HomeWork 5

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1 Complexity Analysis

1.
```
def doNothing(someList):
    return False
```

Answer: O(1)

2.
```
def do Something(someList):
    if len(someList)== 0 :
        return 0
    else if len(list == 1) :
        return 1
    else:
        return doSomething(someList[1:])
```

Answer: O(n)

3.
```
def doSomethingElse(someList):
    n = len(someList)
    for i in range(n):
        for j in range(n):
            if someList[1] > someList[j]:
                temp = someList[1]
                someList[i] = someList[j]
                someList[j] = temp
    return someList
```

Answer: $O(n^2)$

2 Order of Complexity

1. $f(n) = 3n + 2 \in O(n)$

Answer:
$c * n \geq 3n + 2 \quad \forall n_0$
$2n + 3n \geq 3n + 2 \; \forall n > 0$

$2n \geq 2$
$n \geq 1$
inequality holds 2. $g(n) = 7 \in O(1)$

Answer:
g(n) = $7 \in O(1)$
$c * n \geq 7$
$7(n) \geq 7$
$n \geq 1$

3. $h(n) = n^2 + 2n + 4 \in O(n^2)$

Answer:
h(n) = $n^2 + 2n + 4 \in O(n^2)$
$c * n^2 \geq N^2 + 2n + 4 \; \forall > n_0$
$7n^2 \geq n^2 + 2n + 4$
$6n^2 + n^2 \geq n^2 + 2n + 4 \; \forall n > 1$
$6n^2 \geq 2n + 4$
$n^2 \geq \frac{1}{3n} + \frac{2}{3}$

3 Mathematical Induction

1. $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

Answer:
base case $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$
$n = 1$
$1 = \frac{1(1+1)}{2}$
$1 = 1$
is true
inductive hypothesis
assume n = k
$1 + 2 + 3 + ... + k = \frac{k(k+1)}{2}$
$1 + 2 + 3 + ... + k + k + 1 = \frac{k+1(k+1+1)}{2}$
$\frac{k(k+1)}{2} + k + 1 = \frac{k^2+3k+2}{2}$

$\frac{k^2}{2} + \frac{k}{2} + k + 1 = \frac{k^2}{2} + \frac{3k}{2} + 1$

$\frac{k^2}{2} + \frac{3k}{2} + 1 = \frac{k^2}{2} + \frac{3k}{2} + 1$

is true

2. $2 + 2^2 + 2^3 + 2^4 + ... + 2^n = 2^(n + 1) \; -2$

Answer:

base case

$2n^2 + 2n^3 + 2n^4$ ... $+2n^n = 2n^{n+1} - 2$

$2 + 2^2 + 2^3 + 2^4 + ... + 2^n = 2^{n+1} - 2$

$n = 1$

$2^1 = 2^{1+1} - 2$

$2 = 2^2 - 2$

$2 = 4 - 2$

$2 = 2$

is true

inductive hypothesis

assume n = k

$2 + 2^2 + 2^3 + 2^4 + 2^k = 2^{k+1} - 2$

$2^{k+1} - 2 + 2^{k+1+1} - 2$

$2^{k+1} - 2 + 2^{k+1} = 2^{k+2} - 2$

$2^{k+2} - 2 = 2^{k+2} - 2$

is true