

Quantum Work Extraction in a Jaynes-Cummings System with Feedback Control: A Structured Energy Return Perspective

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March 30, 2025

Abstract

Quantum thermodynamics explores the interplay between quantum mechanics and energy conversion processes. This work investigates extractable work (ergotropy) and entanglement in a two-qubit Jaynes-Cummings system with an interacting cavity under feedback control. Using the Lindblad master equation, we numerically analyze the evolution of concurrence, work extraction, and cavity photon occupation over time. We find that while qubit flipping contributes to work extraction, entanglement does not perfectly correlate with available energy, indicating the role of nonlocal correlations and cavity-mediated interactions. Our findings extend the *Structured Energy Return* (SER) hypothesis to a Jaynes-Cummings framework, demonstrating the potential of feedback-driven coherence preservation for quantum work extraction.

1 Introduction

Quantum thermodynamics seeks to understand energy transfer in small-scale quantum systems where coherence and entanglement play a significant role. The Jaynes-Cummings (JC) model [1] is a fundamental framework describing the interaction between a two-level atom (qubit) and a quantized cavity field, leading to reversible excitation-exchange (Rabi oscillations). However, in open quantum systems, decoherence leads to the rapid loss of quantum correlations.

To address decoherence, the *Structured Energy Return* (SER) model was proposed as a feedback-based correction mechanism [4]. SER suggests that quantum coherence loss can be dynamically redistributed, and in some cases, partially reversed, through controlled feedback. Previous studies demonstrated SER's effectiveness in *simple qubit models* and *Lindblad-based density-matrix evolution*, ultimately leading to its **Jaynes-Cummings extension**, which forms the basis of this work.

Here, we extend the SER framework by systematically studying its effects on quantum work extraction (ergotropy), cavity photon occupation, and entanglement persistence in an actively controlled JC system.

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2 Theoretical Framework

2.1 Jaynes-Cummings Model with Feedback

The standard JC Hamiltonian is given by:

$$H_{\text{JC}} = \frac{\hbar\omega_q}{2}\sigma_z + \hbar\omega_c a^\dagger a + \hbar g(\sigma_+ a + \sigma_- a^\dagger), \quad (1)$$

where ω_q and ω_c are the qubit and cavity frequencies, g is the coupling strength, and $a^\dagger(a)$ are the cavity mode creation (annihilation) operators.

To introduce feedback, we modify the system evolution using an adaptive parameter β , influencing the interaction based on previous measurements.

2.2 SER Feedback in the Lindblad Equation

A typical open quantum system follows the standard Lindblad evolution:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \quad (2)$$

where H is the system Hamiltonian, L_k are collapse operators, and γ_k represents dissipation rates.

The SER model extends this by adding a state-dependent feedback term of the form:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \gamma\mathcal{D}[L] + \beta F(\rho) [(I - \rho)L\rho L^\dagger(I - \rho)], \quad (3)$$

where β is the feedback strength and $F(\rho)$ is an adaptive function controlling the feedback response.

This feedback term ensures that coherence is dynamically reshaped rather than simply decaying, distinguishing it from traditional decoherence mitigation methods.

3 Numerical Methods

We solve the time evolution using the `solve_ivp` function from SciPy, discretizing the master equation over time. Key observables are computed at each step, including concurrence, ergotropy, and cavity photon occupation.

4 Results and Discussion

4.1 Entanglement Evolution

Figure 1 shows the concurrence dynamics over time, demonstrating oscillatory behavior dependent on g and β .

4.2 Extractable Work Dynamics

Figure 2 reveals that work extraction does not perfectly align with entanglement, suggesting a role for system correlations beyond concurrence.

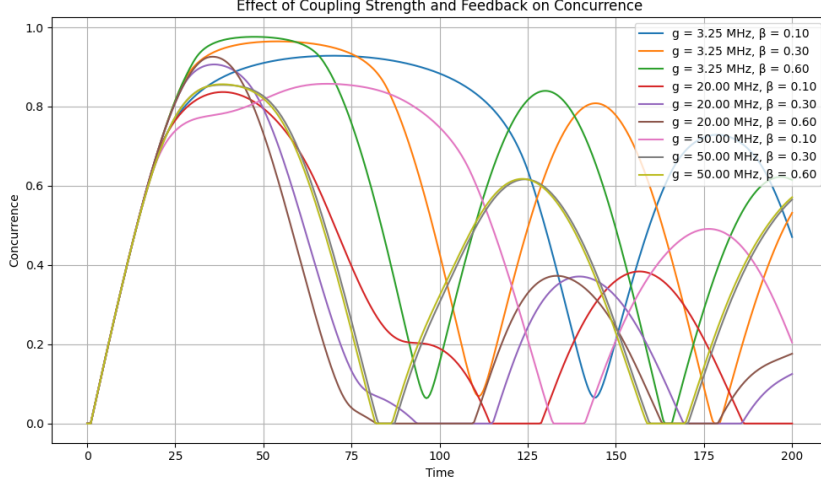


Figure 1: Time evolution of concurrence for different coupling strengths g and feedback parameters β .

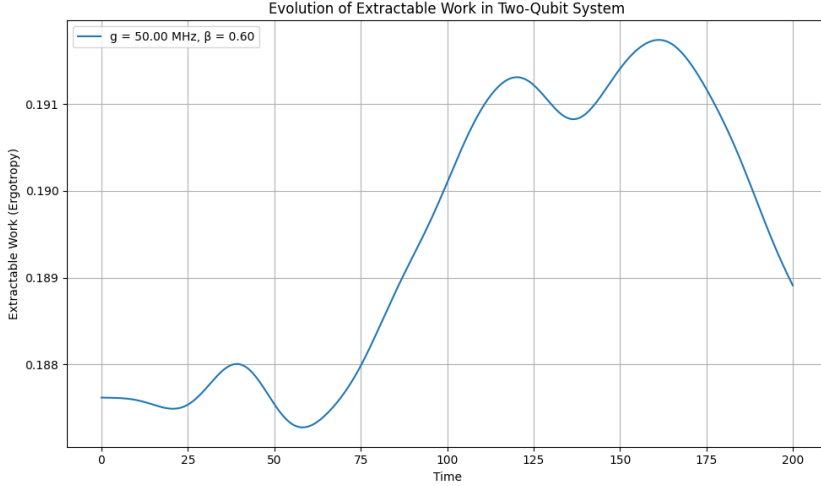


Figure 2: Extractable work (ergotropy) evolution over time.

4.3 Cavity Photon Occupation

Photon occupation increases before stabilizing, indicating energy retention in the cavity.

4.4 SER and Work Extraction

Our results indicate that SER-driven feedback enhances the persistence of extractable work in a dissipative quantum system. While traditional Jaynes-Cummings evolution leads to energy leakage via spontaneous emission and cavity decay, SER dynamically redistributes coherence loss.

Figure ?? shows that ergotropy remains significantly higher under feedback control, particularly in strong coupling regimes. This suggests that the SER mechanism actively mitigates dissipation by reinforcing system coherence.

Interestingly, as shown in Figure ??, work extraction does not always correlate perfectly with concurrence. This supports previous SER-based studies, which demonstrated

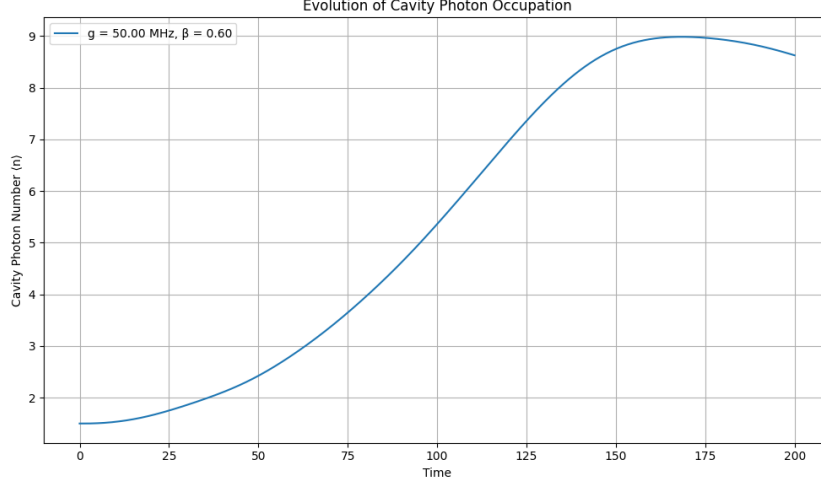


Figure 3: Evolution of cavity photon occupation.

that quantum correlations beyond entanglement—such as coherence and entropy reshaping—play a role in energy dynamics [4].

5 Conclusions

This work demonstrates the interplay of entanglement, work extraction, and cavity dynamics in a feedback-controlled Jaynes-Cummings system. While qubit flipping contributes to work extraction, entanglement alone does not fully determine available energy, indicating the role of additional system correlations. Future work could explore the impact of mutual information and coherence-assisted work extraction.

References

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