COMP S264F

Discrete Mathematics Tutorial 11

Conditional Probability, Random Variables.

Question 1:

Let E be the event that the family has two boys, and let F be the event that the family has at least one boy, then:

$$E = \{BB\} \quad . \quad F = \{BG, GB, BB\} \quad E \cap F = \{BB\}.$$

$$P(F) = \frac{|F|}{|U|} = \frac{3}{4} . \quad P(E \cap F) = \frac{|E \cap F|}{|U|} = \frac{1}{4}$$

$$\therefore P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$$

Question 4: (random variable).

Let X be the random variable that equals the number of people who recieve the correct hat from the checker.

let X_i be the random variable with $X_i = 1$ if the i-th person receives the correct hat and $X_i = 0$ otherwise.

then:

$$X = X_1 + X_2 + X_3 + \cdots + X_n . \Rightarrow \text{for all } i,\ P(X_i = 1) = \frac{1}{n}$$

thus $E(X_i) = 1 \cdot P(X_i = 1) + 0 \cdot P(X_i = 0) = 1 \cdot \frac{1}{n} + 0 = \frac{1}{n}$.

by the linearity of expectations, we have: $E(X) = E(X_1) + E(X_2) + \cdots + E(X_n) = n \cdot \frac{1}{n} = 1.$

Question 5:

$$E\Big(\sum_{1 \le i < j \le n} I_{ij}\Big) = \sum_{1 \le i < j \le n} E(I_{ij}) = \frac{n(n-1)}{2} \cdot \frac{1}{2} = \frac{n(n-1)}{4}.$$

swapped.

Question 6:

		X	Y	XY
H	H	2	0	0
H	T	1	1	1
T	H	1	1	1
T	T	0	2	0.

$P(X) = 1$

$P(Y) = 1.$

$P(XY) = 1 \times \frac{1}{4} + 1 \times \frac{1}{4} = \frac{1}{2}$

$\therefore P(X) \cdot P(Y) \neq P(XY).$

$\therefore X, Y$ are not independent random variables.