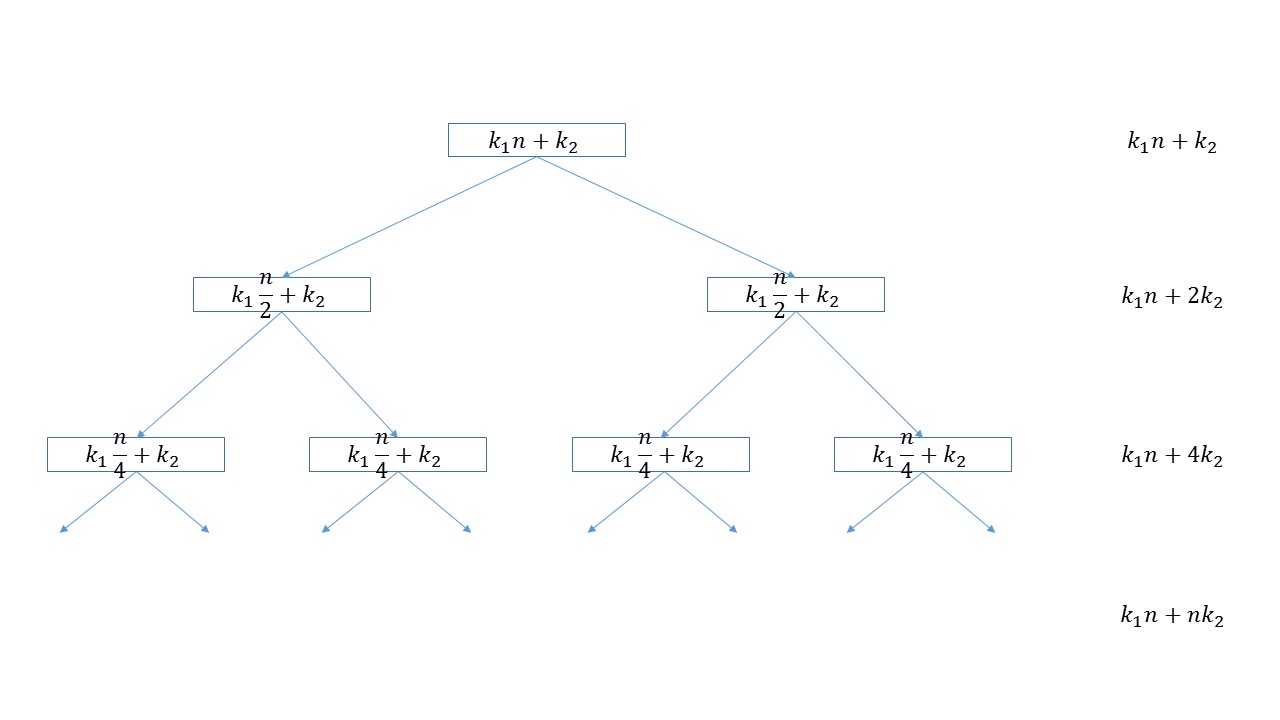
To finish the inductive,

Therefore,

So,

2.

If , then we can assume , then



Sum all the layers up,

Therefore,

5.3

Use brick method,

|  |  |  |
| --- | --- | --- |
| Number of Nodes | Work per Node | Total Work |
|  |  |  |
|  |  |  |
| …… | | |

Work on two layers are the same , this is a balanced tree

Define d = depth then on the leaf node, the work is,

Therefore,

5.4

Similar to 5.3,

|  |  |  |
| --- | --- | --- |
| Number of Nodes | Work per Node | Total Work |
|  |  |  |
|  |  |  |
| …… | | |
|  |  |  |

Total work is decreasing, , this is a root dominant tree, therefore

5.5

Use the brick method:

|  |  |  |
| --- | --- | --- |
| Number of Nodes | Work per Node | Total Work |
|  |  |  |
|  |  |  |
| …… | | |
|  |  |  |

Total work is decreasing, , this is a root dominant tree, therefore

5.6

|  |  |  |
| --- | --- | --- |
| Number of Nodes | Work per Node | Total Work |
|  |  |  |
|  |  |  |
|  |  |  |
| …… | | |
|  |  | n |

Define d = depth, we have

Therefore,

5.7

Guess:

Base Case:

Inductive Case:

To make the above equation valid, let:

Therefore, the parameters should be: