COMP S264F Discrete Mathematics
Tutorial 3: Methods of Proof

Question 1 (Direct proof). Prove the following statements.

(a) The product of any two consecutive integers is even.

(b) For any integer x, $x^2 + 2x + 1$ is even if and only if x is odd.

Question 2 (Proof by contraposition). Prove the following statements.

(a) If a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10.

(b) For all integers m, n, if $m + n$ is even, then m and n are both even or both odd.

Question 3 (Non-proof). Identify the error or errors in the following arguments that supposedly shows that if $\forall x\ (P(x) \vee Q(x))$ is true, then $\forall x\ P(x) \vee \forall x\ Q(x)$ is true.

1. $\forall x\ (P(x) \vee Q(x))$
2. $P(c) \vee Q(c)$ for any element c in the domain
3. $P(c)$, from step 2
4. $\forall x\ P(x)$
5. $Q(c)$, from step 2
6. $\forall x\ Q(x)$
7. $\forall x\ P(x) \vee \forall x\ Q(x)$, from steps 4 and 6

Question 4 (Proof by cases). Prove that for all integers n, $n^2 - n + 3$ is odd.

Question 5 (Proof by cases). Let x and y be some integers. Prove or disprove the following.

(a) (x is odd) $\leftrightarrow$ ($x^2 + 6x + 9$ is even)

(b) (xy is odd) $\leftrightarrow$ (x and y are both odd)

Question 6 (Proof by contradiction). Prove the following statements.

(a) If a, b are integers, then $a^2 - 4b - 2 \neq 0$.

(b) Suppose a, b are real numbers. If a is rational and ab is irrational, then b is irrational.

Question 7 (Mathematical induction). Prove the following statements.

(a) For any positive integer n, $3^n + 1$ is divisible by 2.

(b) For any positive integer n, $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.