Hamiltonian Structure of the Forge Equation: A Pulse-Based Model for Discrete Light Propagation

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Abstract

We extend the Forge framework for light propagation by deriving its Hamiltonian structure from a covariant Lagrangian in curved spacetime. Starting from a variational principle involving a null 4-velocity field and a pulse density scalar, we perform a Legendre transformation to obtain the Hamiltonian density. The result clarifies the energy dynamics of pulse propagation and emission and lays the foundation for future quantization. This paper completes the core dynamical formulation of the Forge model and connects it with canonical field theory approaches.

1 Introduction

The Forge Equation describes light as a pulse-based propagation process through spacetime, governed by discrete emissions rather than continuous waves. In prior work, we developed a covariant variational principle for the Forge framework. In this paper, we derive its Hamiltonian formulation, preparing the way for canonical quantization and deeper analysis.

2 Recap: Variational Form of the Forge Equation

We consider a Lorentzian spacetime $(M, g_{\mu\nu})$ with signature $(-,+,+,+)$. The action functional is:

$$S[\rho, u^\mu, \lambda] = \int d^4x \sqrt{-g} \left(-u^\mu \nabla_\mu \rho + \sigma \ln \rho - \lambda \, g_{\mu\nu} u^\mu u^\nu \right), \tag{1}$$

with the null propagation condition:

$$g_{\mu\nu} u^\mu u^\nu = 0. \tag{2}$$

3 Hamiltonian Derivation

We begin by identifying $\dot{\rho} = \partial_t \rho$ and isolate the time derivative:

$$\mathcal{L} = -u^0 \dot{\rho} - u^i \partial_i \rho + \sigma \ln \rho - \lambda g_{\mu\nu} u^\mu u^\nu. \tag{3}$$

The canonical momentum conjugate to ρ is:

$$\pi_\rho = \frac{\partial \mathcal{L}}{\partial \dot{\rho}} = -u^0. \tag{4}$$

The Hamiltonian density is then:

$$\mathcal{H} = \pi_\rho \dot{\rho} - \mathcal{L} = u^i \partial_i \rho - \sigma \ln \rho + \lambda g_{\mu\nu} u^\mu u^\nu. \tag{5}$$

On-shell (when the null condition holds), the last term vanishes, yielding:

$$\mathcal{H}_{\text{on-shell}} = u^i \partial_i \rho - \sigma \ln \rho. \tag{6}$$

4 Physical Interpretation

Each term in the Hamiltonian has a meaningful role:

- $u^i \partial_i \rho$: spatial flow of pulse density
- $-\sigma \ln \rho$: source emission dynamics, possibly linked to entropy
- $\lambda g_{\mu\nu} u^\mu u^\nu$: constraint term (enforced off-shell)

This structure allows energy-based analysis of the Forge model and introduces a gateway to canonical quantization.

5 Outlook and Future Work

The Hamiltonian formulation opens several pathways:

1. Canonical quantization using field operators $\hat{\rho}, \hat{\pi}_\rho$
2. Path integral formulation in curved spacetime
3. Extensions to include polarization, spin, or gauge symmetry

A Detailed Derivation Steps

[Include step-by-step calculus of variations, covariant derivatives, and Legendre transform here.]