The role of active constraints on the convergence of stochastic uncostrained algorithms

To prove the convergence of algorithms for stochastic uncostrained optimization, there are different levels of progress that must be achieved:

1) Progress in primal space, e.g. $g(x_k) \to 0$, decrease of merit function;
2) Getting correct iterates, so $\{x_k\} \to x^*$;
3) Being able to identifiy the active set;
4) $\{y_k\} \to x^*$

The first experiment requires to build a small scale problem with quadratic objetcive function, two linear constraints and a 2-dimensional input vector, of the following structure

$$\begin{aligned} \min_x \quad & c^T x + \frac{1}{2} x^T Q x \\ s.t. \quad & Ax \leq b \\ & x \in \Re^2 \end{aligned} \tag{1}$$

We want to recreate a stochastic optimization setting. Therefore, the objective is to generate samples from the $[0,1] \times [0,1]$ box, and to solve the SQP subproblem for each point x_k:

$$\begin{aligned} \min_d \quad & c^T (x_k + d) + \frac{1}{2}(x_k + d)^T I (x_k + d) \\ s.t. \quad & A(x_k + d) \leq b \\ & 0 \leq x_k + d \leq 1 \end{aligned} \tag{2}$$

The aim is to compute the optimal movement d, in order to find out if $x_k + d$ has the same number of active constraints of the actual solution of the problem. Then, I plot an heatmap in which 1 is assigned to the points in which active constraints are correctly computed and 0 otherwise.

The second experiment requires to produce white noise with different levels of variance σ and to add it to c, thus having $\tilde{c} = c(x) + \mathcal{N}(0, \sigma^2)$. Perform the same exact optimization with $\tilde{c}$, producing different realizations (e.g., 10), and then compute the average of the points which correctly predict the active constraints. Then I build the heatmap again.

0.1 Experiments results

First of all, it is mandatory to keep in mind that the number of active constraints can be a fallace indicator of the quality of the solution. We can have points which reach different active constraints, even though the number of AC is still the same.

References