

# COMP S264F Discrete Mathematics

## Tutorial 1: Logic (1)

In the following questions, assume  $p$ ,  $q$  and  $r$  are propositions.

**Question 1.** Assuming that  $p$  is true, and  $q$  is false, and  $r$  is true, find the truth value of each proposition. Note that logical operators are evaluated in the order:  $\neg$ ,  $\wedge$ ,  $\vee$ ,  $(\rightarrow, \leftrightarrow)$ .

- (a)  $p \wedge q \rightarrow r$
- (b)  $p \vee q \rightarrow \neg r$
- (c)  $p \wedge (q \rightarrow r)$
- (d)  $p \leftrightarrow (q \rightarrow r)$

**Question 2.** Write the truth table of each proposition in Question 1.

**Question 3.** Identify which of the following propositions are logically equivalent.

- *Implication:*  $p \rightarrow q$
- *Converse:*  $q \rightarrow p$
- *Contrapositive:*  $\neg q \rightarrow \neg p$
- *Inverse:*  $\neg p \rightarrow \neg q$

**Question 4.** Without the use of truth table, determine whether each of the following logical equivalences is true or not. Show your answer clearly with appropriate steps.

- (a)  $\neg(\neg p \wedge q) \wedge (p \vee q) \equiv p$
- (b)  $(p \wedge \neg q) \rightarrow (q \rightarrow \neg r) \equiv (\neg p \vee q) \vee \neg r$

**Question 5.** Determine whether each of the following propositions is a tautology or not.

- (a)  $p \otimes p$
- (b)  $p \otimes \neg p$
- (c)  $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$
- (d)  $[p \wedge (p \rightarrow q)] \rightarrow q$
- (e)  $[(p \vee q) \wedge \neg p] \rightarrow q$

**Question 6.** Each of the following four cards has a letter on one side and a number on the other side. Name those (and only those) cards which have to be turned over so that we can determine the truth value of the statement

“If a card has a vowel on one side, it has an even number on the other side.”.

|        |        |        |        |
|--------|--------|--------|--------|
| +----+ | +----+ | +----+ | +----+ |
| A      | B      | 4      | 7      |
| +----+ | +----+ | +----+ | +----+ |