Discrete Mathematics

Final Exam Specimen

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Question 1 (10 marks)

(a)

p	q	$(q \to \neg p)$	$p \to (q \to \neg p)$	$\neg p$
T	F	T	T	
F	T	T.	~~F~~ T.	
T	T	F	F	
F	F	~~F~~ T.	T.	

if the answer is correct then. statement is right.

(b) $(p \to \neg q) \vee (p \wedge q)$.

$= (\neg p \vee \neg q) \vee (p \wedge q)$.

$= \neg(p \wedge q) \vee (p \wedge q)$.

$= \vee T \quad (\neg a \vee a \equiv T)$.

(c) No

(d). left $\neg \forall x\, (P(x) \to Q(x))$ right $\exists x\, (P(x) \wedge \neg Q(x))$.

$= \neg \exists x\, (\overline{P(x) \to Q(x)})$ $\quad a \to b \equiv \neg a \vee b$.

~~$= \exists x\, \neg(P(x) \wedge Q(x))$~~ So they are logically equivalent.

~~$= \exists x\, (\neg P(x) \vee \neg Q(x))$~~.

$= \exists x\, \neg(\neg P(x) \vee Q(x))$.

$= \exists x\, (P(x) \wedge \neg Q(x))$.

Question 2 (10 marks).

Proof by contradiction:

for the sake of contradiction. $b \neq 0$. $b, a \in \mathbb{Z}$.

suppose x is an rational number. which is $\frac{a}{b}$.

(x^2 is an irrational number.

the $x^2 = \frac{a^2}{b^2}$ which can be represent in fraction.

Therefore x^2 is rational, which contradicts that x^2 is irrational.

$\therefore$ x is an irrational number.

Question 3 (15 marks)

(a). $f \circ g(x) = f(g(x)) = \frac{1}{(\frac{1}{x}+2)-2} = x$.

$g \circ f(x) = g(f(x)) = x-2+2 = x$.

(b). $\because$ if in $f(x)$. $x=2$. then $f(x)$ is meaningless. undefined

in $g(x)$ $x=0$. the $g(x)$ is also meaningless. undefined

(c). (i) $f: X \to Y$ where $X = R - \{2\}$, $Y = R - \{0\}$. $f(x) = \frac{1}{x-2}$

is f_2 one-to-one?

Let x and y that $f(x) = f(y) \Rightarrow \frac{1}{x-2} = \frac{1}{y-2}$ Because $X = R - \{2\}$.

$\therefore$ $x-2$. $y-2 \neq 0$ $\therefore$ $x-2 = y-2 \Rightarrow x = y$. Yes.

is f_2 onto?

For any $b \in Y = R - \{0\}$.

$b = f(a) \Rightarrow \frac{1}{a-2} = b$ $\because X = R - \{2\}$ $\therefore a-2 \neq 0$.

$\because b \neq 0$. $\therefore \frac{1}{b} = a-2$ $a = \frac{1}{b}+2$. $\because (a \neq 2)$. $\therefore a \in X$.

$\because \frac{1}{b} \neq 0$.

(ii) Inverse of f:

Because f is a bijection, then we can define an inversefunction of f:

$x = f(y) = \frac{1}{y-2} \Rightarrow y = \frac{1}{x}+2$.

$\therefore$ Therefore, f is bijective

Question 4 (5 marks).

Let's suppose there are n students, and their birthday is on the same week.

then according to the Generalized pigeonhole principle:

n students placed into 7 days. then at least one day.

that there are $\lceil \frac{n}{7} \rceil = 4$ people have their birthday.

Because it is "$\lceil\,\rceil$" $\therefore$ n $\frac{n}{7} > 3$ $n > 21$ $\therefore$ at least there are 22. students whose birthday are in same week.

Question 5 (10 marks).

(a). 4^8 the number of ways to draw 8 notes of any of the four types is $C(8+3,3) = 165$

(b). $4^8 - 3^8$ $C(7+3,3) = 120$.

(C). $4^8 - 4$ (1 type) $165 - 4 = 161$

Question 6 (10 marks).

We can form a committee of 6 members with a chairperson from 15 people.

De Morgan's law

Propositions. $\neg(p \vee q) \equiv (\neg p \wedge \neg q)$

sets : $(\overline{A} \cup \overline{B}) \equiv (\overline{A \cap B})$.

Question 10 (10 marks).

(a) $P(M \cap B \mid B) = \frac{P(M \cap B)}{P(B)} = \frac{10\%}{15\%} = \frac{2}{3}$.

(b). $P(\overline{M} \cap B \mid \overline{M}) = \frac{P(\overline{M} \cap B)}{P(\overline{M})} = \frac{\overbrace{P(B) - P(M \cap B)}^{5\%}}{1-25\%} = \frac{1}{15}$

(C) $P(M \cup B) = P(M) + P(B) - \underbrace{P(M \cap B)}_{\text{not independent}} = \frac{3}{10}$

(d). $P(\overline{M} \cap \overline{B}) = P(\overline{M \cup B})$

$= 1 - P(M \cup B) = \frac{7}{10}$.

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