COMP S265F Design and Analysis of Algorithms
Online Test
Apr 23 (Fri), 2021
14:00 - 15:45

This test contains 8 questions. You can print this test paper and work on it, or write your answers on blank papers. Show the steps clearly. Please take photos on your work, convert them to a PDF file, and then submit on OLE. You may use the mobile app "Adobe Scan". **Note that computer-typed answers are not accepted.**

Question 1 (10 marks).

(a) Find the time complexity of the following algorithm: [5]

```
for i in range(n):
    x = 0
    for j in range(i, i + 3):
        x += j
    print(x)
```

(b) Let c be a positive constant. Suppose an algorithm takes $T(n)$ time on an input of size n, where

$$T(n) = \begin{cases} T(n-1) + c & \text{if } n > 1, \\ c & \text{if } n = 1. \end{cases}$$

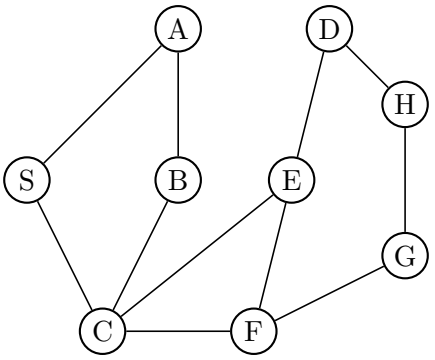
Find the time complexity $T(n)$ of the algorithm. [5]

Question 2 (15 marks). Given a set of characters A, B, C, D, E, F, G and their corresponding frequencies.

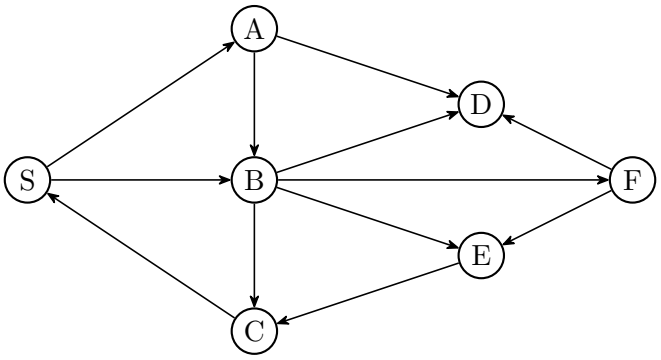
Character	A	B	C	D	E	F	G
Frequency	82	15	27	19	56	10	41

Construct the Huffman code for these characters. Show the merging steps clearly and draw the code tree. Hence, compute the average character length of the Huffman code.

Question 3 (15 marks). Perform a Breadth-First Search (BFS) on the following undirected graph, using vertex S as the source. List the vertices in the visited order of BFS, and show for each vertex v, its distance $dist[v]$ from S, and its breadth-first tree parent $\pi[v]$. Then, draw the breadth-first tree obtained.



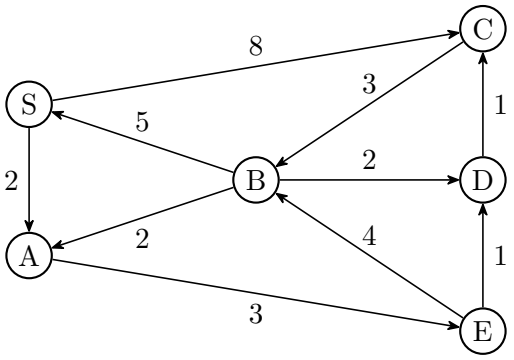
Question 4 (15 marks). Perform a Depth-First Search (DFS) on the following directed graph, using vertex S as the source.



(a) List the vertices in the discovered order of DFS, and show for each vertex v, its discovery time $d[v]$, finish time $f[v]$, and its depth-first tree parent $\pi[v]$ in the DFS. Then, draw the depth-first tree obtained. [9]

(b) Show the classification of each edge (tree edge, back edge, forward edge, cross edge). [6]

Question 5 (15 marks). Apply the Dijkstra's algorithm on the following weighted directed graph, using vertex S as the source.



Show for each vertex v, its $d[v]$ values and its parent $\pi[v]$ in the Shortest Path Subtree P after each iteration in the Dijkstra's algorithm, and then draw the Shortest Path Tree obtained.

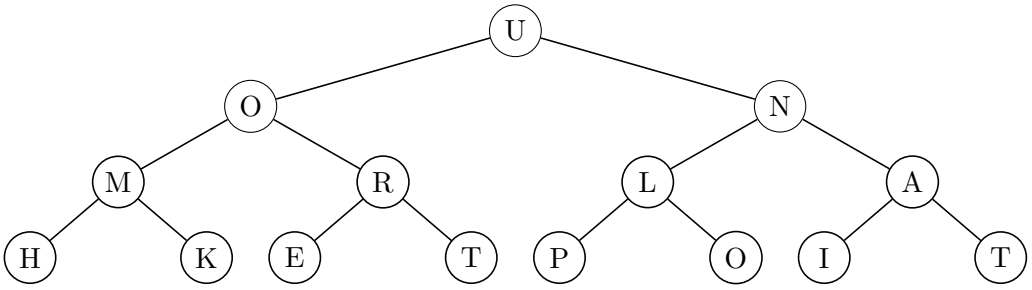
Question 6 (10 marks). Given the following definition of the class `Node`:

```
class Node:
    def __init__(self, label, left, right):
        self.label = label
        self.left = left
        self.right = right
```

Consider the following algorithm in Python that works on a complete binary tree rooted at `node`:

```
def func(node):
    if node == None:
        return
    else:
        func(node.left)
        func(node.right)
        print(node.label, end=" ")
```

(a) Given the following complete binary tree, write the output produced by the algorithm on the root I. [4]



(b) Find the time complexity of the algorithm. [6]

Question 7 (10 marks). Consider the following algorithm in Python that returns a list X such that for any $0 \leq i \leq n-1$, the entry $X[i]$ stores the sum of $0+1+2+\cdots+i$:

```
1  def func(n):
2      X = []
3      for i in range(n):
4          if ①_____:
5              X.append(②_____)
6          else:
7              X.append(X[③_____] + ④_____)
8      return X
```

(a) Fill in the missing underlined parts of the algorithm. [4]

(b) Use mathematical induction to prove the correctness of the algorithm. [6]

Question 8 (10 marks). Given an unweighted graph $G = (V, E)$, two vertices s and v, a distance $d_l \geq 0$, design an algorithm to return the shortest path distance d from vertex s to vertex v, if the path exists and it is within distance d_l; return -1 otherwise.

[End of Paper]