1) Introduction

Motivation

When measuring the temperature of a fluid, thermal equilibrium between the thermometer and the fluid has to be achieved for an accurate reading. Depending on the heat capacity of the thermometer and the rate of heat transfer, the time taken to reach this thermal equilibrium may vary. This waiting time can be reduced by introducing machine learning to create a predictive model based on a given heat capacity and rate of heat transfer, to give an accurate temperature value before thermal equilibrium has been reached.

Objectives

1. Predict temperature of fluid within ± 1.5°C of temperature given by commercial thermometer which has achieved thermal equilibrium with fluid
2. Prediction value should be given out within $25s$ after temperature sensor is in contact with the fluid
3. Working range of temperature prediction should be from $10-60$°C

Assumptions

Some assumptions are made regarding the apparatus and procedures involved in the experiment:

1. The reading of commercial thermometer is assumed to accurately represent the instantaneous temperature of water.
2. Conditions experienced by water body is assumed to be constant throughout data collection and on the day of testing (i.e. wind conditions, ambient temperature).
3. Heat capacity of temperature sensor is assumed to remain constant between $10-60$°C .

Hypothesis

From our pre-analysis, the relationship between the temperature on the sensor, $T_s(t)$, and the actual temperature of the fluid (water is used in our experiments) T_w , is hypothesised to satisfy the following equation:

$$T_s(t) = (T_{amb} - T_w)\, e^{-\frac{t}{\tau}} + T_w \qquad \text{[Eq 1.0]}$$

where,
T_{amb} *is the ambient temperature*
t is the time elapsed after inserting sensor
τ *is the time constant*

$\tau = \frac{C_s}{\lambda}$, where C_s is the heat capacity of the temperature sensor and λ is the total thermal conductance between water and the temperature sensor. For this experiment, we are conducting assuming that C_s remains constant. This allows us to assert that τ is independent of temperature, which results in a linear relationship between the raw output of the sensor, and the actual temperature of the water. We will further elaborate on the justification behind this assertion under the section “Results and Discussion”.

2) Methods

Collection of Training Data Set

- Prepare an insulated water bath of 10°C and insert a commercial thermometer. Let the set-up sit for 5 minutes to reach thermal equilibrium

- A Python program is used for the recording of the raw output of temperature sensor. Start the program and insert the sensor into the water bath simultaneously.
- The program reads and records the raw output of the sensor once every 2 seconds, up till 30 seconds from start of experiment.
- Repeat experiment 10 times with water bath of the same temperature
- Repeat procedure with water bath of temperatures of 15℃, 20℃, 25℃, …. , and 60℃

Data Processing and Building Prediction Model

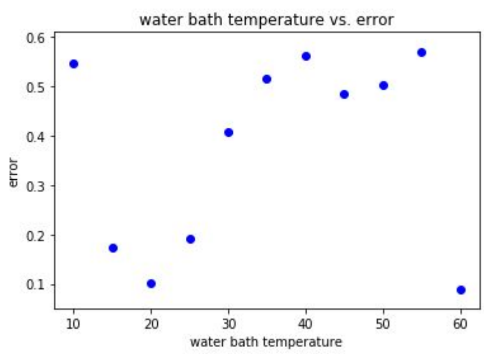
- Linear Regression is used to build prediction model, where there is a linear relationship between the actual temperature of water (T_w) and the raw output of the sensor (T_s).
- A 140 x 2 matrix is built using data collected. The value of all entries in the first column is 1, and the second column consist of the output of the sensor at $25s$ from the start of the experiment. 140 experiments are conducted where we vary the temperature of the water from $10-60$℃ at 5℃ intervals. Each experiment would be represented by a row in the matrix.
- Every 14 rows represents experiments conducted for a fixed temperature of water. The values of all these experiments are averaged, and a row of data now consists of the value 1, as well as the average output of the sensor $25s$ after the start of the experiment.
- Vector B is created, which consist of 11 water bath temperatures from $10-60$℃
- Matrix A is projected on vector B. Using the equation $c = (A^T A)^{-1} A^T * b$
- Vector C is found which represents the coefficients of the linear relationship between T_w and T_s. This relationship is expressed as $T_w = c_1 T_s + c_0$ where $c = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$

Testing our Prediction Model

- Prepare an insulated water bath of 10℃ and insert a commercial thermometer. Let the set-up sit for 5 minutes to reach thermal equilibrium
- Insert temperature sensor and start the prediction algorithm simultaneously
- Record the difference between the predicted and actual temperature of the water bath
- Repeat experiment 2 times
- Repeat the above procedure with temperature increments of 5℃ until $T_w = 60$℃
- Find the average temperature difference , $< \Delta T >$, for each T_w

3) Result and Discussion

Error Margin of Predicted Temperatures



Justification for using τ as a constant

We seek to calculate the difference in the value of τ where the temperature of water is 10℃ and 60℃, to understand its significance in the accuracy of our predictive model.

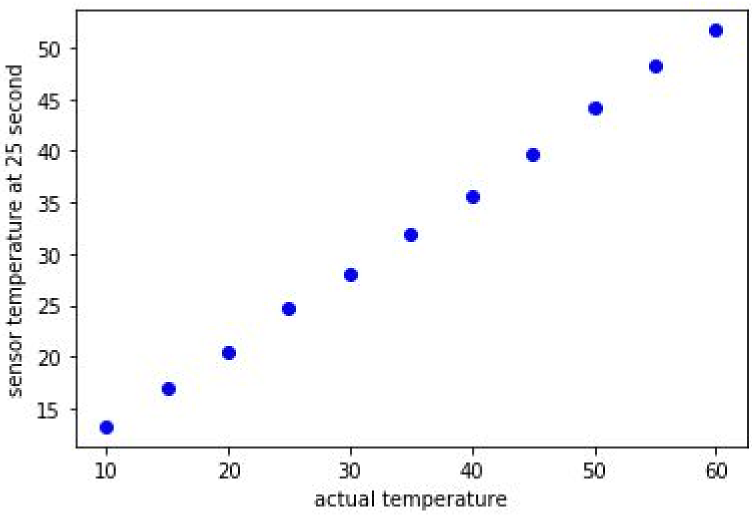
The percentage difference in τ is found to range between 6.67% to 7.15%. *[Refer to Appendix A for details on procedure of experiment]*. Although such a difference in the value of τ may affect the accuracy of our prediction model, our team has decided to accept the margin of error given that the range of temperature this sensor would be used, is small. Hence the percentage difference in τ would not be large enough to significantly compromise the accuracy of our prediction model.

Sources of Error

- *Change in T_w due to heat loss to surroundings*
  While this source of error is reduced by using an insulated container (vacuum flask) for the water bath, but there is still some heat loss due to conduction through the container walls and due to convection from the surface of the water. Hence, predicted temperature may be slightly higher (if water is hot) or lower (if water is cold) than expected. To further reduce heat transfer due to convection, we used flask with a narrow opening for smaller surface area of water exposed, and conducted the experiment in an environment without strong winds.
- *Temperature gradient of the water body is not constant.*
  The output of the sensor varies based on its position relative to the commercial thermometer, and the nearer the two, the more similar their temperature gradients. To minimise this, our team place the sensor close to the thermometer when conducting the experiments
- Errors due to non-constant values of τ
  As the temperature sensor would be applied over a small range of temperature (between 10℃ and 60℃), errors in prediction due to τ would not compromise the accuracy of the predictive model significantly.

4) Conclusion – summarizes all key findings and results

Our prediction model has an accuracy of ___. This model would be applied over a small range between 10℃ and 60℃.



5) Appendix – Supporting data, analysis and document. Place your raw data in the appendix instead of placing them in the main report. Results should be placed in the section 3 and referenced to the appendix where the data is used to arrive at the result.

Appendix A - The values of Tau under different temperatures of water

Method of obtaining the values of τ

To Determine τ for $T_w = 10$℃ and $T_w = 60$℃

- Using data collected for training set, randomly choose 3 sets of data at 10℃, and 60℃ where $t > 25s$.
- Manipulate the equation in hypothesis [Eq 1.0] to obtain: $\tau = \dfrac{t}{ln\left(\frac{T_{amb}-T_w}{T_s(t)-T_w}\right)}$
- Find the average raw output from the temperature sensor for the temperature of water at 10℃ and 60℃, and calculate the value of τ using these average values.
- Then, calculate the percentage difference between the two averages obtained.

Temperature	10℃	60℃
Average τ value	19.08504291	17.81175075
% difference	$\frac{19.08504291-17.81175075}{19.08504291} \times 100\%$ $= 6.67\%$	$\frac{19.08504291-17.81175075}{17.81175075} \times 100\%$ $= 7.15\%$

We conclude that the percentage difference of τ ranges between 6% - 7%.

Appendix B - Pre-Analysis

Pre-Analysis Page 1

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Names of Group Members: Group No.: 8 Cohort Class: 17F01.

2D Pre-Analysis Worksheet

In this 2D project, you will be writing a program that reads data from a temperature sensor and uses machine learning and statistical analysis to predict the actual temperature of a water bath accurately within the shortest time possible. Attach this analysis worksheet to the appendix of your final PW report on week 12 Friday 6 pm.

Analysis of a transient problem

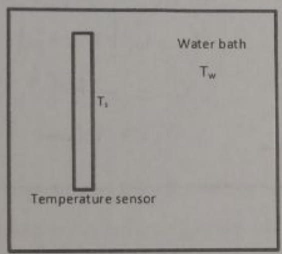

Figure 1

T_s: temperature on sensor which is a function of time

T_w: temperature of the water bath

T_{amb}: ambient temperature

C_s:heat capacity of the sensor

λ: combined thermal conductance (water to sensor)

1) Write the time dependent First Law.

$$\frac{dE}{dt} = \dot{Q}(t) - \dot{W}(t)$$

2) Write the time dependent first Law for your temperature sensor using Figure 2.

$$\frac{dE}{dt} = \dot{Q}(t)$$

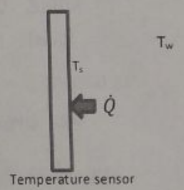

Figure 2

3) Write two separate expressions for $\dot{Q}$ and $\frac{dE}{dt}$

$$\frac{dE}{dt} = C_s \frac{dT_s}{dt}$$

$$\dot{Q} = \lambda \left(T_w - T_s(t) \right)$$

4) Form a first order differential equation using the expressions in (3).

$$C_s \frac{dT_s}{dt} = \lambda \left(T_w - T_s(t) \right)$$

$$\frac{dT_s}{dt} = \frac{\lambda}{C_s} \left(T_w - T_s(t) \right)$$

Pre-Analysis Page 2

Solution: $A(t) = A_f\left(1-e^{-\frac{t}{\tau}}\right)$

$$\frac{dT_s}{dt} = \frac{\lambda}{C_s}\left[T_w \; T_s\right)$$

5) What is the solution for this first-order differential equation? For this pre-analysis, you can assume T_w is constant, and C_s and λ are independent of Temperature.

$$\frac{dT_s}{dt} = -\frac{1}{\frac{C_s}{\lambda}}\left(T_s(t) - T_w\right)$$

$$= -\frac{1}{\tau}\left(T_s(t) - T_w\right), \quad \text{where } \tau = \frac{C_s}{\lambda}$$

$$\cancel{T_s(t) = T_w\left(1 - e^{-\frac{t}{\tau}}\right)}$$

$$\cancel{= T_w\left(1 - e^{-\frac{t}{\left(\frac{C_s}{\lambda}\right)}}\right)}$$

$$\int \frac{1}{\tau}\,dt = \int \frac{1}{T_w - T_s}\,dT_s$$

$$-\frac{t}{\tau} = \ln\left|T_w - T_s\right| + C$$

$$T_s = -Ae^{-\frac{t}{\tau}} + T_w$$

$$T_s = (T_{amb} - T_w)e^{-\frac{t}{\left(\frac{C_s}{\lambda}\right)}} + T_w.$$

when $t=0$, $T_s = T_{amb} \Rightarrow -A + T_w = T_{amb}$

6) The solution to the first-order differential equation should have a $e^{-\frac{t}{\left(\frac{C_s}{\lambda}\right)}}$ term. It is often expressed as $e^{-\left[\frac{t}{\tau}\right]}$ where τ is called the time constant. What is the significance of a large τ vs. a small τ?

$\tau = \frac{C_s}{\lambda}$

As $\tau \to \infty$, $T_s(t) \to 0$.

As $\tau \to 0$, $T_s(t) \to T_w$

Large τ means that temperature of sensor ~~approaches~~ is very small,
small τ means that temperature of senor is very close to temperature of water bath.

7) Given $T_w = 80\ °C$, $T_{amb} = 25\ °C$, plot the T_s vs. time curves for $\tau = 10\ s$ and $\tau = 50\ s$.

see attached graph.

8) Do you think the assumption that τ is independent of temperature is valid? Design a series of experiments to determine τ and prove your assumption.

Yes.

Experimental design:

1. Prepare an insulated water bath of 40°C. Use a thermometer to measure water temperature at intervals of 5 seconds to ensure its temperature is constant.
2. Insert temperature sensor probe and start stopwatch.
3. Stop stopwatch when sensor reading and thermometer reading are the same. Record time elapsed.
4. Using equation $T_s(t) = T_w\left(1 - e^{-\frac{t}{\tau}}\right)$, calculate τ
5. Repeat 1-4 with water bath of temperatures, 20 and 30 °C.

If τ obtained differ by less than 5% from one another, assumption is true.