COMP S264F Discrete Mathematics
Online Mid-term Test
Nov 24, 2020 (Tue) 14:05 - 15:35

You are required to write down your answers **with steps** on papers (**and write your name and student ID on the first page**), take photos on them, convert them to a PDF file, and then submit to OLE. You may use the mobile app CamScanner. **Note that computer-typed answers are not accepted.**

Question 1 (10 marks). Consider the proposition

$$\neg(r \to \neg q) \vee (p \wedge r)$$

(a) Write a truth table for the proposition. [5]

(b) Simplify the proposition. [3]

(c) Write a truth table for your simplified proposition in (b). [2]

Question 2 (10 marks). Determine the truth value of the proposition

$$\forall x \; \exists y \; P(x, y)$$

for each of the following cases. Justify your answer.

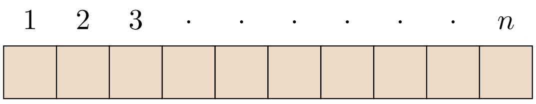
(a) $P(x, y)$: x and y are real numbers such that $x + 2y = 5$. [5]

(b) $P(x, y)$: x and y are real numbers such that $x + 2y = xy$. [5]

Question 3 (10 marks). Prove that if n is an integer, then $3n^2 + n + 14$ is even.

Question 4 (10 marks). Use proof by contradiction to show that if 100 balls are placed in 9 boxes, then some box contains 12 or more balls.

Question 5 (10 marks). Consider a chocolate bar of n squares arranged as a line:



Use mathematical induction to show that no matter how the chocolate bar is split into the n square pieces, it takes $P(n) = n - 1$ splits.

Question 6 (20 marks). Let A, B, C be any sets.

(a) Prove or disprove that $A - (B \cap C) = (A - B) \cap (A - C)$. [10]

(b) Prove that $A \subseteq B$ and $A \subseteq C$ if and only if $A \subseteq B \cap C$. [10]

Question 7 (10 marks). Consider the following function.

$$f : \mathbb{N} \to \mathbb{N} \text{ such that } f(x) = x^2 - 1$$

(a) Determine whether the function f is one-to-one. Justify your answer. [5]

(b) Determine whether the function f is onto. Justify your answer. [5]

Question 8 (10 marks).

(a) Find $(f \circ g)(n)$ for the following functions from $\mathbb{Z}$ to $\mathbb{Z}$: [5]

$$f(n) = n^2 - 4n + 1 \quad \text{and} \quad g(n) = 2n + 1$$

(b) Suppose $g : A \to B$ and $f : B \to C$. Prove or disprove that if $f \circ g$ is onto, then g is also onto. [5]

Question 9 (10 marks). Identify any error with justifications in the following arguments that supposedly shows that $1 = 2$.

1. Let y be a positive integer.
2. Let $x = y$.
3. $x^2 = xy$ (multiply both sides by x)
4. $x^2 - y^2 = xy - y^2$ (subtract both sides by y^2)
5. $(x - y)(x + y) = y(x - y)$ (factor both sides)
6. $x + y = y$ (divide both sides by $x - y$)
7. $2y = y$ (as $x = y$)
8. $2 = 1$ (divide both sides by y)

[End of Paper]