COMP S264F Discrete Mathematics
Tutorial 4: Set Theory (1) – Suggested Solution

Question 1.

(a) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $\{2, 3, 5, 7\}$

(c) $\{4, 25, 64\}$

(d) $\{6, 7, 9\}$

Question 2.

(a)
$$\begin{aligned}(A \cup B) \cap C &= (\{1, 4, 7, 10\} \cup \{1, 2, 3, 4, 5\}) \cap \{2, 4, 6, 8\} \\ &= \{1, 2, 3, 4, 5, 7, 10\} \cap \{2, 4, 6, 8\} \\ &= \{2, 4\}\end{aligned}$$

Thus, $|(A \cup B) \cap C| = 2$.

(b)
$$\begin{aligned}A \cup (B \cap C) &= \{1, 4, 7, 10\} \cup (\{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8\}) \\ &= \{1, 4, 7, 10\} \cup \{2, 4\} \\ &= \{1, 2, 4, 7, 10\}\end{aligned}$$

Thus, $|A \cup (B \cap C)| = 5$.

(c)
$$\begin{aligned}A - B &= \{1, 4, 7, 10\} - \{1, 2, 3, 4, 5\} \\ &= \{7, 10\}\end{aligned}$$

Thus, $|A - B| = 2$.

(d)
$$\begin{aligned}B - A &= \{1, 2, 3, 4, 5\} - \{1, 4, 7, 10\} \\ &= \{2, 3, 5\}\end{aligned}$$

Thus, $|(B - A)| = 3$.

(e) $\overline{U} = \varnothing$.

Thus, $|\overline{U}| = 0$.

(f)
$$\begin{aligned}\overline{(A \cap B)} \cup C &= \overline{(\{1, 4, 7, 10\} \cap \{1, 2, 3, 4, 5\})} \cup \{2, 4, 6, 8\} \\ &= \overline{\{1, 4\}} \cup \{2, 4, 6, 8\} \\ &= \{2, 3, 5, 6, 7, 8, 9, 10\} \cup \{2, 4, 6, 8\} \\ &= \{2, 3, 4, 5, 6, 7, 8, 9, 10\}\end{aligned}$$

Thus, $|\overline{(A \cap B)} \cup C| = 9$.

(g)
$$\begin{aligned}\overline{A} \cup \overline{B} \cup C &= \overline{\{1, 4, 7, 10\}} \cup \overline{\{1, 2, 3, 4, 5\}} \cup \{2, 4, 6, 8\} \\ &= \{2, 3, 5, 6, 8, 9\} \cup \{6, 7, 8, 9, 10\} \cup \{2, 4, 6, 8\} \\ &= \{2, 3, 4, 5, 6, 7, 8, 9, 10\}\end{aligned}$$

Thus, $|\overline{A} \cup \overline{B} \cup C| = 9$.

(h)
$$\begin{aligned}(A \cup \overline{C}) - (B - \overline{A}) &= (\{1, 4, 7, 10\} \cup \overline{\{2, 4, 6, 8\}}) - (\{1, 2, 3, 4, 5\} - \overline{\{1, 4, 7, 10\}}) \\ &= (\{1, 4, 7, 10\} \cup \{1, 3, 5, 7, 9, 10\}) - (\{1, 2, 3, 4, 5\} - \{2, 3, 5, 6, 8, 9\}) \\ &= \{1, 3, 4, 5, 7, 9, 10\} - \{1, 4\} \\ &= \{3, 5, 7, 9, 10\}\end{aligned}$$

Thus, $|(A \cup \overline{C}) - (B - \overline{A})| = 5$.

Question 3.

(a) True

(b) False

(c) False

(d) True

(e) False

(f) True

(g) True

(h) True

Question 4.

(a) By solving $x^2 + x = 2$, we have $A = \{1, -2\}$. Then, $-2 \in A$ and $-2 \notin B$, so $A \nsubseteq B$.

(b) $B = A \cap C = \varnothing$. Hence, $A \nsubseteq B$.

(c) $A = \{2, 4, 6, 8, \cdots\}$ is the set of positive even integers.
$B = \{1, 2, 3, 4, \cdots\}$ is the set of positive integers.
Hence, $A \subseteq B$.
Formal proof: Assume $y \in A$. Then $y = 2x$ for some positive integer x. Thus, y is also a positive integer, i.e., $y \in B$. $A \subseteq B$ follows.

(d) $B = \{1, 2, 3\}$. Then, $4 \in A$ and $4 \notin B$, so $A \nsubseteq B$.

Question 5.

(a) B must contain all of the elements in A so that $A \cap B = A$.
Hence, we can deduce that $A \subseteq B$.

(b) A must contain all of the elements in B so that $A \cup B = A$.
Hence, we can deduce that $B \subseteq A$.

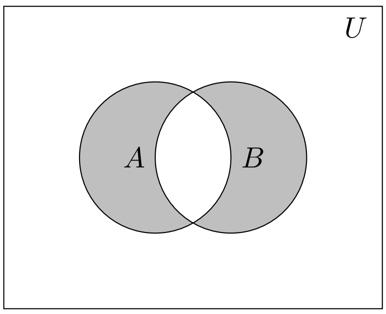
(c) $\overline{A}$ and B should be disjoint, i.e., having completely different elements.
In other words, $A = U - \overline{A}$ contain all of the elements in B, where U is the domain (i.e., universal set).
Hence, $B \subseteq A$.

(d) By De Morgan's Law, $\overline{A \cap B} = \overline{A} \cup \overline{B} = \overline{B}$
From (b), we can deduce that $\overline{A} \subseteq \overline{B}$.
Therefore, $B = U - \overline{B} \subseteq U - \overline{A} = A$, i.e., $B \subseteq A$.

Question 6.

(a)
$$\begin{aligned} A \triangle B &= (A \cup B) - (A \cap B) \\ &= (\{1,2,3\} \cup \{2,3,4,5\}) - (\{1,2,3\} \cap \{2,3,4,5\}) \\ &= \{1,2,3,4,5\} - \{2,3\} \\ &= \{1,4,5\} \end{aligned}$$

(b) Let U be the universal set. The Venn diagram is



(c) $A \triangle B$ is the set of elements in either sets A or B, but not both.