# COMP S264F Discrete Mathematics
# Tutorial 5: Set Theory (2) – Suggested Solution

**Question 1.** There are many possible combinations. Here are some suggestions.

(a) $A = \{1\}$
$B = \{2\}$
$C = \{1, 2\}$

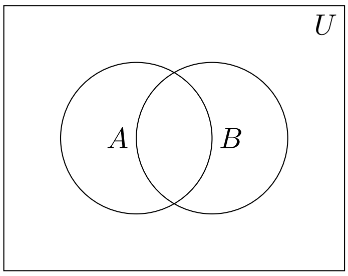
(b) $A = \{1\}$
$B = \{2\}$
$C = \{3\}$

(c) $A = \{1\}$
$B = \{2\}$
$C = \{1, 2\}$

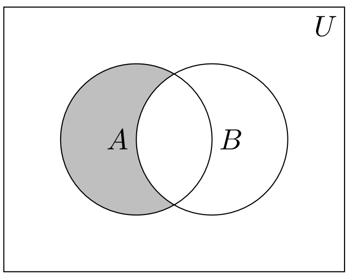
(d) $A = \{1, 2, 3\}$
$B = \{1, 2\}$
$C = \{2, 3\}$

**Question 2.**

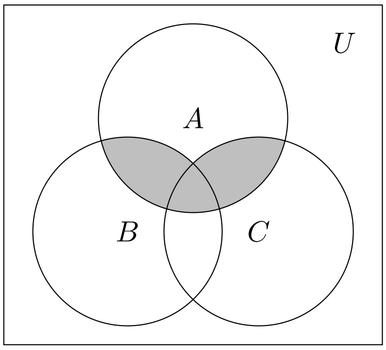
(a)
$$\begin{aligned} A \cap (B - A) &= A \cap (B \cap \overline{A}) \\ &= A \cap \overline{A} \cap B \\ &= \varnothing \cap B \\ &= \varnothing \end{aligned}$$



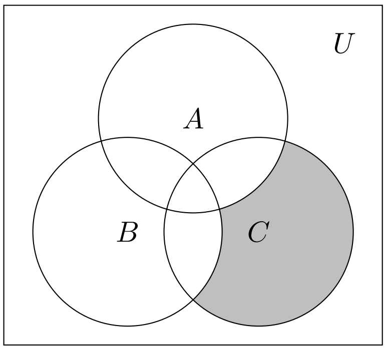

(b)
$$\begin{aligned} A \cap \overline{(A \cap B)} &= A \cap (\overline{A} \cup \overline{B}) && \text{(by De Morgan's law)} \\ &= (A \cap \overline{A}) \cup (A \cap \overline{B}) && \text{(by distributive law)} \\ &= \varnothing \cup (A \cap \overline{B}) \\ &= A \cap \overline{B} \\ &= A - B \end{aligned}$$



(c) $(A - \overline{B}) \cup (A - \overline{C}) = (A \cap B) \cup (A \cap C)$
$= A \cap (B \cup C)$ (by distributive law)



(d) $(\overline{A} - B) \cap \overline{(A \cup \overline{C})} = (\overline{A} \cap \overline{B}) \cap \overline{(A \cup \overline{C})}$
$= (\overline{A} \cap \overline{B}) \cap (\overline{A} \cap C)$ (by De Morgan's law)
$= \overline{A} \cap \overline{A} \cap \overline{B} \cap C$
$= \overline{A} \cap \overline{B} \cap C$



**Question 3.**

(a) $A \times A = \{(1,1),(1,2)\ ,\ (2,1),(2,2)\}$
Thus, $|A \times A| = 4$.

(b) $A \times B = \{(1,x),(1,y),(1,z)\ ,\ (2,x),(2,y),(2,z)\}$
Thus, $|A \times B| = 6$.

(c) $B \times A = \{(x,1),(x,2)\ ,\ (y,1),(y,2)\ ,\ (z,1),(z,2)\}$
Thus, $|B \times A| = 6$.

(d) $B \times B = \{(x,x),(x,y),(x,z)\ ,\ (y,x),(y,y),(y,z)\ ,\ (z,x),(z,y),(z,z)\}$
Thus, $|B \times B| = 9$.

**Question 4.**

(a) $P(A) = \{\varnothing\ ,\ \{a\},\{\varnothing\}\ ,\ \{a,\varnothing\}\}$

(b)

| | | | |
|---|---|---|---|
| (i) True | (v) False | (ix) True | (xiii) False |
| (ii) False | (vi) True | (x) True | (xiv) True |
| (iii) False | (vii) True | (xi) True | (xv) True |
| (iv) False | (viii) False | (xii) True | (xvi) True |

**Question 5.**

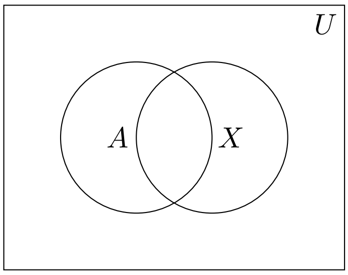
(a) Assume $A \subseteq B$. Then,

$$\begin{aligned} x \in A \cap C &\implies x \in A \text{ and } x \in C \\ &\implies x \in B \text{ and } x \in C \qquad \text{(as } A \subseteq B\text{)} \\ &\implies x \in B \cap C \end{aligned}$$

Thus, $A \cap C \subseteq B \cap C$.

(b) Assume $P(A) \subseteq P(B)$.
By definition, $A \in P(A)$.
Since $P(A) \subseteq P(B)$, we have $A \in P(B)$, which implies $A \subseteq B$.

(c) Assume $A \cap B = A \cap C$ and $A \cup B = A \cup C$.




By the above Venn diagram, we can deduce that $X = [(A \cup X) - A] \cup (A \cap X)$ for any set $X$.
This can be formally proven, as follows:

$$\begin{aligned} [(A \cup X) - A] \cup (A \cap X) &= [(A \cup X) \cap \overline{A}] \cup (A \cap X) \\ &= [(A \cap \overline{A}) \cup (X \cap \overline{A})] \cup (A \cap X) \qquad \text{(by distributive law)} \\ &= [\varnothing \cup (X \cap \overline{A})] \cup (A \cap X) \\ &= (X \cap \overline{A}) \cup (A \cap X) \\ &= X \cap (\overline{A} \cup A) \qquad \text{(by distributive law)} \\ &= X \cap U = X\ . \end{aligned}$$

$$\begin{aligned} \text{Therefore, } B &= [(A \cup B) - A] \cup (A \cap B) \\ &= [(A \cup C) - A] \cup (A \cap C) \qquad \text{(by assumptions)} \\ &= C\ . \end{aligned}$$

(d) We need to prove the biconditional statement in both directions.

(i) We first prove that $A \subseteq C$ and $B \subseteq C \implies A \cup B \subseteq C$.
Assume $A \subseteq C$ and $B \subseteq C$.

$$\begin{aligned} \text{If } x \in A \cup B, \text{ then } x \in A \text{ or } x \in B &\implies x \in C \text{ or } x \in C \qquad \text{(as } A \subseteq C \text{ and } B \subseteq C\text{)} \\ &\implies x \in C \end{aligned}$$

Thus, $A \cup B \subseteq C$.
Hence, $A \subseteq C$ and $B \subseteq C \implies A \cup B \subseteq C$ follows.

(ii) Next, we prove that $A \cup B \subseteq C \implies A \subseteq C$ and $B \subseteq C$.
Assume $A \cup B \subseteq C$. We consider prove the following two cases.

**Case 1:** $A \cup B \subseteq C \implies A \subseteq C$.
If $x \in A$, then $x \in A \cup B \implies x \in C$ (as $A \cup B \subseteq C$).
Thus, $A \subseteq C$.

**Case 2:** $A \cup B \subseteq C \implies B \subseteq C$.
Similarly, if $x \in B$, then $x \in A \cup B \implies x \in C$ (as $A \cup B \subseteq C$).
Thus, $B \subseteq C$.

To sum up, $A \cup B \subseteq C \implies A \subseteq C$ and $B \subseteq C$ follow.

Therefore, the bidirectional statement follows.