

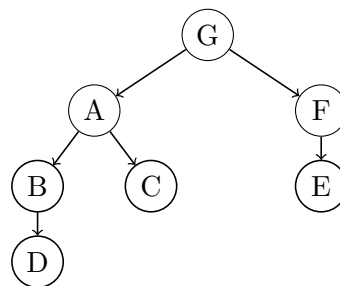
COMP S265F Design and Analysis of Algorithms
Lab 8: Depth-First Search and Topological Sort – Suggested Solution

Question 1.

(a) The required information of each vertex is shown below:

visited order	1	2	3	4	5	6	7
vertex v	G	A	F	B	C	E	D
$dist[v]$	0	1	1	2	2	2	3
parent in BF tree	–	G	G	A	A	F	B

(b) Below is the breadth-first tree obtained:

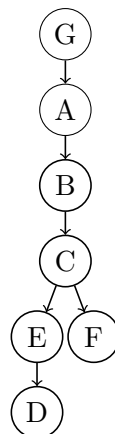


Question 2.

(a) The required information of each vertex is shown below:

discovered order	1	2	3	4	5	6	7
vertex v	G	A	B	C	E	D	F
$d[v]$	1	2	3	4	5	6	9
$f[v]$	14	13	12	11	8	7	10
$\pi[v]$	–	G	A	B	C	E	C

(b) Below is the depth-first tree obtained:



(c) Classification of edges:

edge	(G, A)	(A, B)	(B, C)	(C, E)	(E, D)	(C, F)	(F, E)	(B, D)	(A, C)	(G, F)
type	tree	tree	tree	tree	tree	tree	cross	forward	forward	forward

Question 3.

- (a)
- 1st DFS: source = A, Vertices in finishing order = (D, E, F, C, B, A)
 - 2nd DFS: source = G, Vertices in finishing order = (G)
- (b) The ordering of the vertices obtained by the topological sort is (G, A, B, C, F, E, D).

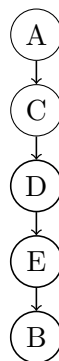
Question 4.

- (a) As shown in Unit 4 Slides 73 and 74, a directed graph G is acyclic if and only if a DFS on G yields no back edges. Each vertex has three states: 0 (not discovered)/ 1 (discovered but not finished)/ 2 (finished). Then, we can check whether G contains cycle, as follows:

```
1: procedure HASCYCLE( $G$ )
2:   Mark all vertices  $u$  in state 0
3:   for each vertex  $u$  that is in state 0 do
4:     if DFS( $u$ ) then
5:       return true
6:     end if
7:   end for
8:   return false
9: end procedure

10:
11: procedure DFS( $x$ )
12:   if vertex  $x$  is in state 2 then
13:     return false
14:   end if
15:   if vertex  $x$  is in state 1 then
16:     return true
17:   end if
18:   Mark  $x$  to be in state 1
19:   for each neighbor  $y$  of  $x$  do
20:     if DFS( $y$ ) then
21:       return true
22:     end if
23:   end for
24:   Mark  $x$  to be in state 2
25:   return false
26: end procedure
```

- (b) Below is the depth-first tree obtained by DFS(A) on the directed graph G :



In the recursive call DFS(B), when traversing the edge (B, A), vertex A is found to be in state 1, i.e., (B, A) is a back edge. Thus, HASCYCLE(G) will return true. Therefore, the directed graph has a cycle, which is ((A, C), (C, D), (D, E), (E, B), (B, A)).