

HOME ASSIGNMENT-2

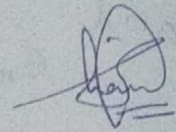
ME: 325 - Smart Materials
& Structures

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P-1



Q.1) _____?

Solⁿ: Let

$\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}$ } stresses in the three
directions corresponding to
their subscripts.

and $\epsilon_x, \epsilon_y, \epsilon_z, \epsilon_{xy}, \epsilon_{yz}, \epsilon_{zx}$ } strains in the respective
three directions.

Now, from constitutive eqⁿ of Piezoelectric materials,
we can say that

$$\epsilon_i = S_{ij} \sigma_j + d_{ij} E_j$$

where $\{S_{ij} \sigma_j\} \rightarrow$ Mechanical contribution

and $\{d_{ij} E_j\} \rightarrow$ Electrical contribution

Also, we know that

$$\left\{ \begin{array}{l} \epsilon_x = \frac{1}{E_p} \left[(\sigma_x - \mu(\sigma_y + \sigma_z)) \right] - d_{3x} E_3 \\ \epsilon_y = \frac{1}{E_p} \left[\sigma_y - \mu(\sigma_x + \sigma_z) \right] - d_{3y} E_3 \\ \epsilon_z = \frac{1}{E_p} \left[\sigma_z - \mu(\sigma_x + \sigma_y) \right] - d_{3z} E_3 \end{array} \right. \begin{array}{l} \text{--- (i)} \\ \text{--- (ii)} \\ \text{--- (iii)} \end{array}$$

and

$$\left\{ \begin{array}{l} \epsilon_{xy} = \frac{2(1+\mu)}{E_p} \sigma_{xy} \\ \epsilon_{yz} = \frac{2(1+\mu)}{E_p} \sigma_{yz} \\ \epsilon_{zx} = \frac{2(1+\mu)}{E_p} \sigma_{zx} \end{array} \right\}$$

Since,

$$E_3 = \frac{V}{t} = \frac{g_{3i} F}{wt}$$

We get

$$\left\{ \begin{array}{l} \epsilon_x = \frac{1}{E_p} \left(\sigma_x - \mu(\sigma_y + \sigma_z) \right) - \frac{d_{3x} g_{3x} F}{wt} \\ \epsilon_y = \frac{1}{E_p} \left[\sigma_y - \mu(\sigma_x + \sigma_z) \right] - \frac{d_{3y} g_{3y} F}{wt} \\ \epsilon_z = \frac{1}{E_p} \left[\sigma_z - \mu(\sigma_x + \sigma_y) \right] - \frac{d_{3z} g_{3z} F}{wt} \end{array} \right\} \rightarrow \text{Required Set of Equations}$$

Q.2) _____?

Solⁿ:(a) Relation b/w voltage (V) and gSince, $g = \frac{\text{Electric field developed}}{\text{Stress in material}}$

$$\Rightarrow g = \frac{E}{\sigma}$$

$$\Rightarrow E = g\sigma = g \frac{F}{tW} = \frac{V}{t}$$

$$\therefore \boxed{V = \frac{gF}{W}}$$

(b) Charge Accumulation (q) and dSince, $d = \frac{\text{Electric Displacement vector}}{\text{Stress developed}}$

$$\Rightarrow d = \frac{D}{\sigma} \text{ or } D = d\sigma$$

$$\Rightarrow \frac{q}{A} = d\sigma \quad \left(\because F = \sigma A \right)$$

$$\therefore \boxed{q = dF}$$

(c) Relation b/w coupling coefficient k, g, dcoupling coeff. , $k = \sqrt{\frac{\text{Electrical energy}}{\text{Mechanical work}}}$

$$\Rightarrow k = \sqrt{\frac{W_E}{W_M}}$$

$$= \sqrt{\frac{Q^2 \times 2}{2C_p F \Delta_z}}$$

$$\left(\because \Delta_z \text{ is deflection in } P_z \text{ material} \right)$$

$$\Rightarrow k^2 = \frac{Q^2}{F \Delta_z C_p}$$

Now, $Q = dF \Rightarrow Q^2 = d^2 F^2$

we get $k^2 = \frac{d^2 F^2}{F \Delta_z C_p} = \frac{d^2 F}{\Delta_z C_p}$

Also, since, $C_p = \frac{(\epsilon_0 \times \text{area})}{\text{thickness}}$ permittivity

we get, $k^2 = \frac{d^2 F}{\Delta_z \epsilon_0 \frac{lw}{t}} = \frac{d^2}{\left(\frac{\Delta_z}{t}\right) \times \epsilon_0 \left(\frac{F}{lw}\right)} = \frac{d^2}{S \times \epsilon_0}$

where, S is compliance

Hence, $k^2 = \frac{d^2}{S \epsilon_0}$ ————— (A)

Since, $g = \frac{\text{charge density}}{\text{Permittivity}} = \frac{d}{\epsilon_0}$ ————— (a)

and $S = \frac{1}{E_p}$ ————— (b)

Using (a), (b) in (A)

$k^2 = d g E_p$ ————— Required Relation