Convolution Algorithm

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Realizing convolution algorithm

$$X_m = \sum_{n=0}^{N-1} x_n w^{nm},$$

where N is the size of the vectors, $w = e^{2i\pi/N}$

$$X_m = \sum_{n=0}^{N/2-1} x_n w^{nm} + w^{mN/2} \sum_{n=0}^{N/2-1} x_{n+N/2} w^{nm},$$

$$X_m = \sum_{n=0}^{N/2-1} x_{2n} w^{2nm} + w^m \sum_{n=0}^{N/2-1} x_{2n+1} w^{2nm}.$$

Matix representation

$$\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ 1 & w^2 & w^4 & w^6 & w^8 & w^{10} & w^{12} & w^{14} \\ 1 & w^3 & w^6 & w^9 & w^{12} & w^{15} & w^{18} & w^{21} \\ 1 & w^4 & w^8 & w^{12} & w^{16} & w^{20} & w^{24} & w^{28} \\ 1 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} & w^{30} & w^{35} \\ 1 & w^6 & w^{12} & w^{18} & w^{24} & w^{30} & w^{36} & w^{42} \\ 1 & w^7 & w^{14} & w^{21} & w^{28} & w^{35} & w^{42} & w^{49} \end{bmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

$$X_m = \sum_{n=0}^{N-1} x_n w^{nm},$$

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Matix representation – arranging in even terms

$$
\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{pmatrix}
=
\left[\begin{array}{cccc|cccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & w^2 & w^4 & w^6 & w & w^3 & w^5 & w^7 \\
1 & w^4 & w^8 & w^{12} & w^2 & w^6 & w^{10} & w^{14} \\
1 & w^6 & w^{12} & w^{18} & w^3 & w^9 & w^{15} & w^{21} \\
\hline
1 & w^8 & w^{16} & w^{24} & w^4 & w^{12} & w^{20} & w^{28} \\
1 & w^{10} & w^{20} & w^{30} & w^5 & w^{15} & w^{25} & w^{35} \\
1 & w^{12} & w^{24} & w^{36} & w^6 & w^{18} & w^{30} & w^{42} \\
1 & w^{14} & w^{28} & w^{42} & w^7 & w^{21} & w^{35} & w^{49}
\end{array}\right]
\left(\begin{array}{c} x_0 \\ x_2 \\ x_4 \\ x_6 \\ \hline x_1 \\ x_3 \\ x_5 \\ x_7 \end{array}\right)
$$

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Matix representation – arranging in even terms

$$
\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{pmatrix}
=
\left[\begin{array}{cc|cc|cc|cc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & w^4 & w^2 & w^6 & w & w^5 & w^3 & w^7 \\
\hline
1 & w^8 & w^4 & w^{12} & w^2 & w^{10} & w^6 & w^{14} \\
1 & w^{12} & w^6 & w^{18} & w^3 & w^{15} & w^9 & w^{21} \\
\hline
1 & w^{16} & w^8 & w^{24} & w^4 & w^{20} & w^{12} & w^{28} \\
1 & w^{20} & w^{10} & w^{30} & w^5 & w^{25} & w^{15} & w^{35} \\
\hline
1 & w^{24} & w^{12} & w^{36} & w^6 & w^{30} & w^{18} & w^{42} \\
1 & w^{28} & w^{14} & w^{42} & w^7 & w^{35} & w^{21} & w^{49}
\end{array}\right]
\left(\begin{array}{c} x_0 \\ x_4 \\ \hline x_2 \\ x_6 \\ \hline x_1 \\ x_5 \\ \hline x_3 \\ x_7 \end{array}\right)
$$

Replacing with twiddle factors (mod 8)

$$
\left[\begin{array}{cc|cc|cc|cc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & w^{4} & w^{2} & w^{6} & w & w^{5} & w^{3} & w^{7} \\
\hline
1 & w^{8} & w^{4} & w^{12} & w^{2} & w^{10} & w^{6} & w^{14} \\
1 & w^{12} & w^{6} & w^{18} & w^{3} & w^{15} & w^{9} & w^{21} \\
\hline
1 & w^{16} & w^{8} & w^{24} & w^{4} & w^{20} & w^{12} & w^{28} \\
1 & w^{20} & w^{10} & w^{30} & w^{5} & w^{25} & w^{15} & w^{35} \\
\hline
1 & w^{24} & w^{12} & w^{36} & w^{6} & w^{30} & w^{18} & w^{42} \\
1 & w^{28} & w^{14} & w^{42} & w^{7} & w^{35} & w^{21} & w^{49}
\end{array}\right]
$$

$$
\begin{pmatrix}
W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 \\
W^0 & W^4 & W^2 & W^6 & W^1 & W^5 & W^3 & W^7 \\
W^0 & W^0 & W^4 & W^4 & W^2 & W^2 & W^6 & W^6 \\
W^0 & W^4 & W^6 & W^2 & W^3 & W^7 & W^1 & W^5 \\
W^0 & W^0 & W^0 & W^0 & W^4 & W^4 & W^4 & W^4 \\
W^0 & W^4 & W^2 & W^6 & W^5 & W^1 & W^7 & W^3 \\
W^0 & W^0 & W^4 & W^4 & W^6 & W^6 & W^2 & W^2 \\
W^0 & W^4 & W^6 & W^2 & W^7 & W^3 & W^5 & W^1
\end{pmatrix}
$$

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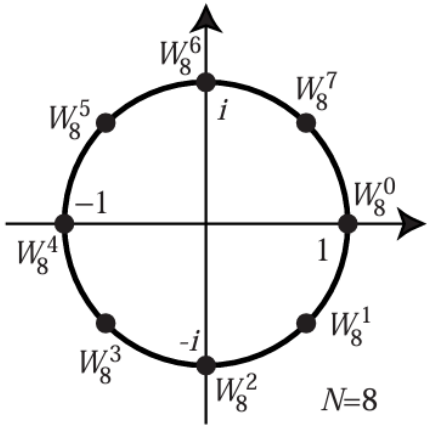
FFT algorithm



$$
\begin{bmatrix}
W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 \\
W^0 & W^4 & W^2 & W^6 & W^1 & W^5 & W^3 & W^7 \\
W^0 & W^0 & W^4 & W^4 & W^2 & W^2 & W^6 & W^6 \\
W^0 & W^4 & W^6 & W^2 & W^3 & W^7 & W^1 & W^5 \\
W^0 & W^0 & W^0 & W^0 & W^4 & W^4 & W^4 & W^4 \\
W^0 & W^4 & W^2 & W^6 & W^5 & W^1 & W^7 & W^3 \\
W^0 & W^0 & W^4 & W^4 & W^6 & W^6 & W^2 & W^2 \\
W^0 & W^4 & W^6 & W^2 & W^7 & W^3 & W^5 & W^1
\end{bmatrix}
\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{pmatrix}
=
\left[\begin{array}{cc|cc|cc|cc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & w^2 & -w^2 & w & -w & w^3 & -w^3 \\
\hline
1 & 1 & -1 & -1 & w^2 & w^2 & -w^2 & -w^2 \\
1 & -1 & -w^2 & w^2 & w^3 & -w^3 & w & -w \\
\hline
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & w^2 & -w^2 & -w & w & -w^3 & w^3 \\
\hline
1 & 1 & -1 & -1 & -w^2 & -w^2 & w^2 & w^2 \\
1 & -1 & -w^2 & w^2 & -w^3 & w^3 & -w & w
\end{array}\right]
\left(\begin{array}{c} x_0 \\ x_4 \\ \hline x_2 \\ x_6 \\ \hline x_1 \\ x_5 \\ \hline x_3 \\ x_7 \end{array}\right)
$$

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Recursive matrix multiplications

The recursive sum can be represented as a sequence of matrix transformations:

$$(X) = [A_2][A_1][A_0][P](x),$$

$$\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{pmatrix} = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & W^0 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & W^1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot & W^2 & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & W^3 \\ 1 & \cdot & \cdot & \cdot & W^4 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & W^5 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot & W^6 & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & W^7 \end{bmatrix} \begin{bmatrix} 1 & \cdot & W^0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & W^2 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & W^4 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & W^6 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & W^0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & W^2 \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & W^4 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & W^6 \end{bmatrix} \begin{bmatrix} 1 & W^0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & W^4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & W^0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & W^4 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & W^0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & W^4 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & W^0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & W^4 \end{bmatrix} \left(\begin{array}{c} x_0 \\ x_4 \\ \hline x_2 \\ x_6 \\ \hline x_1 \\ x_5 \\ \hline x_3 \\ x_7 \end{array}\right)$$

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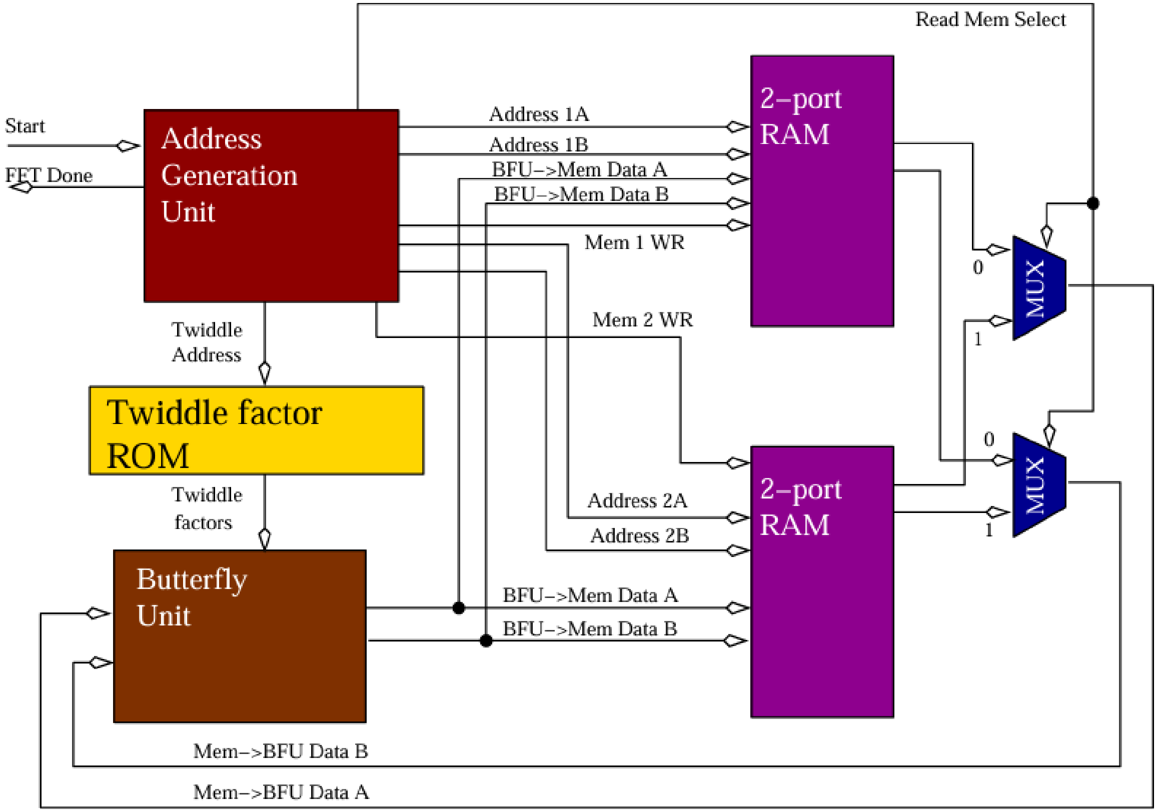
The full transform requires – step 1

1) an address generator

ILLUSTRATION OF THE BIT-REVERSED INDICES.

Index	binary	Bit reversed index	binary
0	000	0	000
1	001	4	100
2	010	2	010
3	011	6	110
4	100	1	001
5	101	5	101
6	110	3	011
7	111	7	111



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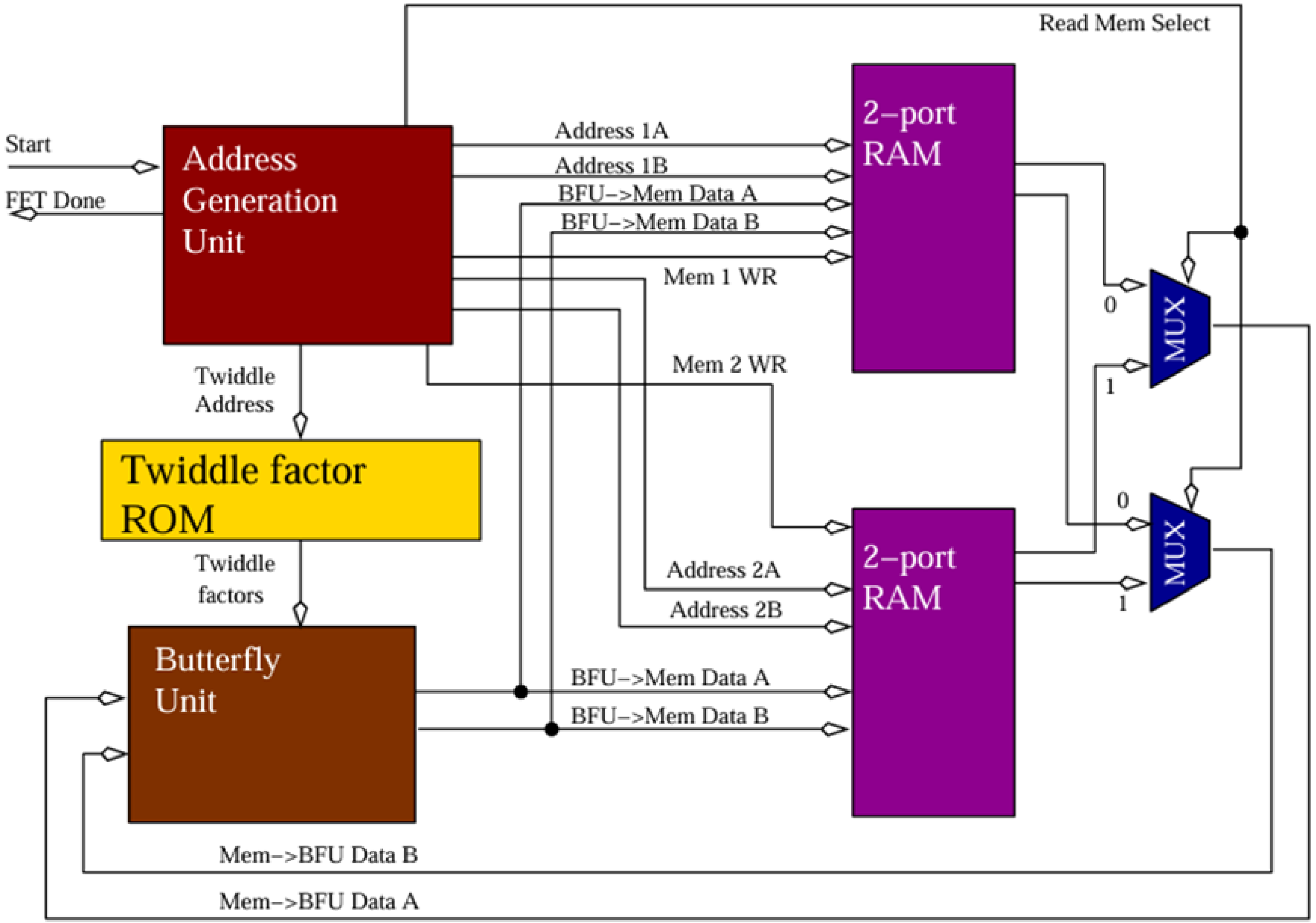
The full transform requires – step 2

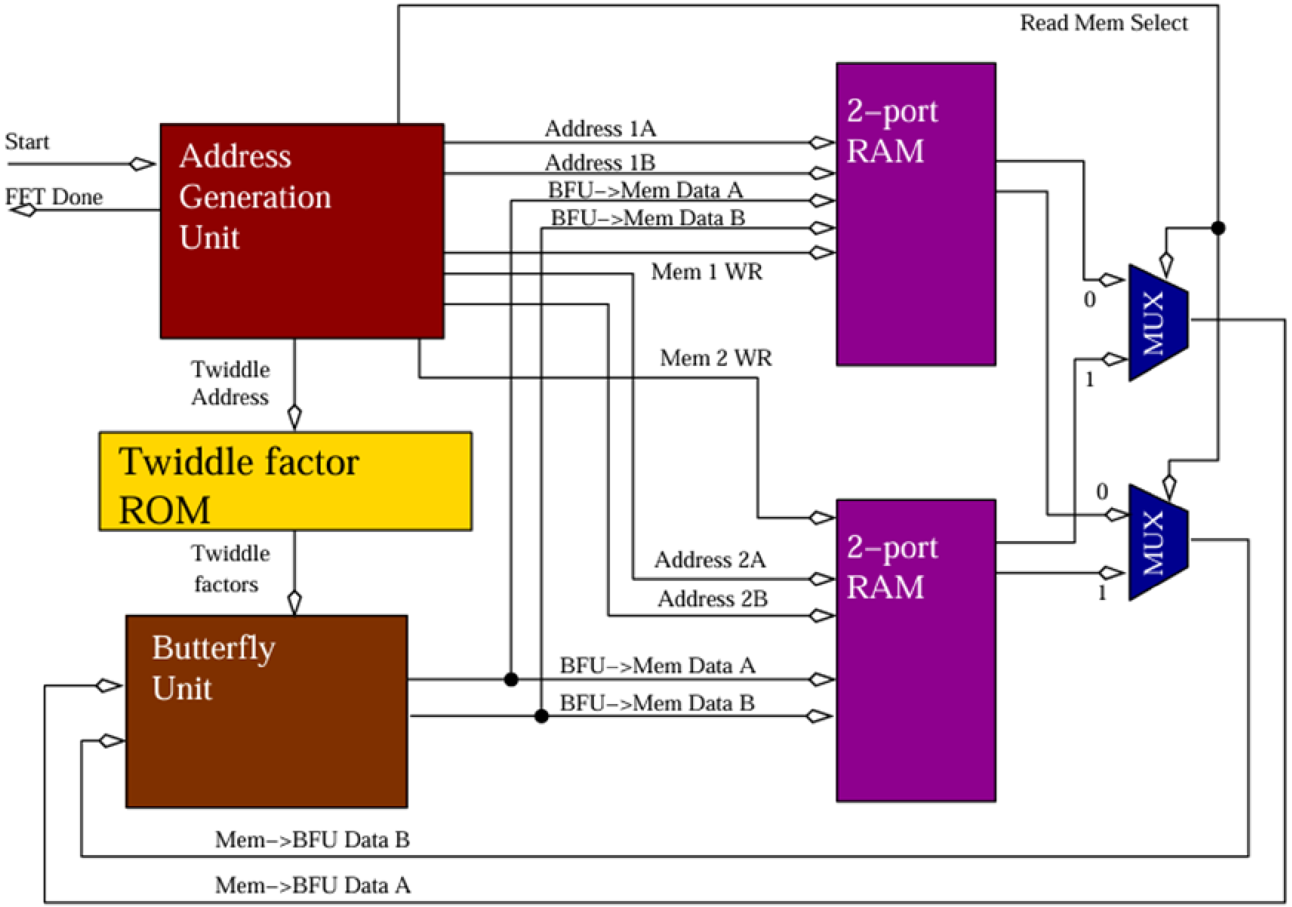
2) a “butterfly” operator to do the complex multiply/add,



Limited

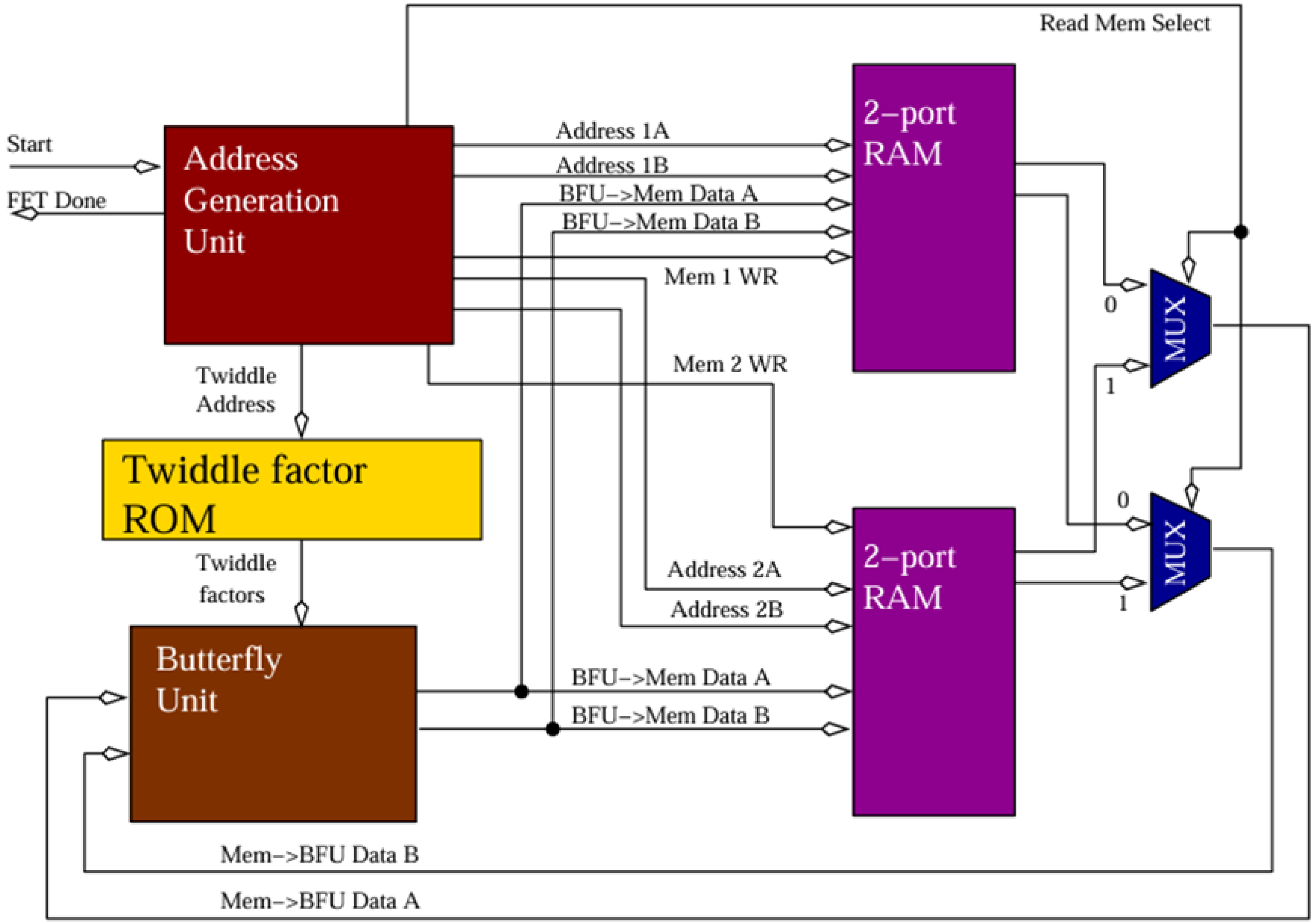
The full transform requires – step 3

3) A memory and



The full transform requires – step 4

4) roots-of-unity(twiddle factor) generator



Limited

Computation time

N	1000	10^6	10^9
N^2	10^6	10^{12}	10^{18}
$N\log_2 N$	10^4	$20\ x\ 10^6$	$30\ x\ 10^9$

$$10^{18} ns \rightarrow 31.2\ years$$

$$30\ x\ 10^9 ns \rightarrow 30\ seconds$$

N	$r = \log_2 N$	**BRUTE FORCE** $4N^2$	**FFT** $2N\log_2 N$	speedup
2	1	16	4	4
4	2	64	16	4
8	3	256	48	5
1,024	10	4,194,304	20,480	205
65,536	16	$1.7 \cdot 10^{10}$	$2.1 \cdot 10^6$	~10^4

$$\left[\begin{array}{cc|cc|cc|cc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & w^2 & -w^2 & w & -w & w^3 & -w^3 \\
\hline
1 & 1 & -1 & -1 & w^2 & w^2 & -w^2 & -w^2 \\
1 & -1 & -w^2 & w^2 & w^3 & -w^3 & w & -w \\
\hline
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & w^2 & -w^2 & -w & w & -w^3 & w^3 \\
\hline
1 & 1 & -1 & -1 & -w^2 & -w^2 & w^2 & w^2 \\
1 & -1 & -w^2 & w^2 & -w^3 & w^3 & -w & w
\end{array}\right]$$

$$\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & w^2 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -w^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & w^2 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -w^2
\end{bmatrix}$$

$$[A_2] = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & w & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & w^2 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & w^3 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -w & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -w^2 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -w^3
\end{bmatrix}$$

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