COMP S264F Discrete Mathematics
Tutorial 2: Logic (2)

Question 1. Consider the following predicates and proposition, where the domain for x consists of all Computing students:

- $p(x)$: x go to school by bus.
- $q(x)$: x go to school by train.
- r: It is a rainy day.

Express each of the following statements in terms of $p(x), q(x), r$, quantifiers, and logical operators.

(a) When Keith goes to school, he takes a bus but not a train.

(b) Tom goes to school by either bus or train.

(c) It is sunny, so John walks to school.

(d) Paul takes a bus to school only if it rains.

(e) All Computing students go to school by bus.

(f) Some Computing students do not go to school by train.

Question 2. Let the domains for x and y be groups of male and female students, respectively. Consider the following propositions.

- $\forall x\ \exists y\ (x \text{ loves } y)$
- $\exists y\ \forall x\ (x \text{ loves } y)$

(a) Translate the above propositions into English sentences.

(b) Explain the difference in their meaning with an example.

Question 3. Determine the truth value of the proposition $\exists x\ \forall y\ (x \leq y^2)$ if the domain is the set of

(a) positive real numbers.

(b) integers.

(c) non-zero real numbers.

Question 4. Prove or disprove each of the following propositions.

(a) $\exists x\ (x + 1 \text{ is an odd number})$, where the domain is the set of prime numbers.

(b) $\forall x\ \exists y\ ((x \leq y) \to (x^2 \leq y - x))$, where the domain is the set of positive integers.

(c) $\exists x\ \forall y\ (x^2 + y \leq 0)$, where the domain is the set of negative integers.

(d) $\forall x\ \forall y\ (x^2 + y^2 \geq 0)$, where the domain is the set of real numbers.

(e) $\exists x\ \exists y\ (xy \text{ is a prime number})$, where the domain is the set of real numbers.

(f) $\forall x\ (x + 1 \text{ is not divisible by } x)$, where the domain is the set of prime numbers.

(g) $\forall x\ (x \text{ and } x + 1 \text{ are relatively prime})$, where the domain is the set of prime numbers.
Note that two integers a and b are relatively prime if 1 is the only positive common factor of a and b.