COMP S264F

Discrete Mathematics

Tutorial 9. Combinatorics

Question 2: Find the number of strings that can be formed by ordering the letters in the String "ETHANE"

(a) In the 6-letter string, there are two occurences of "E". Hence, there are $\frac{6!}{2!} = 360$ ways to order the letters.

(b). We consider "EE" as one unit, and there are 5 distinct units to permute and thus $5! = 120$ ways to order the letters.

(c). There are 7 different lengths of the formed string

Length 0: 1   Length 1: 5.   Length 2: ① contains at most one E: $C(5,2) \times 2! = 10 \times 2 = 20$. ② contains two E: 1. } 21

Length 3: Contains at most one E: $C(5,3) \times 3! = 60$.
Contains two E: $C^1_4 \times \underbrace{\frac{3!}{2!}}_{\text{repetition}} = 4 \times 3 = 12$ } 72

Length 4: Contains at most one E: $C(5,4) \cdot 4! = 5 \times 24 = 120$.
Contains two E: $C(4,2) \times \frac{4!}{2!} = 6 \times 12 = 72$ } 192

Length 5: contains at most one E: $C(5,5) \times 5! = 4 \times 60 = 240$.
Contains two E: $C(4,3) \times \frac{5!}{2!} = 4 \times 60 = 240$.

Length 6: $\frac{6!}{2!} = 360$.   sum = 1011.

(d). the formed string must be in the form → $X_1 N X_2 A X_3$, where X_1, X_2, X_3 are three strings (which can be null) and they are formed togethers by the remaining 4 letters. E, T, H, E.

two steps:

① Assign the 4 letters * to the three strings X_1, X_2, X_3.

e.g., ***||* , *|*|** ....

the number of ways to do step 1 is. $C(4+3-1, 4) = C(6,4) = 15$.

② then permute the 4 letters.

$$\frac{4!}{2!} = 12.$$

by product rule, the number of ways is. $12 \times 15 = 180$

Question 3

(a) transfer $\Downarrow$

select 10 balls from three different number of color balls

$\Downarrow$

$* * * | * * * * * * | *$ arrange 10 balls with different colors.

$\therefore$ $C(10+2, 2) = 66$ ways.

(b). Let A be the set of ways to select 10 balls with at least one red ball.

So: $|A| = |U| - |\bar{A}|$

$= 66 - C(10+1, 1) = 11 = 55$ · 66-

(c). $C(9+1, 1) = 10$ ways. (d)(e).

Question 4. Find the number of non-negative integer solutions to the equation $x+y+z=18$.

(a). this problem $\Downarrow$

assigning 18 balls to 3 buckets x, y, z.

$C(18+2, 2) = 190$.

(b). $x \geq 3$, $y \geq 2$, $z \geq 1$. "at least", is pretty easy.

Just means that. if we assigned 3 balls in X, 2 balls in y 1 ball in Z, then. we can assigning 18 − 6 = 12 balls as we want: any

$C(12+2, 2) = 91$.

(c). $\bar{A}$ $x<7$, $\bar{B}$ $y<8$, $\bar{C}$ $z<9$. we caculate "at least" A $x \geq 7$, $y \geq 8$, $z \geq 9$.

$|\bar{A} \cap \bar{B} \cap \bar{C}| = |\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C|$. $|U| = 190$. (in (a)).

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

$= C(11+2, 2) + C(10+2, 2) + C(9+2, 2) - C(3+2, 2) - C(2+2, 2) - C(1+2, 2) + 0$

$= 10$.