COMP S265F Design and Analysis of Algorithms
Lab 12: Finite Automata and Regular Expressions – Suggested Solution

Question 1.

(i) For the NFA, the transition table f_ε with the lambda closures is:

f_ε	s	a	b	c	ε	$\lambda(s)$
start	0	$\emptyset$	$\emptyset$	$\emptyset$	$\{1\}$	$\{0,1,2,3,4,5,6,7\}$
	1	$\{1\}$	$\emptyset$	$\emptyset$	$\{2\}$	$\{1,2,3,4,5,6,7\}$
	2	$\emptyset$	$\emptyset$	$\emptyset$	$\{3,4\}$	$\{2,3,4,5,6,7\}$
	3	$\emptyset$	$\{3\}$	$\emptyset$	$\{5\}$	$\{3,5,6,7\}$
	4	$\emptyset$	$\emptyset$	$\{4\}$	$\{5\}$	$\{4,5,6,7\}$
	5	$\emptyset$	$\emptyset$	$\emptyset$	$\{6\}$	$\{5,6,7\}$
	6	$\{6\}$	$\emptyset$	$\emptyset$	$\{7\}$	$\{6,7\}$
final	7	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{7\}$

(ii) We construct the transition table f_D of the DFA, as follows:

- start state $= \lambda(0) = \{0,1,2,3,4,5,6,7\}$ (also a final state, as it contains NFA final state 7)

- $$\begin{aligned} f_D(\{0,1,2,3,4,5,6,7\},a) &= \lambda(\{1,6\}) \\ &= \{1,2,3,4,5,6,7\} \\ f_D(\{0,1,2,3,4,5,6,7\},b) &= \lambda(\{3\}) \\ &= \{3,5,6,7\} \\ f_D(\{0,1,2,3,4,5,6,7\},c) &= \lambda(\{4\}) \\ &= \{4,5,6,7\} \end{aligned}$$

  Therefore, we have the following table:

f_D	s	a	b	c
start, final	$\{0,1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$

- There are three new final states (all containing the NFA final state 7):
  $\{1,2,3,4,5,6,7\}, \{3,5,6,7\}, \{4,5,6,7\}$.

- $$\begin{aligned} f_D(\{1,2,3,4,5,6,7\},a) &= \lambda(\{1,6\}) \\ &= \{1,2,3,4,5,6,7\} \\ f_D(\{1,2,3,4,5,6,7\},b) &= \lambda(\{3\}) \\ &= \{3,5,6,7\} \\ f_D(\{1,2,3,4,5,6,7\},b) &= \lambda(\{4\}) \\ &= \{4,5,6,7\} \end{aligned}$$

  Therefore, we have the following table:

f_D	s	a	b	c
start, final	$\{0,1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$
final	$\{1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$

- $$\begin{aligned} f_D(\{3,5,6,7\},a) &= \lambda(\{6\}) \\ &= \{6,7\} \end{aligned}$$

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\begin{aligned}
f_D(\{3,5,6,7\}, b) &= \lambda(\{3\}) \\
&= \{3,5,6,7\} \\
f_D(\{3,5,6,7\}, c) &= \lambda(\emptyset) \\
&= \emptyset
\end{aligned}
$$

Therefore, we have the following table:

f_D	s	a	b	c
start, final	$\{0,1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$
final	$\{1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$
final	$\{3,5,6,7\}$	$\{6,7\}$	$\{3,5,6,7\}$	$\emptyset$

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\begin{aligned}
f_D(\{4,5,6,7\}, a) &= \lambda(\{6\}) \\
&= \{6,7\} \\
f_D(\{4,5,6,7\}, b) &= \lambda(\emptyset) \\
&= \emptyset \\
f_D(\{4,5,6,7\}, c) &= \lambda(\{4\}) \\
&= \{4,5,6,7\}
\end{aligned}
$$

  Therefore, we have the following table:

f_D	s	a	b	c
start, final	$\{0,1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$
final	$\{1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$
final	$\{3,5,6,7\}$	$\{6,7\}$	$\{3,5,6,7\}$	$\emptyset$
final	$\{4,5,6,7\}$	$\{6,7\}$	$\emptyset$	$\{4,5,6,7\}$

- There are two new states: $\{6,7\}$ (final state as it contains NFA final state 7) and $\emptyset$.

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\begin{aligned}
f_D(\{6,7\}, a) &= \lambda(\{6\}) \\
&= \{6,7\}
\end{aligned}
$$

  Therefore, we have the following table:

f_D	s	a	b	c
start, final	$\{0,1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$
final	$\{1,2,3,4,5,6,7\}$	$\{1,2,3,4,5,6,7\}$	$\{3,5,6,7\}$	$\{4,5,6,7\}$
final	$\{3,5,6,7\}$	$\{6,7\}$	$\{3,5,6,7\}$	$\emptyset$
final	$\{4,5,6,7\}$	$\{6,7\}$	$\emptyset$	$\{4,5,6,7\}$
final	$\{6,7\}$	$\{6,7\}$	$\emptyset$	$\emptyset$
	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

- We rename the six states to $0, 1, \ldots, 5$:

f_D	s	a	b	c
start, final	0	1	2	3
final	1	1	2	3
final	2	4	2	5
final	3	4	5	3
final	4	4	5	5
	5	5	5	5

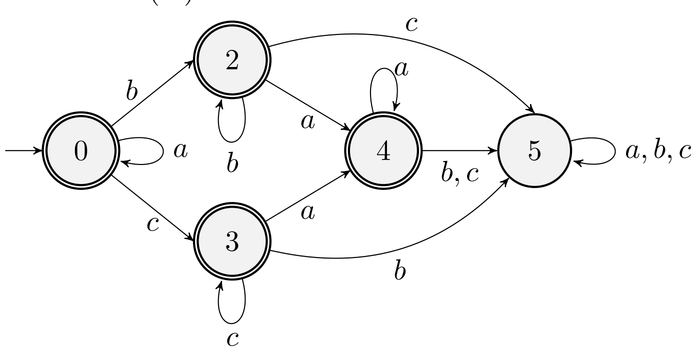
(iii) States 0 and 1 are both final state, and have the same transitions for all inputs.

Therefore, we merge states 0 and 1 to a merged state (state 0).

The new transition table f_D of the DFA is:

f_D	s	a	b	c
start, final	0	**0**	2	3
final	2	4	2	5
final	3	4	5	3
final	4	4	5	5
	5	5	5	5

(iv) The DFA in (iii) is shown below:



Question 2.

(i) Suppose, for the sake of contradiction, that L is regular. Thus, L can be accepted by a DFA with m state for some $m \in \mathbb{N}$.
Consider the string $a^m bc^m$. Since $|a^m bc^m| \geq m$, by the pumping lemma, there are string x, y, z such that

- $a^m bc^m = xyz$
- $|y| > 0$
- $|xy| \leq m$
- $xy^i z \in L$ for any $i \in \mathbb{N}$

As $|xy| \leq m$, x and y contain a's only.
Then, we can pump y to y^i for any $i > 1$, which will increase the number of a's only. Thus, the number of a's will not equal the number of c's, i.e., $xy^i z \notin L$, which is a contradiction. Therefore, L is not regular.

(ii) Suppose, for the sake of contradiction, that M is regular. Thus, M can be accepted by a DFA with k state for some $k \in \mathbb{N}$.
Consider the string $a^k b^k$. Since $|a^k b^k| \geq k$, by the pumping lemma, there are string x, y, z such that

- $a^k b^k = xyz$
- $|y| > 0$
- $|xy| \leq k$
- $xy^i z \in M$ for any $i \in \mathbb{N}$

As $|xy| \leq k$, x and y contain a's only.
Then, we can pump y to y^i for any $i > 1$, which will increase the number of a's only. For sufficiently large i, the difference between the numbers of a's and c's will exceed 10, i.e., $xy^i z \notin M$, which is a contradiction. Therefore, M is not regular.

Question 3. The NFA with ε moves for the regular expression is: