COMP S264F Discrete Mathematics
Tutorial 2: Logic (2) – Suggested Solution

Question 1.

(a) $p(\text{Keith}) \land \neg q(\text{Keith})$

(b) $p(\text{Tom}) \otimes q(\text{Tom})$

(c) $\neg r \to \neg(p(\text{John}) \lor q(\text{John}))$

(d) $p(\text{Paul}) \to r$

(e) $\forall x\ p(x)$

(f) $\exists x\ \neg q(x)$

Question 2.

(a)
- $\forall x\ \exists y\ (x \text{ loves } y)$
  Each male student loves a female student.
- $\exists y\ \forall x\ (x \text{ loves } y)$
  There is a female student that all male students love.

(b) Suppose we have two male students, Brown and Leonard, and two female students, Cony and Sally.

The first proposition allows each male student to love a different female student, e.g., Brown loves Cony, and Leonard loves Sally. But the second proposition does not allow such case (all male students must love the same female student).

Question 3.

(a) False. For any positive real number x, we can set $y = \sqrt{\dfrac{x}{2}}$ such that $y^2 = \dfrac{x}{2} < x$.

(b) True. We can set $x = 0$ because $\forall y\ (y^2 \geq 0)$.

(c) True. When x is negative, say $x = -1$, $\forall y\ (y^2 \geq 0 > -1 = x)$.

Question 4.

(a) True. When $x = 2$, $x + 1 = 3$ which is odd.

(b) True. We can find a value y such that $x \leq y$ and $x^2 \leq y - x$, i.e., $y \geq x^2 + x$. Thus, we can set $y = x^2 + x$ such that $y = x^2 + x \geq x$.

(c) True. When $x = -1$,
$$\begin{aligned} x^2 + y &= (-1)^2 + y \\ &= 1 + y \\ &\leq 1 + (-1) \quad (\text{as } y \leq -1) \\ &= 0\ . \end{aligned}$$

(d) True. $n^2 \geq 0$ for any real number n, so $x^2 + y^2 \geq 0$.

(e) True. When $x = 1.5$, $y = 2$, we have $xy = 3$ which is a prime number.

(f) True. Suppose, for the sake of contradiction, $x + 1$ is divisible by x for some prime number x. Then, $x + 1 = kx$ for some integer k, i.e., $(k - 1)x = 1$. As both k and x are positive integers, we must have $k - 1 = 1$ and $x = 1$. But $x = 1$ is not a prime number, which contradicts that x is a prime number.

(g) True. Suppose, for the sake of contradiction, there exists a prime number x such that $x + 1$ and x are not relatively prime. Then, $x + 1$ and x has a common factor $n > 1$ such that $x + 1 = an$ and $x = bn$ for some integers a, b where $a > b$. It follows that $(x + 1) - x = (a - b)n \Rightarrow (a - b)n = 1$. As both $a - b$ and n are positive integers, we must have $a - b = 1$ and $n = 1$, which contradicts that $n > 1$.