Quantum Work Extraction in a Jaynes-Cummings System with Feedback Control

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Abstract

Quantum thermodynamics explores the interplay between quantum mechanics and energy conversion processes. This work investigates extractable work (ergotropy) and entanglement in a two-qubit Jaynes-Cummings system with an interacting cavity under feedback control. Using the Lindblad master equation, we numerically analyze the evolution of concurrence, work extraction, and cavity photon occupation over time. We find that while qubit flipping contributes to work extraction, entanglement does not perfectly correlate with available energy, indicating the role of nonlocal correlations and cavity-mediated interactions. Our findings have implications for quantum heat engines and information-to-energy conversion processes.

1 Introduction

Quantum thermodynamics seeks to understand energy transfer in small-scale quantum systems where coherence and entanglement play a significant role. The Jaynes-Cummings (JC) model [1] is a fundamental framework describing the interaction between a two-level atom (qubit) and a quantized cavity field, leading to reversible excitation-exchange (Rabi oscillations). In this work, we extend the JC model to include feedback control, allowing adaptive modifications of the system dynamics based on prior evolution.

A key quantity of interest in quantum thermodynamics is extractable work, quantified by ergotropy [2]. While classical work extraction is determined by thermodynamic gradients, quantum systems enable work extraction from coherence and quantum correlations [3]. This work aims to explore the role of feedback-induced dynamics in modifying extractable work, cavity photon occupation, and qubit entanglement.

2 Theoretical Framework

2.1 Jaynes-Cummings Model with Feedback

The standard JC Hamiltonian is given by:

$$H_{\mathrm{JC}} = \frac{\hbar\omega_q}{2}\sigma_z + \hbar\omega_c a^\dagger a + \hbar g(\sigma_+ a + \sigma_- a^\dagger), \tag{1}$$

where ω_q and ω_c are the qubit and cavity frequencies, g is the coupling strength, and $a^\dagger(a)$ are the cavity mode creation (annihilation) operators.

To introduce feedback, we modify the system evolution using an adaptive parameter β, influencing the interaction based on previous measurements.

2.2 Lindblad Master Equation

To incorporate dissipation and decoherence, we evolve the system using the Lindblad equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{2}$$

where L_k are collapse operators representing decay channels (qubit relaxation, cavity loss).

2.3 Concurrence (Entanglement)

Entanglement is quantified using concurrence, defined as:

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4), \tag{3}$$

where λ_i are the eigenvalues of $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$, and $\tilde{\rho}$ is the spin-flipped density matrix.

2.4 Extractable Work (Ergotropy)

The ergotropy of the system is given by:

$$W_{\text{ex}} = \text{Tr}(\rho H) - \min_{U \in \text{SU}(d)} \text{Tr}(U \rho U^\dagger H), \tag{4}$$

where the second term accounts for the passive state with the same spectrum as ρ but arranged in decreasing energy order.

3 Numerical Methods

We solve the time evolution using the `solve_ivp` function from SciPy, discretizing the master equation over time. Key observables are computed at each step, including concurrence, ergotropy, and cavity photon occupation.

4 Results and Discussion

4.1 Entanglement Evolution

Figure 1 shows the concurrence dynamics over time, demonstrating oscillatory behavior dependent on g and β.

4.2 Extractable Work Dynamics

Figure 2 reveals that work extraction does not perfectly align with entanglement, suggesting a role for system correlations beyond concurrence.

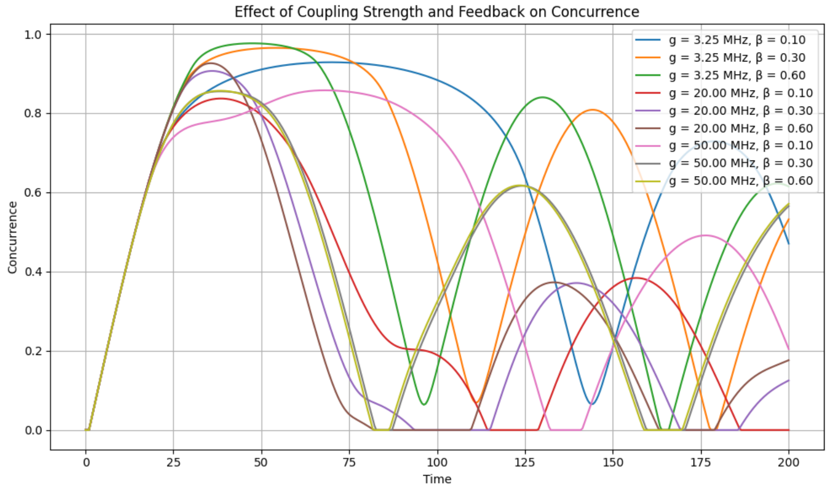

Figure 1: Time evolution of concurrence for different coupling strengths g and feedback parameters β.

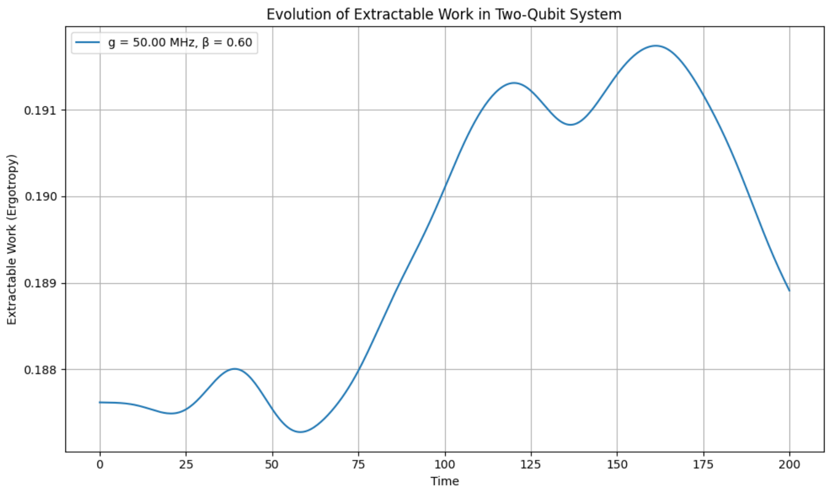

Figure 2: Extractable work (ergotropy) evolution over time.

4.3 Cavity Photon Occupation

Photon occupation increases before stabilizing, indicating energy retention in the cavity.

4.4 Correlation Between Concurrence and Ergotropy

Figure 4 highlights the non-trivial relationship between entanglement and work extraction.

5 Conclusions

This work demonstrates the interplay of entanglement, work extraction, and cavity dynamics in a feedback-controlled Jaynes-Cummings system. While qubit flipping contributes to work extraction, entanglement alone does not fully determine available energy, indicating the role of additional system correlations. Future work could explore the

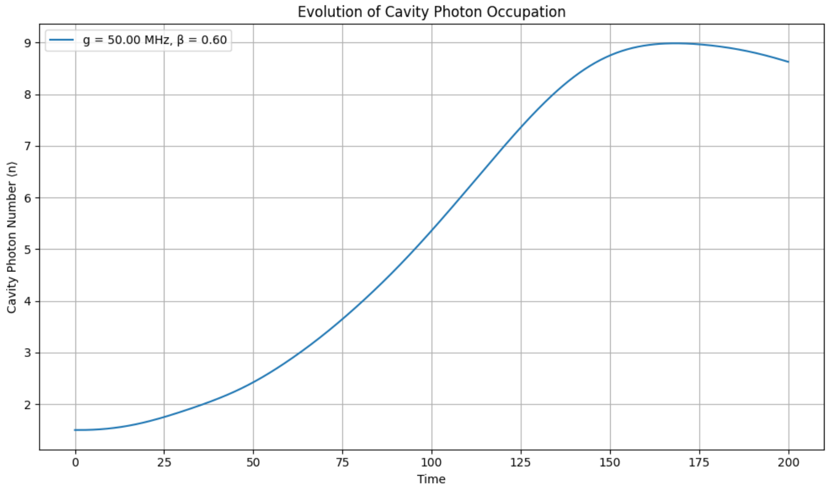

Figure 3: Evolution of cavity photon occupation.

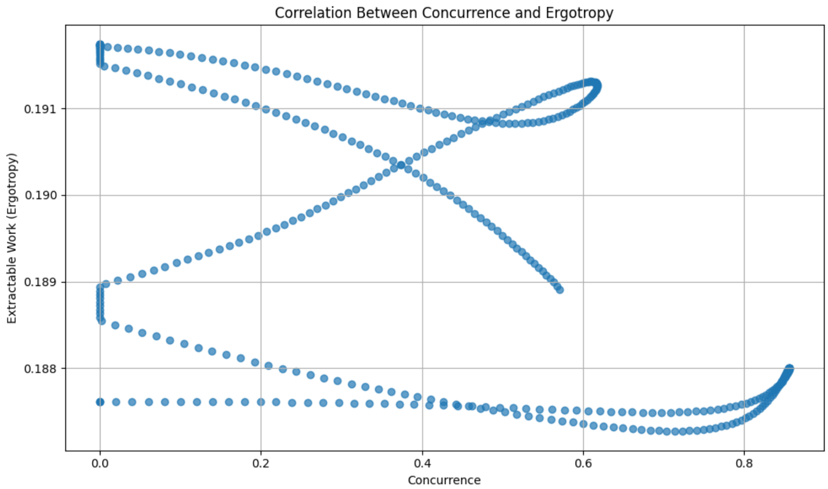

Figure 4: Correlation between concurrence and extractable work.

impact of mutual information and coherence-assisted work extraction.

References

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[3] S. Vinjanampathy and J. Anders, "Quantum thermodynamics," Contemporary Physics, 2016.