HomeWork 4

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1 Mathematical Proofs

1. The sum of two odd integers is even.

Proof:

Let a and b be odd integers. By definition $a = 2n + 1$ and $b = 2m + 1$ the sum of $a + b = (2n+1) + (2m+1) = 2n + 2m + 2 = 2k$ where $k = n + m + 1$ is an integer. Therefore by definition of even we have shown that $a + b$ is even and the hypothesis is true.

2. The sum of two even integers is even.

Proof:

Let a and b be even integers. By definition of even we have that $a = 2n$ and $b = 2m$ Consider $a + b = 2n + 2m = 2(n + m) = 2k$ where $k = n + m$ is an integer. Therefore by definition we have shown the hypothesis is true.

3. The square of an even number is even.

Proof:

Let $a = 2n$ be an even integer. the square of $a^2 = (2n)^2$ $a^2 = (2n)^2 = 4n^2 = 2(2n^2)$ Since $2n^2$ is in the form of $2n$ we have proven that the square of an even integer is even.

4. The product of two odd integers is odd.

Proof:

Let n and m be two odd integers. $n = 2a + 1$ and $m = 2b + 1$. Consider the product $nm = (2a + 1)(2b + 1) = 4ab + 2a + 2b + 1 = 2(2ab + a + b) + 1 = 2k + 1$, $k = (2ab + a + b)$ is an integer. Therefore by definition of odd we have shown that the product is odd.

5. If $n^3 + 5$ is odd then n is even, for any $n \in Z$.

Proof:

Assume n is odd and $k = (2k+1)$ so $a = n^3 + 5 = (2k+1)^3 + 5 = 8k^3 + 12K^3 + 6k + 6 = 2(4k^3 + 6k^2 + 3k + 3)$ .Thus $n^3 + 5$ is two times some integer so it is even.

6. If $3n + 2$ is even then n is even, for any $n \in Z$.

Proof:

n is an integer, therefore $a = 3n + 2$ is even, so $n = 2k + 1$ therefore, $a = 3(2k+1) + 2 = 6k + 5 = 2(3k+2) + 1$ This contradicts $3n+2$ is even, however we know it is even therefore n cannot be odd and is also even. 7. The sum of a rational number and an irrational number is irrational.

Proof:

Assume x is an irrational number, and the sum of x and a rational $\frac{a}{b}$ is a rational $\frac{c}{d}$, where a, b, c and d are integers $(b, d \neq)$ Then $x + \frac{a}{b} = \frac{c}{d}$ By subtraction, $x = \frac{c}{d} - \frac{a}{b}$, and $x = \frac{cb - ad}{bd}$ Since integers are closed under multiplication and subtraction cb, ad, bd and $cb - ad$ are integers, making $\frac{cb - ad}{bd}$ a rational number by definition. This is a contradition given that x is a irrational number. The assumption is wrong therefore proving the sum of a rational and a irrational is an irrational number.

8. The product of two irrational numbers is irrational.

Proof:

This statement is false because of the case: $\sqrt{2}\sqrt{2} = 2$ 2 is a rational number and $\sqrt{2}$ is irrational.

2 Basic Counting Principles

1. How many different three-letter initials can people have?

Answer:

26^3

2. How many different arrangements of the English alphabet are there?

Answer:

26!

3. There are 18 mathematics majors and 325 computer science majors at a college. In how many ways can two representative be picked so that one is a mathematics major and the other is a computer science major.

Answer:

$18 \cdot 325 = 5850$

4. A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?

Answer:

$12 \cdot 2 \cdot 3 = 72$

5. A multiple-choice test contains 10 questions. There are four possible answers for each questions. In how many ways an a student answer the questions on the test if the student answers every questions?

Answer:

4^{10}

6. Suppose we have the same multiple choice test as described in question 5, but we relax the assumption that the student has to answer all questions. In other words, how many ways are there for a student answer the questions on the test if the student can leave answers blank? Answer:

5^{10}