National Math Camp 2020 Exam 1

August 11, 2020

Problem 1. Let m, n, k be integers such that

$$(m - n)^2 + (n - k)^2 + (k - m)^2 = mnk$$

Prove that $m^3 + n^3 + k^3$ is divisible by $m + n + k + 6$.

Problem 2. Let ABC be a triangle with incenter I and circumcenter O. Let D, E, F be the touch points of the incircle with BC, CA, AB respectively. Then prove that OI is the *euler line*[1] of $\triangle DEF$.

Problem 3. Call a permutation of the numbers $1, 2, 3, \cdots, n$ *stable* if it starts with a 1 and its consecutive terms differ by at most 2. Let T_n be the number of stable permutations. If $1 \leq n \leq 2020$, for how many values of n is T_n divisible by 3?

Problem 4. There is given a trapezoid $ABCD$ in the plane with $BC||AD$. We know that the angle bisectors of the angles of the trapezoid are concurrent at O. Let T be the intersection of the diagonals AC, BD. Let Q be on CD such that $\angle OQD = 90°$.

Prove that if the circumcircle of the triangle OTQ intersects CD again at P then $TP||AD$.

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[1]It's the line joining the orthocenter and the circumcenter of a triangle