

COMP S264F Discrete Mathematics
Tutorial 5: Set Theory (2)

Question 1. Give an example of sets A, B and C for the followings.

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| (a) $A \cup C = B \cup C$ but $A \neq B$. | (c) $A - C = B - C$ but $A \neq B$. |
| (b) $A \cap C = B \cap C$ but $A \neq B$. | (d) $(A - B) - C \neq A - (B - C)$ |

Question 2. Simplify the following sets. Hence, draw the Venn diagram.

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| (a) $A \cap (B - A)$ | (c) $(A - \overline{B}) \cup (A - \overline{C})$ |
| (b) $A \cap \overline{(A \cap B)}$ | (d) $(\overline{A} - B) \cap \overline{(A \cup \overline{C})}$ |

Question 3. Let $A = \{1, 2\}$ and $B = \{x, y, z\}$. Find the following sets and hence their cardinality.

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| (a) $A \times A$ | (c) $B \times A$ |
| (b) $A \times B$ | (d) $B \times B$ |

Question 4. Consider the set $A = \{a, \emptyset\}$.

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| (a) Find $P(A)$. | | | |
| (b) Answer true or false for the followings. | | | |
| (i) $a \in A$ | (v) $\{a\} \in A$ | (ix) $\emptyset \in A$ | (xiii) $\{\emptyset\} \in A$ |
| (ii) $a \subset A$ | (vi) $\{a\} \subset A$ | (x) $\emptyset \subset A$ | (xiv) $\{\emptyset\} \subset A$ |
| (iii) $a \in P(A)$ | (vii) $\{a\} \in P(A)$ | (xi) $\emptyset \in P(A)$ | (xv) $\{\emptyset\} \in P(A)$ |
| (iv) $a \subset P(A)$ | (viii) $\{a\} \subset P(A)$ | (xii) $\emptyset \subset P(A)$ | (xvi) $\{\emptyset\} \subset P(A)$ |

Question 5. Prove the followings for all sets A, B and C .

- (a) If $A \subseteq B$, then $A \cap C \subseteq B \cap C$.
- (b) If $P(A) \subseteq P(B)$, then $A \subseteq B$.
- (c) If $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$.
- (d) $A \subseteq C$ and $B \subseteq C$ if and only if $A \cup B \subseteq C$.