COMP S264F Discrete Mathematics
Specimen

You are required to write down your answers on papers, take photos on them, convert them to a PDF file, and then submit on OLE. You may use the mobile app CamScanner.
Computer-typed answers will not be accepted.

Question 1 (10 marks).

(a) Write a truth table for the proposition $p \to (q \to \neg p)$. [3]

(b) Simplify $(p \to \neg q) \vee (p \wedge q)$. [3]

(c) Is $p \to (q \to \neg p) \equiv (p \to \neg q) \vee (p \wedge q)$? State your reason. [1]

(d) Show that $\neg\forall x\ (P(x) \to Q(x))$ and $\exists x\ (P(x) \wedge \neg Q(x))$ are logically equivalent. [3]

Question 2 (10 marks). Use proof by contradiction to show that for all $x \in \mathbb{R}$, if x^2 is irrational, then x is irrational.

Question 3 (15 marks). Consider the following functions $f : X \to Y$ and $g : Y \to X$.

$$f(x) = \frac{1}{x-2} \qquad \text{and} \qquad g(x) = \frac{1}{x} + 2$$

(a) Find $f \circ g(x)$ and $g \circ f(x)$. [4]

(b) Explain why X and Y cannot be the set of real numbers $\mathbb{R}$? [2]

(c) Let $X = \mathbb{R} - \{2\}$ and $Y = \mathbb{R} - \{0\}$.

(i) Show that f is bijective. [7]

(ii) Is the inverse of f well-defined? If yes, show the inverse of f. [2]

Question 4 (5 marks). What is the smallest number of students in a class to guarantee that at least four students were born on the same day of the week? Justify your answer.

Question 5 (10 marks). Consider a pool of \$20, \$50, \$100, and \$500 notes.

(a) How many ways are there to draw 8 notes of any of the four types? [2]

(b) How many ways are there to draw 8 notes if at least one \$20 note has to be drawn? [3]

(c) How many ways are there to draw 8 notes if notes of at least 2 types must be drawn? [5]

Question 6 (10 marks). Give a combinatorial argument to prove that

$$6 \cdot C(15, 6) = 15 \cdot C(14, 5)\ .$$

Note that a non-combinatorial proof will receive 0 marks.

Question 7 (10 marks). Use mathematical induction to prove that for any integer $n \geq 4$, $2^n < n!$. Note that this inequality is not true for $n = 1, 2, 3$.

Question 8 (10 marks). Let A, B, C be sets.

(a) Simplify $(A \cap \overline{B}) \cup (A \cap \overline{C})$. Hence, draw its Venn diagram. [5]

(b) Show that $(A - B) - C \subseteq A - C$. [5]

Question 9 (10 marks). The probability that A attends the lecture is $\frac{1}{4}$ and the probability that B attends the lecture is $\frac{2}{5}$. Find the probability that at least one of them attends the lecture, i.e., A or B (or both) attends the lecture.

Question 10 (10 marks). 25% of the students failed discrete mathematics (denoted as event M), 15% failed basic programming (denoted as event B), and 10% failed both discrete mathematics and basic programming. A student is selected at random.

(a) If the student failed basic programming, find the probability that the student also failed discrete mathematics. [2]

(b) If the student did not fail discrete mathematics, find the probability that the student failed basic programming. [3]

(c) Find the probability that the student failed discrete mathematics or basic programming. [3]

(d) Find the probability that the student failed neither discrete mathematics nor basic programming. [2]

[End of Paper]