COMP S264 F

Discrete Mathematics

Online Midterm Test

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Question 1 (10 marks)

(a) The following is the truth table of $\neg(r \to \neg q) \vee (p \wedge r)$:

r	q	$r \to \neg q$	$\neg(r \to \neg q)$	p	$r \wedge p$
T	T	F	T	T	T
T	F	T	F	F	F
F	T	T	F	T	F
F	F	T	F	F	F

0

(b) $\neg(r \to \neg q) \vee (p \wedge r) = \neg(\neg r \vee \neg q) \vee (p \wedge r)$ (as $a \to b \equiv \neg a \vee b$).

$= r \wedge q \vee p \wedge r$ ✓   (De Morgan's law).   3

$= r \wedge (q \vee p)$ (Destribute law).

(c) truth table of $r \wedge (q \vee p) \to$

r	q	p	$(q \vee p)$	$r \wedge (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	F	T	F
F	F	F	F	F

0

Question 2 (10 marks)

(a) True

Proof: $\forall x \exists y$ that $x+2y=5$

For any $x \in R$ which means there always exist $y=\frac{5-x}{2}$ ✓ to let the statemet establishd.

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(b) False

Prove by Counterexample.

$\forall x \exists y$ that $x+2y=xy$.

✓✗ 2

Counterexample: when $x=2$. the statement will be $x=0$. it is contradict.

Question 3 (10 marks)

Proof: ① n is an odd integer: let $n=2k+1$ (k is integer).

$3n^2+n+14 = 3(2k+1)^2+2k+1+14 = 12k^2+14k+18 = 2(6k^2+7k+9)$

which is always even. ✓

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② n is an even integer: let $n=2k$ (k is integer).

$3n^2+n+14 = 12k^2+2k+14 = 2(6k^2+k+7)$ ✓ which is always even.

Thus, the statement $3n^2+n+14$ is always even when n is an integer.

Question 4 (10 marks)

Suppose, for the sake of contradiction, that there no exist a box which contains 12 or more balls.

Then: if we just give each boxes 11 balls, which is the max number in the contradiction

Thus, the total ball of 9 boxes will be $11 \times 9 = 99$ ✓ 10

Note that $99 < 100$. which means it is impossible of the contradiction.

Therefor which contradicts that there must have some box contains 12 or more balls.

Question 5 (10 marks)

Proof:

Basis step, $n=1$: no need to be split (times $= 0$).

$n=2$: The chocolate need takes $2-1=1$ splits to split into 2 pieces.

Inductive step: Consider a chocolate bar of n squares can be split into n pieces with $(n-1)$ splits.

when there are $(n+1)$ squares in this chocolate bar

1 2 3 . . . . . . n $n+1$

the former n squares can be divided into n pieces with $(n-1)$ splits. 7

and the $(n+1)$ squre need to be divided with 1 more split.

which means for $(n+1)$ squres chocolate bar. it can be split into $n+1$ pieces with $(n-1+1) = n$ steps.

By the principle of mathematical induction. for any positive integer n,

if the chocolate bar is split into the n square pieces, It takes $P(n) = n-1$ steps.

Question 6 (10 marks)

(a) prove:

$x \in A-(B\cap C) \Rightarrow (x\in A)\wedge(x\notin B-C).$

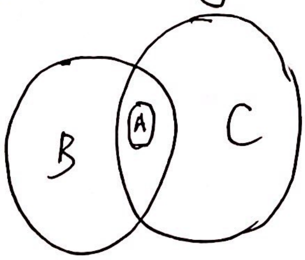
$\Rightarrow (x\in A)\wedge(x\notin B).\wedge(x\in A)\wedge(x\notin C).$ 1

$\Rightarrow x\in(A-C)(A-B).$

(b). Prove: From the Venn diagram, we can know that:

① B and C musthave intersection.

② $A \subseteq B\cap C$.



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Question 7 (10 marks)

(1) injective $x, y \in N$

let $f(x) = f(y) \Rightarrow x^2 - 1 = y^2 - 1$

$x^2 = y^2$. $x = \pm y$.

So $f(x)$ is not a injective function.

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so f is not one-to-one.

(2) surjective :

let $b \in N = f(a) = a^2 - 1$

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$a^2 = b + 1$

$a = \pm\sqrt{b+1}$. So f is not a surjective function).

Question 8 (10 marks).

(a). $(f \circ g)(n) = (2n+1)^2 - 4(2n+1) + 1$

$= 4n^2 + 4n + 1 - 8n - 4 + 1 = 4n^2 - 4n - 2$.

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