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Abstract

We here relate the occurrence of extreme market shares, close to either 0 or 100%, in the media industry to a percolation phenomenon across the social network of customers. We further discuss the possibility of observing self-organized criticality when customers and cinema producers adjust their preferences and the quality of the produced films according to previous experience. Comprehensive computer simulations on square lattices do indeed exhibit self-organized criticality towards the usual percolation threshold and related scaling behaviour.

1. Introduction

Methods borrowed from physics have been recently applied to a wide range of subjects [1–7]. In particular, very simple models were adopted as mathematical metaphors for generic classes of collective phenomena in social systems.

We are proposing here to answer the following set of questions:

- "Why does one observe either hits or failures in certain markets such as toys, gadgets, cinema industries [8], but also in the adoption of technological changes [9], of political and economical measures [10], and in the political arena, rather than a featureless distribution of partial successes ?"

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- A possible answer to the first question being percolation, the next question is “since economic agents are adaptive and profit seeking, should we observe some meta-dynamics driving the market either close to the percolation threshold or away from it?”

The general explanation for the extremely bi-modal distribution of market shares in certain commodities markets is often discussed in evolutionary economics in terms of bounded rationality [9,11,12]: in those cases, decisions are not taken upon complete a priori information, but rather, decision makers are submitted to social influence of earlier adopters. The resulting dynamics is a dynamics of propagation across a social network. But apart from a seminal paper by Föllmer [11], most economics literature about propagation of ideas, or adoption of new products and new techniques is inspired from epidemiology and only considers single traits that propagate like viruses infecting new people on the occasion of random binary encounters in otherwise homogeneous populations [9,12].

In the present study, we wish to take into account the fact that possible adoption of new ideas, products, or technologies concern agents, whose tastes and interests vary across the population and whose mutual influence is a priori pre-determined in a sparse network of social interactions. In terms of physics, the social network of influences is more like a disordered network rather than a well stirred vessel of homogeneous material. A further important notion, often neglected by economists, is the importance of the diversity of economics actors; as suggested by Kirman [13], this notion departs from the standard notion of a “representative” agent.

2. The static case of fixed customer preferences and product quality

We here present the simplest model (to our knowledge) which relates the emergence of collective social/economic phenomena to the existence of a self-organized percolation transition. We use the cinema language of deciding to go and see a given film, but the model can be used for other binary economic, technological or political choices.

Our basic assumption is the existence of a social network which supports information transfer in those cases when not all information is public. Our knowledge of actual social networks is pretty limited and most empirical determinations concern kinship networks [14] which might not be the most relevant for modern commodities markets. But as physicists, we can test whether simple models such as lattices or random graphs display some generic properties which can be of relevance to our problem, with the idea that they are the two extreme cases for possible real social systems.

We then suppose a lattice or a random graph with percolation critical density p_c [15–18]. On each site there is an agent i (“film-goer”) which can communicate information about the film and its quality q to its “nearest neighbours” (agents located on sites to which the present site is joined by a link). The dynamics of the model consists in having each agent i viewing or not a particular film according to the procedure described below.

One time step $t \rightarrow t + 1$ of the model consists in simulating the spread of cinema viewing across the lattice after a small number of initial agents have been informed about the cinema.

Each of the agents i initially informed about the film will decide to go to it, if and only if the quality q (a real number between zero and one) of the film is larger than his/her personal preference p_i, i.e., if $q > p_i$. The agents which decide not to go to the film do not play any further role until the end of this time step.

However, the initial agents which decided to go to the film become themselves sources of information about the film: Now, their first neighbours j are informed about the film and will have to decide (according to the $q > p_j$ criterion) whether to see it or not. One could continue until the procedure stops by itself i.e., until all the neighbours of all the agents which went to the film up to now, either went to the film already or decided already not to go.

Since the agent preferences are frozen, the present model is a classical percolation problem. For instance, if the personal preferences p_i on various sites i take independent random values distributed uniformly between 0 and 1, then the average probability for an agent to go to the film, once one of its neighbours went, is the film quality q. Consequently, if a film happens to have a quality q lower than the percolation threshold p_c, i.e., $q < p_c$, then after a certain short time its diffusion among the public will stop. The film will then be a flop and will not reach any significant percentage of its potential public (which is a fraction q of the lattice).

On the other hand, when the film quality q is larger than p_c (and not too close to p_c), the film will reach most of its potential viewers as the islands of interest will percolate. The film will be viewed by roughly a fraction q of the entire viewers population. Up to here the model predicts the existence of a percolation transition regime for some values of the quality q and preferences p_i.

This is in sharp contrast with the prediction that could be made in the full rational perspective of standard economics: since agents are supposed to possess a full knowledge about the film, they should go to see the films in proportion to q without any threshold effect. But as mentioned above, the standard economics prediction is in contradiction with the observed distribution of hits and flops.

3. Meta-dynamics of preferences and quality

3.1. The model

Since we are considering human economic agents, further interesting elements, missing in traditional percolation theory are introduced: the natural evolution of the tastes of the viewers and the business interest of the film makers, might keep this system close to criticality. Indeed,

- after opportunities during which they went to the cinema, the agents will be more demanding and typically increase their preferences p_i; on the opposite, those who preferred not to go, lower their expectations and preferences. Uninformed agents keep their preferences.

- after hits (resp. flops) the film producers will decrease (resp. increase) the quality q of the offered film(s), in their effort to remain above the threshold while minimizing expenses.

Introducing such local and egoistic reactions, might keep the system globally in a state in which the percolation is neither exceedingly frequent nor exceedingly rare: at the transition point.

3.2. Computer simulations

The simulations were done on square lattices (e.g. assuming the information to spread two-dimensionally, as in a large city; we got similar results on a simple cubic lattice). In our Monte Carlo simulations of $L \times L$ square lattices, $p_c = 0.593$. We restricted ourselves to the simplest dynamics: The quality of the film increased by δq if no cluster spanned from top to bottom, while it decreased by δq otherwise. The viewer's preference p_i, initially distributed randomly between 0 and 1, changed by $\pm\delta p$ depending on whether i went to the film or not. A Leath-type [19] algorithm was used to find which sites were connected to the top (all-occupied) line, and stopped once one site of the cluster reached the bottom line; one such Leath cluster growth corresponds to one time step of the preference and quality dynamics $t \to t+1$, occurring on the occasion of the release of a new film.

The results can be summarized as

- Adaptive film quality: for $\delta p = 0$, $\delta q > 0$, we observed that the quality q moves to the usual percolation threshold; in this limit, the dynamics of our system is reminiscent of self-organization mechanisms arising in thermal critical phenomena [20], where a suitable feedback mechanism may push the temperature towards the critical temperature.
- Adaptive customer preferences: for $\delta p > 0$, $\delta q = 0$, Fig. 1, the p_i distribution drifts towards a single peak centred on the fixed q value, taken equal to 0.5 (no spanning cluster) or 0.593 (some spanning clusters).
- When both δp and δq are positive, p_i and q drift towards $p_c = 0.593$, as observed in Fig. 2, even if the initial q was 0.5. Thus, generalizing invasion percolation, our dynamic percolator shows self-organized criticality: whereas standard percolation is observed at a given threshold of 0.593, the present social percolation model drifts towards this critical point automatically.

More precisely, Fig. 1a shows how the number of sites having preferences p_i different from p_c decreases in time; for smaller systems and longer times ($L = 71$ and 50 million cycles) these numbers even become zero. (The numbers away from p_c go to zero faster if in the case of spanning clusters we continue the simulation until no more neighbours of cinema goers were undecided; in Fig. 1 and most of our calculation we already stopped the simulation when the growing cluster touched the opposite boundary and thus signalled a successful film). Consequently, the number of sites having $p_i = p_c$

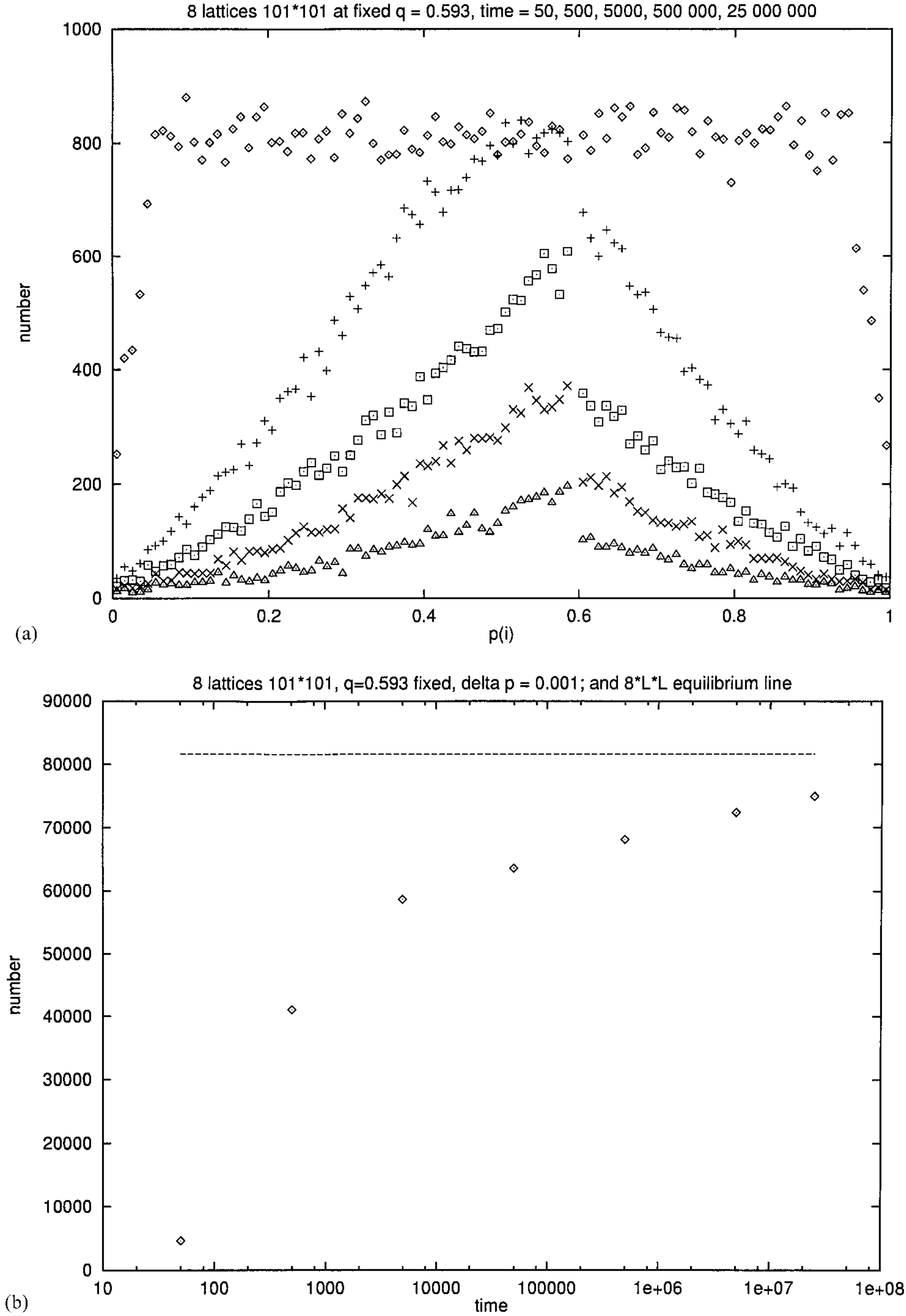
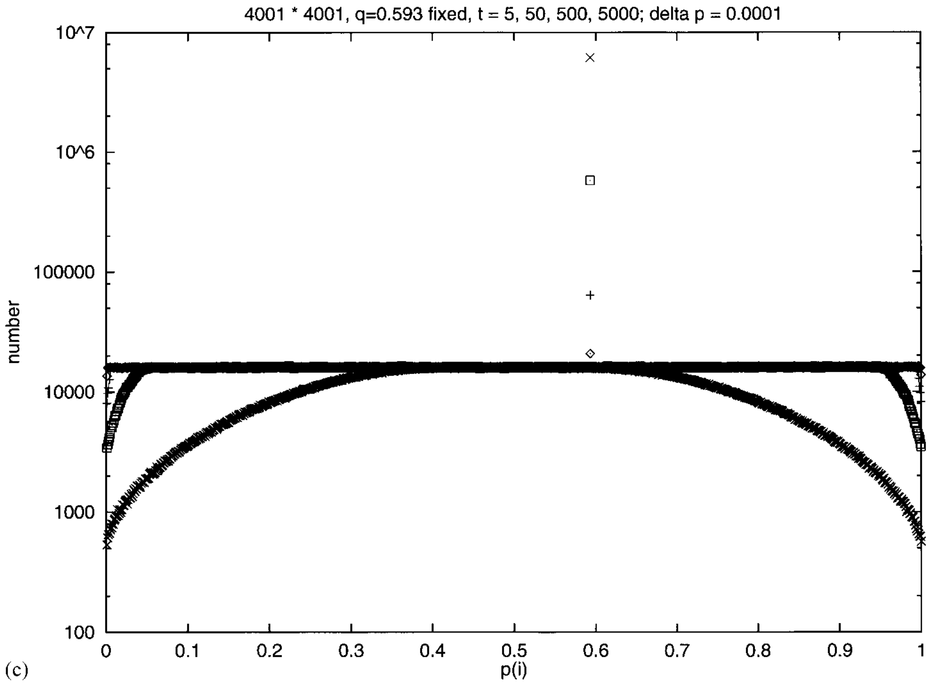


Fig. 1. Time evolution of consumer preferences with fixed film quality. This figure shows how the viewer preferences approach the film quality at the percolation threshold in the simplest case when $\delta q = 0$. (a) time dependence of $N(p_i)$ histogram for p_i different from film quality $q = 0.593$, when the step for change in preference $\delta p = 0.001$; the times are given in the figure headlines: (b) Amplitude of the $N(p_i \simeq 0.593)$ bin versus time in the same simulations as in part a; (c) initial time dependence of $N(p_i)$ (on a logarithmic scale) for $L = 4001$ and $\delta p = 0.0001$.



(c)

Fig. 1. Continued.

goes up, first rapidly and then slowly with time, Fig. 1b. For much larger lattices, Fig. 1c shows that the depletion due to the movement of preferences p_i towards p_c starts near the extreme regions close to $p=0$ and 1.

In Fig. 1, we kept $q=0.593$ constant; if instead q starts from $\frac{1}{2}$ and drifts by $\pm\delta q$ depending on whether or not a spanning cluster exists, then both p_i and q approach p_c=0.593. Fig. 2 shows the distribution of final q for each of eight independent samples of the same size L, as a function of L. Even though one notices some scattering around the average size-dependent value, one can still conclude that the larger the lattice is, the closer q approaches the percolation threshold 0.593. In the trivial limit $\delta p=0$ this value is reached already for smaller lattices, as is seen by the crosses connected by lines in Fig. 2. (After reaching a plateau in q, for $t\delta q \gg 1$, an instability sets in: $q \to 1$; it can be reduced when we start with occupying only one randomly selected site, and it depends on parameters. The metastable plateau before the onset of the instability may come from the quality q moving much faster towards p_c than the individual p_i, since the p_i's change only if touched by a cluster while q always changes [24].)

Fig. 3 represents the observed histogram of N_s, the number of clusters of size s, without the tail at large s coming from the spanning lattices (the number s of cinema visitors is never smaller than L since we start with occupying one full boundary of length L). N_s is shown to follow a power law truncated by finite-size effects.

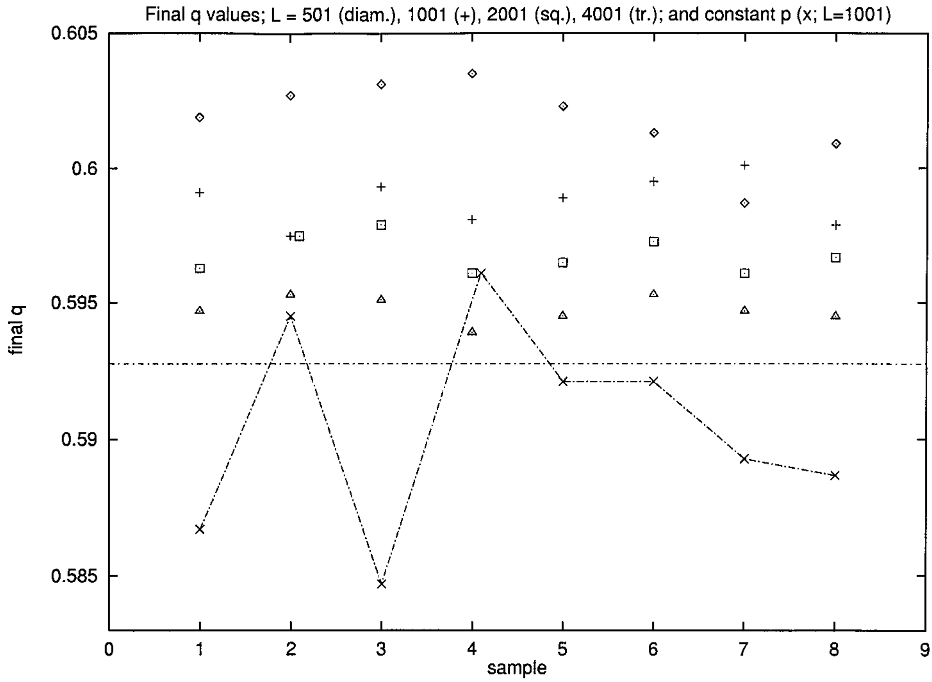


Fig. 2. Effective percolation thresholds, $t \to \infty$, for eight independent samples, with both film quality q and personal preferences p_i varying by ± 0.0001, for $501 \leqslant L \leqslant 4001$; the straight horizontal line gives the known asymptotic percolation threshold. The crosses connected by lines give the thresholds in the limit $\delta p = 0$. This figure shows how for sufficiently large systems the quality and preferences approach the random percolation threshold 0.593, and that the scattering from one sample to the next (i.e., from left to right) does not obscure the systematic downward trend with increasing L.

3.3. Discussion

We have checked that under reasonable assumption concerning consumers behaviour on certain commodities market, percolation is a relevant paradigm to explain the occurrence of double peaked market share distributions. We further verified that for certain types of preferences and quality adjustments, the market "naturally" evolves towards self-organized criticality in the neighbourhood of the percolation threshold. Let us at this stage be more specific about the conditions when the model applies and how it can be used for marketing purposes before going to possible generalizations.

In fact, percolation is not the only possible mechanism to give rise to double peaked market share distributions. One of us (GW in Ref. [10]) has shown that in the case when agents are influenced by the number of their neighbours which take any of the two decisions, similar effects happen due to the presence or absence of seed configurations from which growth can start. The two models differ by their hypotheses and predictions: the Weisbuch and Boudjema model is a model of social influence and the outcome of the adoption process that they described depends upon the initial

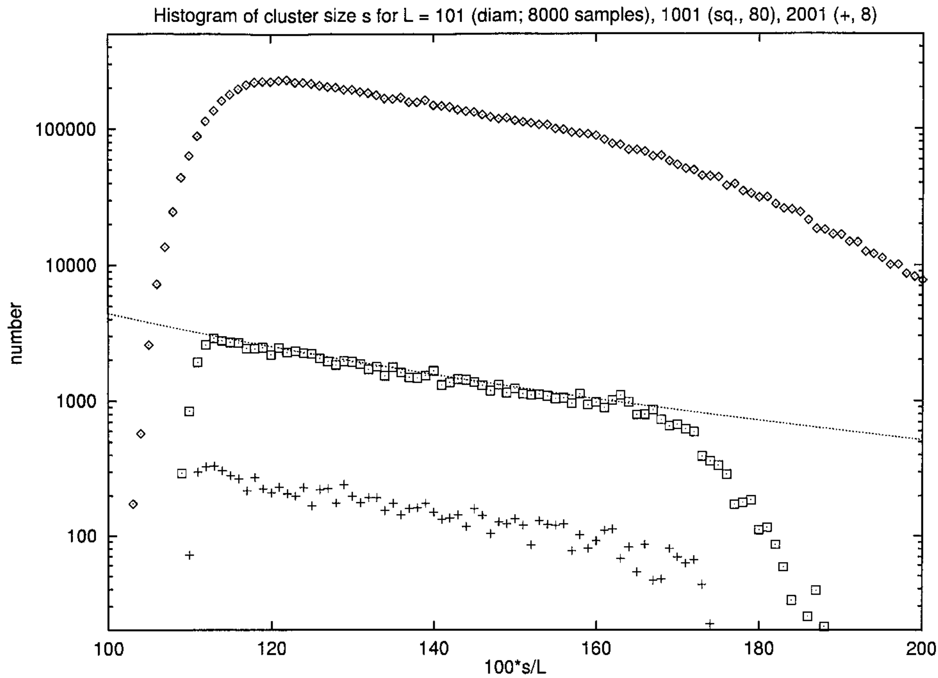


Fig. 3. Number N_s of clusters containing s cinema goers, versus scaled size $100s/L$ for $L = 101$–2001. The straight line has an exponent -3.1. This figure thus shows a not yet explained power law for intermediate cluster sizes.

configurations of adopters; their model is probably appropriate in issues about adoption of new technologies. The present model is a propagation of information model the outcome of which depends on the global density of preferences with respect to the quality of the film; it is applicable to toys, gadgets and media industries. Of course the present version of the model does not take into account an important feature of the real world which is the time overlap of presented films between releases. On the other hand, we conjecture that the phenomenon is generic to all kind of networks, and not limited to the lattices that we used for numerical simulations. Finally, knowledge about the percolation dynamics might be used by producers to dynamically optimize their marketing effort by targeting the diffusion front at any given time, to ensure actual percolation.

The possible existence of self-organized criticality is also discussed with a different perspective by Ishii et al. [21]. These authors claim that self-organized criticality is detrimental to economic agents because of the extra uncertainty that it induces, and that under "normal" social conditions, it should be dealt with by appropriate institutions. Although their views make some sense, we all know of cases when distributions of avalanches are observed in markets. It thus is tempting to extend this "Social Percolation" model to other applications in financial, economic and political systems. In a financial model of herding dynamics [22,23] e.g., q may represent the current attractiveness of a strategy and p_i the pickiness of a trader in adopting/changing strategies. The factors limiting the herding will be mediated by the market impact which lowers the efficiency q of strategies which attracted excessive herding.

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