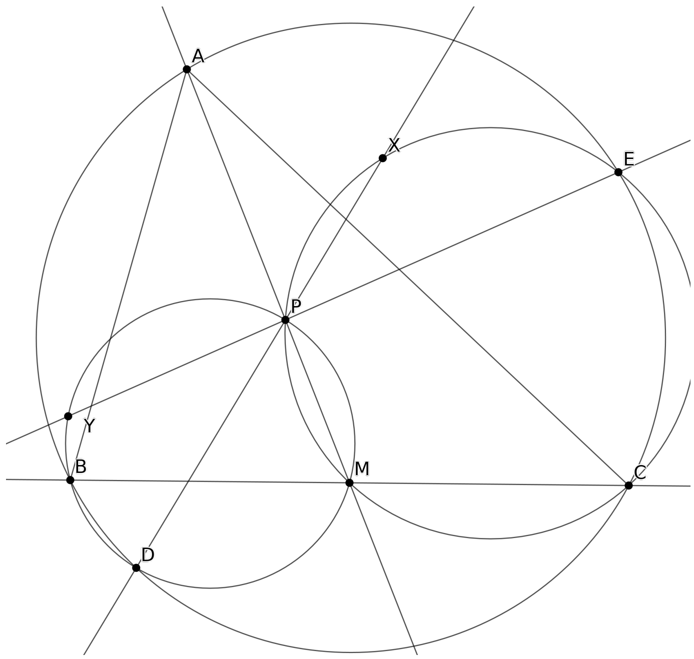

Figure 1

If we invert around the point A with an arbitrary radius, the problem translates to:

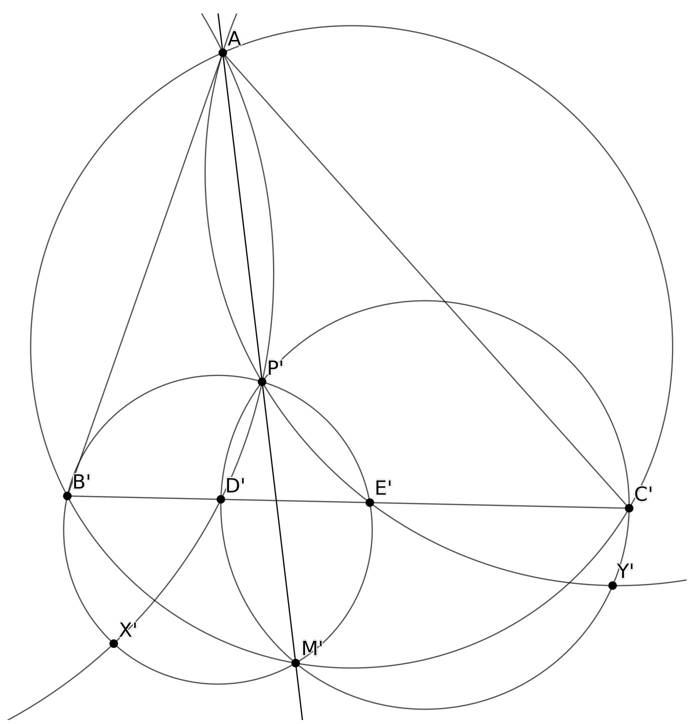

Figure 2

Now rename the whole configuration by removing the primes. Define points U, V on

BC such that $AU \parallel CM$ and $AV \parallel BM$. Define $Q = AU \cap BM, R = AV \cap CM$

Lemma 1

A, P, D, U are cyclic, and so are A, P, E, V

Proof.

$$\begin{aligned} &\angle DPM \\ &\quad = \angle DCM \\ &\quad = \angle UAE \end{aligned}$$

Lemma 2

$\odot UQB$ is tangent to $\odot ABC$ at B. And $\odot CRV$ is tangent to $\odot ABC$ at C.

Proof. by Reim's theorem on $\triangle UQB$ and $\triangle BMC$

Again by Reim's theorem we get $DP \parallel AB,\ PE \parallel AC$. Let X' be the intersection of $\odot ADP$ and $\odot UQB$.

Lemma 3

$BXMP$ is cyclic.

Proof.

$$\angle X'BM = \angle XUA = \angle X'PM$$

Lemma 4

AM is the radical axis of $\odot UQB$ and $\odot CRV$

Proof. Let $AM \cap BC = N$. Since

$$\angle BAN = \angle BUA$$

We have

$$NA^2 = NB \cdot NU$$

Similarly $NA^2 = NC \cdot NV$. And since TB is tangent to $\odot UQB$, and TC is tangent to $\odot CRV$, TN is the radical axis of $\odot UQB$ and $\odot CRV$.

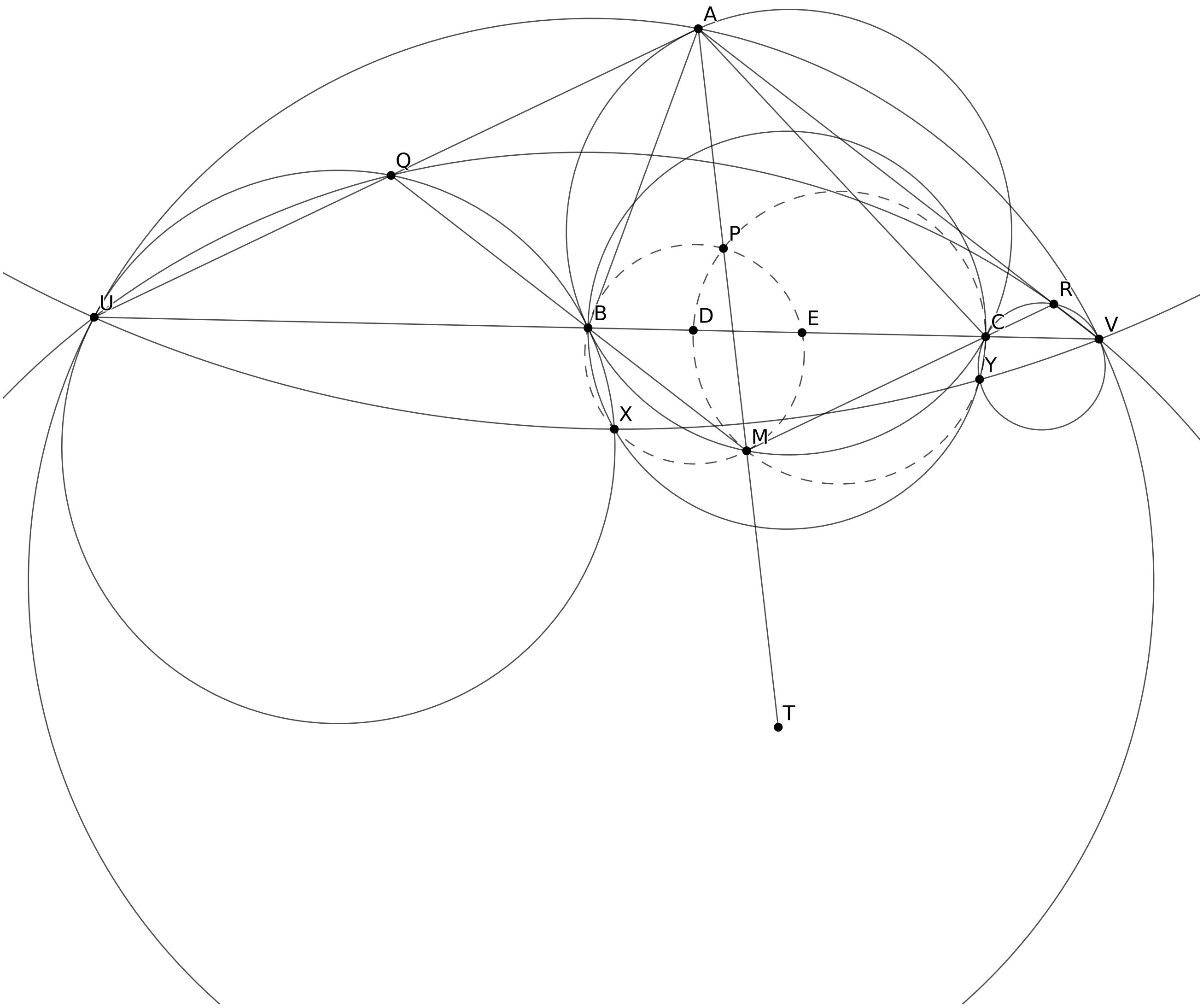

Figure 3

Lemma 5

BXY is cyclic.

Proof. Let BX meet AM at S. And let SC meet $\odot PMC$ and $\odot CYU$ at $Y_1, and Y_2$, so,

$$SY_1 \cdot SC = SM \cdot SP = SX \cdot SB = SC \cdot SY_2$$

which means $Y_1 \equiv Y_2$. Meaning $Y_1 = Y_2 = Y$.

so,

$$SX \cdot SB = SY \cdot SC$$

Lemma 6

$UXYV$ is cyclic.

Proof. the same as before.

Now, if we take the three circles, $\odot AUV, \odot ABC, \odot UXYV$, let the radical center be K. Let KX meet $\odot UXV$ at Y'. then $KY' \cdot KX = KB \cdot KC$. So $BXY'C$ is cyclic, implying that $Y' = Y$.

So, XY passes through the intersection point of BC and the radical axis of $\odot ABC$ and $\odot AUV$.