COMP S264F Discrete Mathematics
Tutorial 4: Set Theory (1)

Question 1. Assume the universal set $U = \mathbb{N}$. Find each of the following sets.

(a) $A = \{x \mid 1 \leq x \leq 10\}$

(b) $B = \{x \mid x \in A \text{ and } x \text{ is prime}\}$

(c) $C = \{x^2 \mid x \in A \text{ and } x \bmod 3 = 2\}$

(d) $D = \{x + y \mid x \in B \text{ and } y \in C \text{ and } x + y \in A\}$

Question 2. Let $U = \{x \mid x \in \mathbb{N} \text{ and } 1 \leq x \leq 10\}$ be the universal set. Consider the following sets.

- $A = \{1, 4, 7, 10\}$
- $B = \{1, 2, 3, 4, 5\}$
- $C = \{2, 4, 6, 8\}$

Find the cardinality of each of the following sets.

(a) $(A \cup B) \cap C$

(b) $A \cup (B \cap C)$

(c) $A - B$

(d) $B - A$

(e) $\overline{U}$

(f) $\overline{(A \cap B)} \cup C$

(g) $\overline{A} \cup \overline{B} \cup C$

(h) $(A \cup \overline{C}) - (B - \overline{A})$

Question 3. Suppose x is an integer. Determine the truth value of each of the followings.

(a) $x \in \{x\}$

(b) $x \subseteq \{x\}$

(c) $\{x\} \subset \{x\}$

(d) $\{x\} \subseteq \{x\}$

(e) $\{x\} \in \{x\}$

(f) $\{x\} \in \{x, \{x\}\}$

(g) $\{x\} \subset \{x, \{x\}\}$

(h) $\{\{x\}\} \subset \{x, \{x\}\}$

Question 4. For each of the followings, determine if $A \subseteq B$.

(a) $A = \{x \mid x^2 + x = 2\}$
$B = \{1, -1\}$

(b) $A = \{1, 3, 5, 7\}$
$C = \{2, 4, 6, 8\}$
$B = A \cap C$

(c) $A = \{2x \mid x \in \mathbb{Z}^+\}$
$B = \{x \mid x \in \mathbb{Z}^+\}$

(d) $A = \{1, 2, 3, 4\}$
$C = \{5, 6, 7, 8\}$
$B = \{x \mid x \in A \text{ and } x + y = 8 \text{ for some } y \in C\}$

Question 5. State the relation between the sets A and B such that the given condition is true.

(a) $A \cap B = A$

(b) $A \cup B = A$

(c) $\overline{A} \cap B = \varnothing$

(d) $\overline{A \cap B} = \overline{B}$

Question 6. The *symmetric difference* of two sets A and B is the set $A \triangle B = (A \cup B) - (A \cap B)$.

(a) Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$. Find $A \triangle B$.

(b) Draw the Venn diagram of $A \triangle B$.

(c) Describe the meaning of $A \triangle B$ in words.