COMP S264F Discrete Mathematics
Tutorial 8: Basics of Counting

Question 1. A committee composed of Brown, Cony, Sally and Leonard is going to select two officials, namely, president and secretary. Find the number of selections in which

(a) Brown is the president.

(b) Leonard is not an official.

(c) Cony is the secretary or not an official.

(d) Sally is an official and Cony is not the president.

Question 2. Edward lives in Tseung Kwan O and studies in OUHK. He interchanges at Kwun Tong on the way to/from school. Suppose there are *four* routes he travels between his home and Kwun Tong, and *three* routes between Kwun Tong and OUHK. Find the number of ways that he can travel

(a) from home to OUHK via Kwun Tong.

(b) round trip from home to OUHK via Kwun Tong.

(c) round trip from home to OUHK via Kwun Tong but without using a route more than once.

Question 3. There are 70 students in this class. Among 53 students who own game console (PS5, NS, or both), there are 41 who own PS5 and 26 who own NS. Find the number of students who

(a) do not own any game console.

(b) own both consoles.

(c) own PS5 only.

(d) own NS only.

Question 4. In a group of 191 computing students,

- 10 are studying cryptography, machine learning and database;
- 36 are studying cryptography and machine learning;
- 20 are studying cryptography and database;
- 18 are studying machine learning and database;
- 65 are studying cryptography;
- 76 are studying machine learning; and
- 63 are studying database.

Find the number of students who are not studying any one of the three topics.

Question 5. Find the number of integers between 1 and 10000 (inclusive) that are multiples of 3 or 5 or 11.

Question 6. A student must take five courses from three schools. Numerous courses are offered by the schools, but the student cannot take more than two courses under the same school. No course is jointly offered by two or more schools.

(a) Show that each student will take at least two courses offered by the same school.

(b) Show that each student will take at least one courses offered by each of the three schools.

Question 7. A standard deck of 52 cards has 13 kinds of cards, with four cards of each kind, one in each of the four suits: hearts, diamonds, spades and clubs. How many cards must be selected from a standard deck of 52 cards to guarantee that

(a) at least three cards of the same suit are selected?

(b) at least three hearts are selected?

Question 8. An inventory consists of a list of 89 items, each marked "available" or "unavailable." There are 50 available items. Show that there are at least two available items in the list exactly nine items apart. (For example, available items at positions 13 and 22, or positions 69 and 78 satisfy the condition.)

Question 9. Consider an equilateral triangle with sides of length 1 cm. Suppose points are arbitrarily placed inside the triangle. Determine the least number of points in the triangle such that there exist two points whose distance is at most 0.5 cm.

Question 10. The Internet is made up of interconnected physical networks of computers. Each computer (or more precisely, each network connection of a computer) is assigned an Internet address. In Version 4 of the Internet Protocol (IPv4), an address is a string of 32 bits. It begins with a network number (*netid*). The *netid* is followed by a host number (*hostid*), which identifies a computer as a member of a particular network. Three forms of addresses are used, with different numbers of bits used for *netids* and *hostids*.

- **Class A** addresses, used for the largest networks, consist of a prefix 0, followed by a 7-bit *netid* and a 24-bit *hostid*.
- **Class B** addresses, used for medium-sized networks, consist of a prefix 10, followed by a 14-bit *netid* and a 16-bit *hostid*.
- **Class C** addresses, used for the smallest networks, consist of a prefix 110, followed by a 21-bit *netid* and an 8-bit *hostid*.

Bit Number	0	1	2	8	16	24	31
Class A	0	netid		hostid			
Class B	1	0	netid		hostid		
Class C	1	1	0	netid		hostid	

Table 1: IPv4 address.

There are several restrictions on addresses because of special usages: 1111111 is not available as the *netid* of a **Class A** network, and the *hostids* consisting of all 0s and all 1s are not available for use in any classes. A computer on the Internet has either a Class A, a Class B, or a Class C address. Find the number of IPv4 addresses available for computers on the Internet.

Question 11. A password for an e-commerce system must have at least 8, but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters $*$, $>$, $<$, !, $+$, and $=$.

(a) How many different passwords are available for this system?

(b) A new rule is applied: the password must contain at least one of the special characters. How many different passwords are available for this system now?

(c) There is a hacker trying to hack the system. Assume it takes one nanosecond (1×10^{-9} seconds) for him to check each possible password. Determine the longest possible time (i.e., the worst case of trying all the combinations) for him to successfully hack into the system in (a) and (b).

Question 12. The North American numbering plan (NANP) specifies the format of telephone numbers in the U.S., Canada, and many other parts of North America. A telephone number in this plan consists of 10 digits, which are split into a three-digit area code, a three-digit office code, and a four-digit station code. Because of signaling considerations, there are certain restrictions on some of these digits. To specify the allowable format,

- let X denote a digit that can take any of the values 0 through 9,
- let N denote a digit that can take any of the values 2 through 9, and
- let Y denote a digit that must be a 0 or a 1.

In the **old plan**, the formats of the area code, office code, and station code are *NYX*, *NNX*, and *XXXX*, respectively, so that telephone numbers had the form *NYX-NNX-XXXX*.
In the **new plan**, the formats of these codes are *NXX*, *NXX*, and *XXXX*, respectively, so that telephone numbers have the form *NXX-NXX-XXXX*.

(a) How many different North American telephone numbers are possible under the **old plan**?

(b) How many different North American telephone numbers are possible under the **new plan**?

(c) Find the least number of area codes needed to guarantee that the 25 million $(25,000,000)$ phones in a state can be assigned distinct 10-digit telephone numbers under the **new plan**.