HOME ASSIGNMENT — 2

ME : 325 - Smart Materials & Structures

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P-1

Q.1) ——————?

Sol^n: Let

$\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}$ } → stresses in the three directions corresponding to their subscripts.

and $\epsilon_x, \epsilon_y, \epsilon_z, \epsilon_{xy}, \epsilon_{yz}, \epsilon_{zx}$ } → strains in the respective three directions.

Now, from constitutive eq^n of Piezoelectric materials, we can say that

$$\epsilon_i = S_{ij}\sigma_j + d_{ij}E_j$$

where $\{S_{ij}\sigma_j\}$ → Mechanical contribution

and $\{d_{ij}E_j\}$ → Electrical contribution

Also, we know that

$$\left\{\begin{aligned} \epsilon_x &= \frac{1}{E_p}\left[\left(\sigma_x - \mu(\sigma_y + \sigma_z)\right)\right] - d_{3x}E_3 && \text{(i)} \\ \epsilon_y &= \frac{1}{E_p}\left[\sigma_y - \mu(\sigma_x + \sigma_z)\right] - d_{3y}E_3 && \text{(ii)} \\ \epsilon_z &= \frac{1}{E_p}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right] - d_{3z}E_3 && \text{(iii)} \end{aligned}\right.$$

and

$$\begin{cases} \epsilon_{xy} = \dfrac{2(1+\mu)}{E_p} \sigma_{xy} \\ \epsilon_{yz} = \dfrac{2(1+\mu)}{E_p} \sigma_{yz} \\ \epsilon_{xz} = \dfrac{2(1+\mu)}{E_p} \sigma_{zx} \end{cases}$$

Since,

$$E_3 = \frac{V}{t} = \frac{g_{3i} F}{wt}$$

We get

$$\begin{cases} \epsilon_x = \dfrac{1}{E_p}\left(\sigma_x - \mu(\sigma_y + \sigma_z)\right) - \dfrac{d_{3x}\, g_{3x} F}{wt} \\ \epsilon_y = \dfrac{1}{E_p}\left[\sigma_y - \mu(\sigma_x + \sigma_z)\right] - \dfrac{d_{3y}\, g_{3y} F}{wt} \\ \epsilon_z = \dfrac{1}{E_p}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right] - \dfrac{d_{3z}\, g_{3z} F}{wt} \end{cases}$$

Required set of Equations

Q.2) ——————?

Sol^n:

(a) Relation b/w voltage (v) and g

Since, $g = \frac{\text{Electric field developed}}{\text{Stress in material}}$

$\Rightarrow g = \frac{E}{\sigma}$

$\Rightarrow E = g\sigma = g\frac{F}{tW} = \frac{V}{t}$

$\therefore \boxed{V = \frac{gF}{W}}$

(b) Charge Accumulation (q) and d

Since, $d = \frac{\text{Electric Displacement vector}}{\text{stress developed}}$

$\Rightarrow d = \frac{D}{\sigma}$ or $D = d\sigma$

$\Rightarrow \frac{q}{A} = d\sigma$ $\quad (\because F = \sigma A)$

$\therefore \boxed{q = dF}$

(c) Relation b/w coupling coefficient k, g, d.

coupling coeff., $k = \sqrt{\frac{\text{Electrical energy}}{\text{Mechanical work}}}$

$\Rightarrow k = \sqrt{\frac{W_E}{W_M}}$

$= \sqrt{\frac{Q^2 \times 2}{2 C_p F \Delta_z}}$ $\quad (\because \Delta_z$ is deflection in Pz material$)$

$$\Rightarrow K^2 = \frac{Q^2}{F \Delta_z C_p}$$

Now, $Q = dF \Rightarrow Q^2 = d^2F^2$

we get $K^2 = \frac{d^2F^2}{F\Delta_z C_p} = \frac{d^2F}{\Delta_z C_p}$

Also, since, $C_p = \frac{\varepsilon_0 \times \text{area}}{\text{thickness}}$ permittivity

we get, $K^2 = \frac{d^2F}{\Delta_z \frac{\varepsilon_0 \times lw}{t}} = \frac{d^2}{\frac{\left(\frac{\Delta_z}{t}\right)}{\left(\frac{F}{lw}\right)} \times \varepsilon_0} = \frac{d^2}{S \times \varepsilon_0}$

where, S is compilance

Hence, $K^2 = \frac{d^2}{S\varepsilon_0}$ —— (A)

Since, $g = \frac{\text{charge density}}{\text{Permittivity}} = \frac{d}{\varepsilon_0}$ —— (a)

and $S = \frac{1}{E_p}$ —— (b)

Using (a), (b) in (A)

$\boxed{K^2 = d\,g\,E_p}$ → Required Relation.