Homework 3

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1 Knights and Knaves

1. One day a traveller was wandering around the island of Knights and Knaves, when he encountered two local inhabitants, P and Q. The traveller asked: "Is any of you a knave?". P replied: "At least one of us is a knave".

Can you find out what P and Q are? If so, what are they? If not, explain why not, and what other information you would need to know.

Yes you can find out what P and Q are.

P is a Knight

Q is a Knave

2. Later on, the traveller met two other locals , A and B. He asked whether either of them is a knight . A replied: "If B is a knave, then I am a knave too".

What are A and B?

A is a Knight

B is a Knight

2 Logical Identities

1. $\neg(p \rightarrow (q \rightarrow p))$

$\neg(p \rightarrow (\neg p \wedge q))$ Implication Rule

$\neg((p \to \neg p) \wedge (p \to q))$ Distribution Rule

$\neg((\neg p \wedge \neg p) \wedge (\neg p \wedge q))$ Implication Rule

$((\neg\neg p \wedge \neg\neg p) \wedge (\neg\neg p \wedge q))$ DeMorgans Rule

$((p \wedge p) \wedge (p \wedge q))$ Double Negative

$(T) \wedge (p \wedge q)$ Idempotent Laws

Simplified Answer

$p \wedge q$

2. $\neg((p \wedge q) \to (q \vee p))$

$(\neg p \vee \neg q) \to (\neg q \wedge \neg p)$ DeMorgans Law

$\neg(\neg p \vee \neg q) \vee (\neg q \wedge \neg p)$ Implication Rule

$(\neg\neg p \wedge \neg\neg q) \vee (\neg q \wedge \neg p)$ DeMorgans Law

$(p \wedge q) \vee (\neg q \wedge \neg p)$ Double Negative

$(p \wedge \neg p) \vee (q \wedge \neg q)$ Associative

$F \vee F$ Identity Laws

Simplified Answer:

Contradiction

3 Logical Equivalences

1. $p \to (q \to r) and (p \vee q) \to r$

LeftSide:

$p \to (\neg q \vee r)$ Implication Rule

$(p \to \neg q) \vee (p \to r)$ Distribution Law

$(\neg p \vee \neg q) \vee (\neg p \vee r)$ Implication Rule

$(\neg p \vee \neg p) \vee (\neg q \vee r)$ Associative Law

$F \vee (\neg q \vee r)$ Identity Law

Left Side Answer : $(\neg q \vee r)$

Right Side:

$(p \to r) \vee (p \to q)$ Distributive Law

$(\neg p \vee r) \wedge (\neg p \vee q)$ Implication Law

$(\neg p \vee \neg p) \wedge (r \vee q)$ Identity Laws

$F \wedge (q \vee r)$

Answer:

$(\neg q \vee r)$ and $(q \vee r)$

They are not equivalent

2. $p \to (q \to r) and (p \to q) \to r$

LeftSide:

$p \to (\neg q \vee r)$ Implication Rule

$(p \to \neg q) \vee (p \to r)$ Distribution Law

$(\neg p \vee \neg q) \vee (\neg p \vee r)$ Implication Rule

$(\neg p \vee \neg p) \vee (\neg q \vee r)$ Associative Law

$F \vee (\neg q \vee r)$ Identity Law

Left Side Answer : $(\neg q \vee r)$

Right Side:

$(\neg p \vee q) \to r$ Implication Rule

$(\neg p \to r) \vee (q \to r)$ Distribution Rule

$(\neg \neg p \vee r) \vee (\neg q \vee r)$ Implication Rule

$(p \vee r) \vee (\neg q \vee r)$ double negative

$(r \vee r) \vee (\neg q \vee p)$ Associative property

$T \vee (\neg q \vee p)$ Identity law

$(\neg q \vee p)$ Answer:

$(\neg q \vee r)$ and $(\neg q \vee p)$

NOT EQUAL

4 Logical Consequence

1. $\dfrac{\begin{array}{c}\text{Jimmy is smart} \\ \text{Smart people are rich}\end{array}}{\text{Jimmy is rich}}$

The Statement is valid by Hypothetical Syllogism.

$$\frac{\begin{array}{c}P \to Q \\ Q \to R\end{array}}{P \to R}$$

p	q	r	$p \to q$	$q \to r$	$p \to r$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

2. $\dfrac{\begin{array}{c}\text{Islands are surrounded by water} \\ \text{Puerto Rico is surrounded by water}\end{array}}{\text{Puerto Rico is an island}}$

The Statement is valid because of the conjunction rule

$$\frac{\begin{array}{c}\text{P} \\ \text{Q}\end{array}}{P \wedge Q}$$

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1