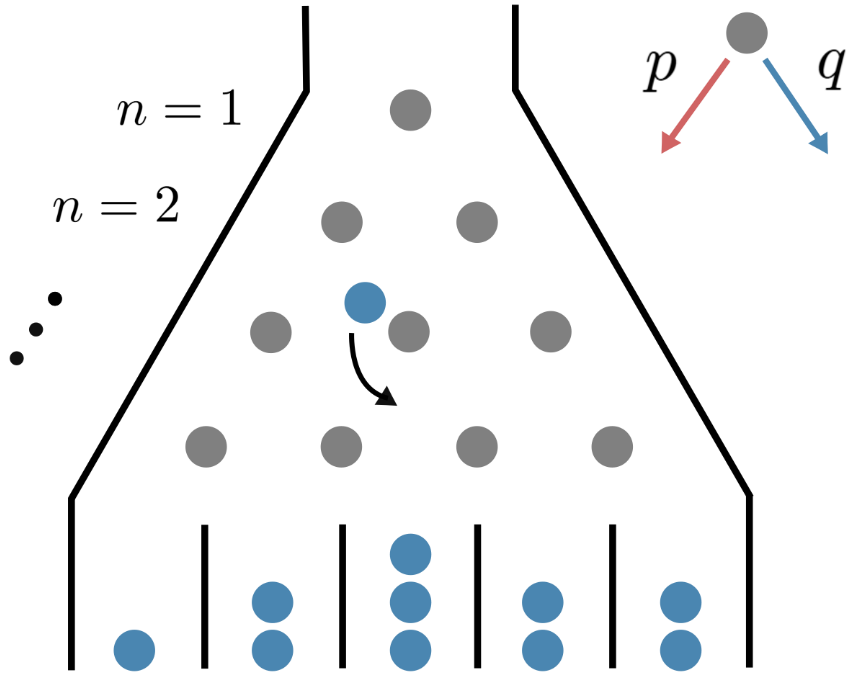
QUANTUM WALKS

AND QUANTUM MONTE CARLO

Angus Crookes | Presentation Deck

Galton Board

A statistical Simulator

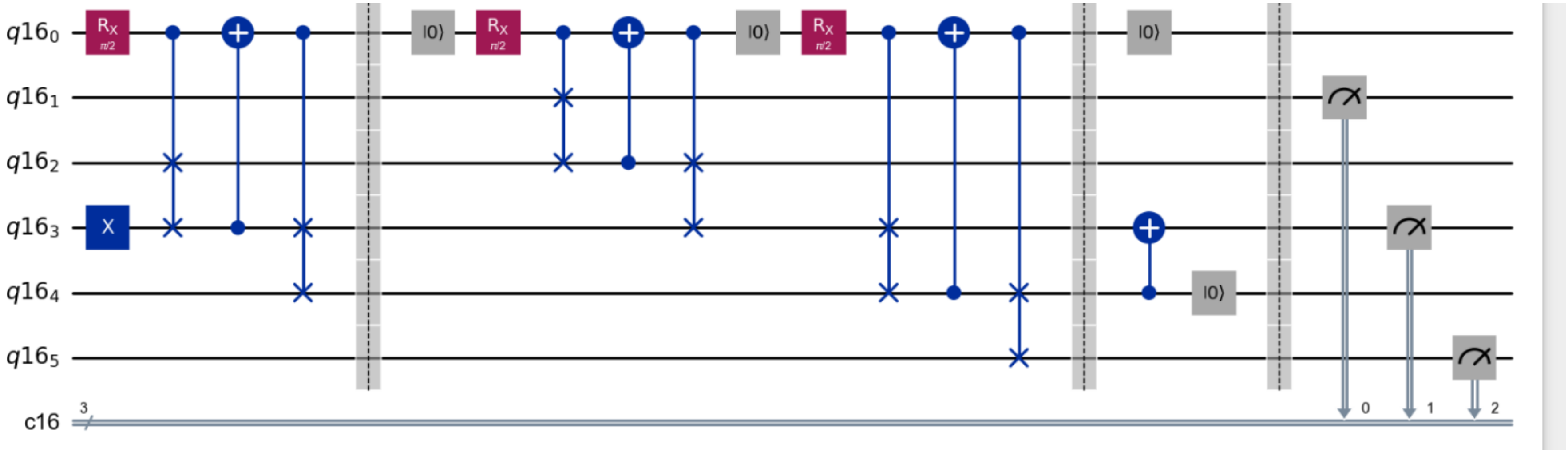


- Galton Board can simulate statistical distributions
- n layers of pegs, $n + 1$ buckets
- Buckets collect beads and form a distribution (i.e. normal)

Quantum Galton Board

Quantum analogue of classical board

- The general Quantum Galton Board for $n = 2$ and $p = 0.5$

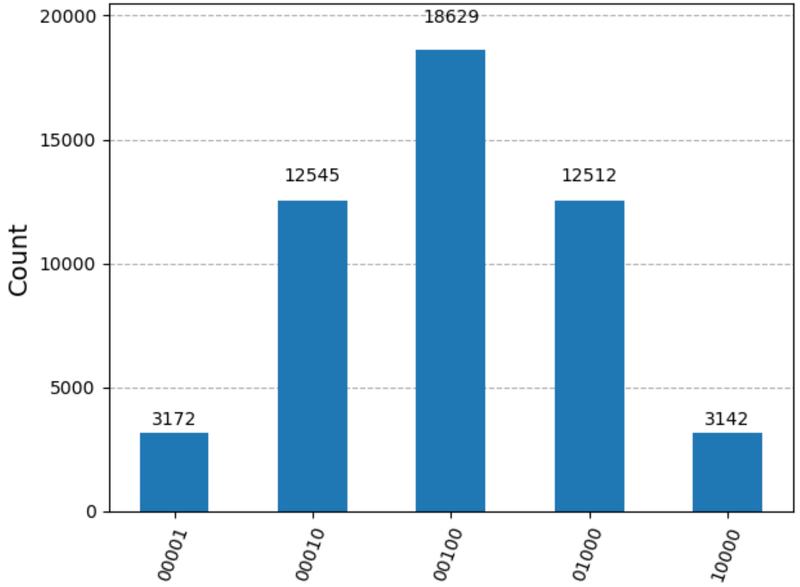


- Each set of Rx, CNOT and CSWAP represent a PEG
- Different p obtained by changing rotation angle: $\theta = 2\arcsin(\sqrt{p})$

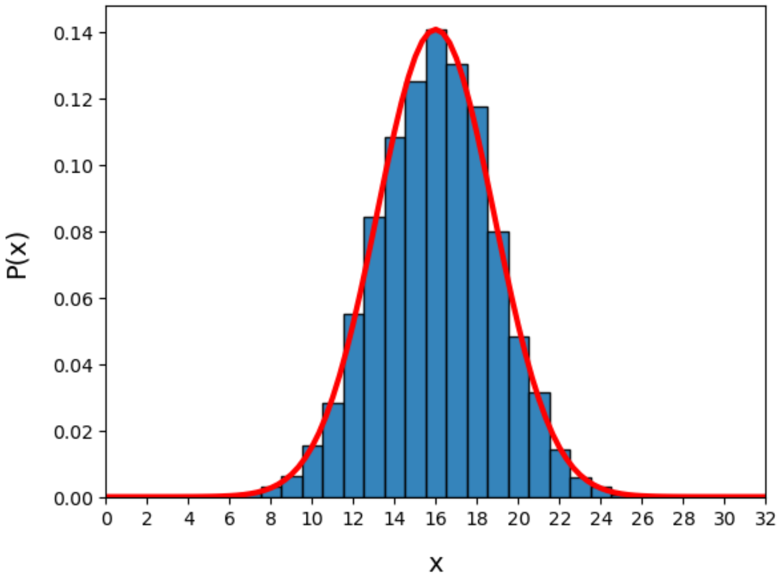
Normal distribution

n-layer board

For identical pegs with $p = 0.5$ we get a **normal distribution**



post-processing

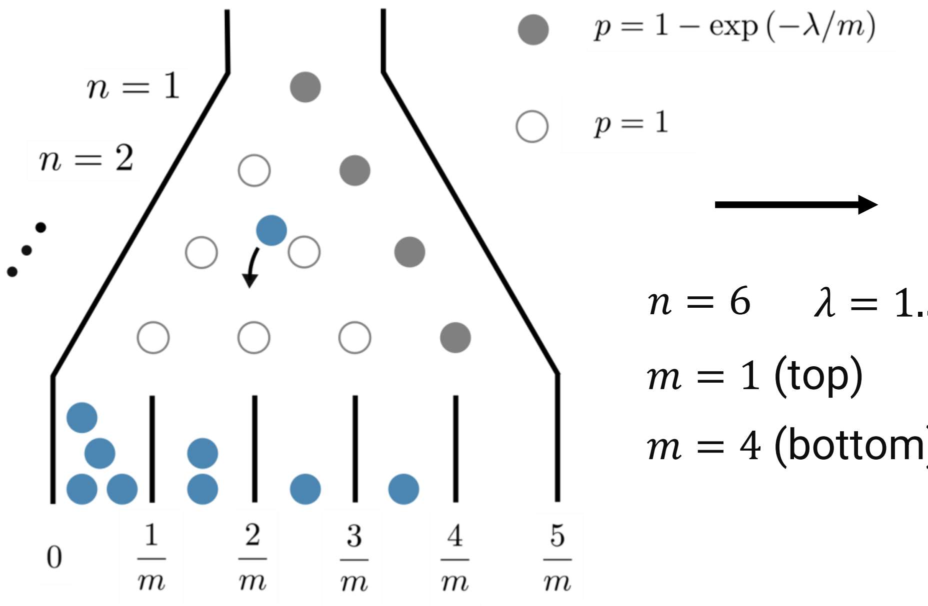


On the right is a simulation with four layers with 10000 shots. Post processing takes the quantum state outputs 00001, 00010, 00100, 01000, 10000 and maps them to 0,1,2,3,4 respectively. For each eight shots these numbers are summed, and the counts of each (x) are compared to a Gaussian distribution.

Exponential distribution

n-layer board

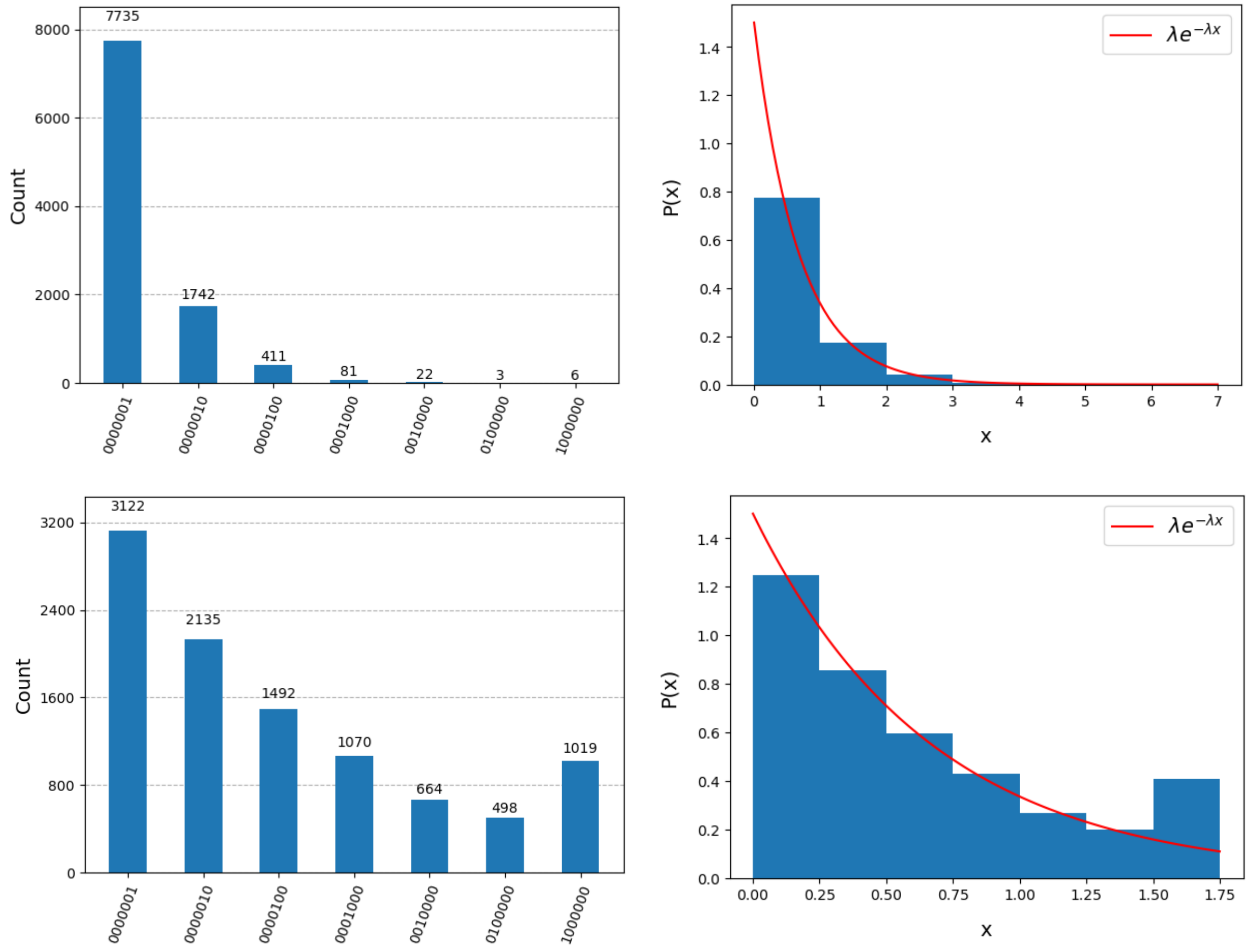
The outcome can be made an exponential distribution:



$n = 6 \quad \lambda = 1.5$

$m = 1$ (top)

$m = 4$ (bottom)

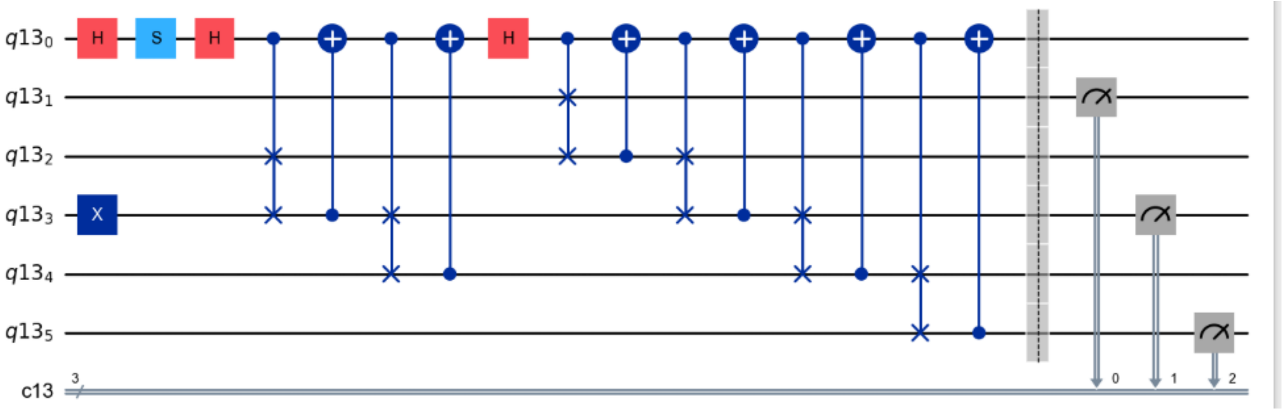


- m decreases the bucket sizes
- Makes finer and more accurate, but requires more layers to fix ‘overflow’ at end

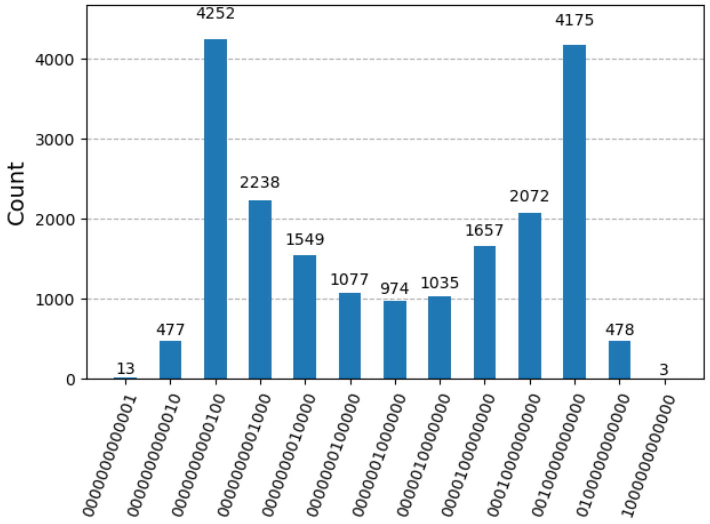
WISER Quantum Project

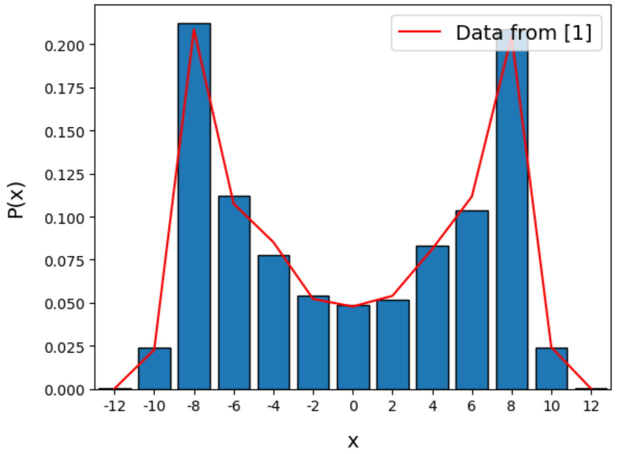
Hadamard Quantum Walk

Distribution with a QGB



A Hadamard Quantum Walk using a Quantum Galton Board for $n = 2$





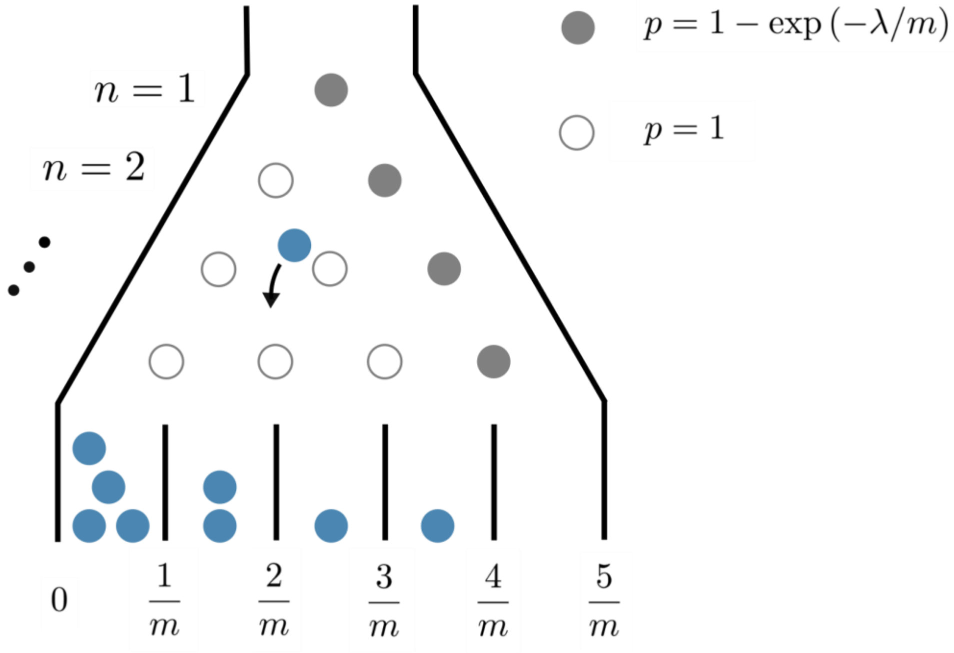
Reset gates removed and replaced with CNOT gates. Initial state is $\frac{|0\rangle+|1\rangle}{2}$

Distribution is compared to date obtained from the code in [1] [Quantum walk | Cirq | Google Quantum AI]

Above for $n = 12$ layers (i.e. 12 iterations of Walk)

Future directions

And aims in this work



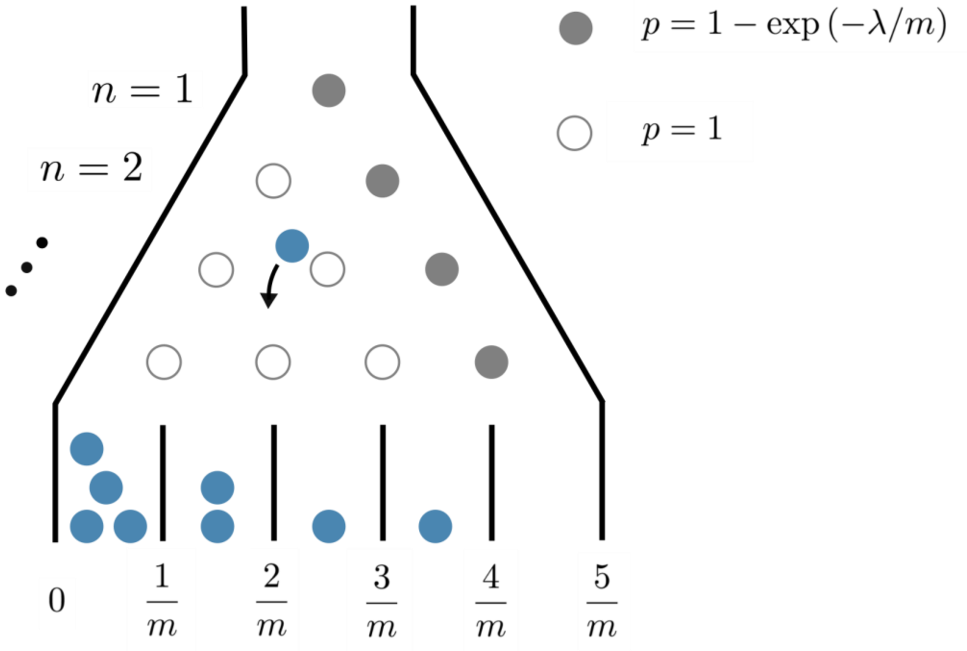
- Optimisation of the exponential distribution circuit e.g. some *CNOT* and *CSWAP* gates can be removed
- Simulate circuits on real quantum hardware, how does noise effect distance to the desired distribution.
- Can the Quantum Galton Board be adapted to simulate neural networks or other graphs and distributions.

Supplementary Slides

Exponential distribution

n-layer board

Motivation behind the exponential QGB design



For an exponential distribution:

$$\int_{\frac{i}{m}}^{\frac{i+1}{m}} \lambda e^{-\lambda x} dx = e^{-i\lambda/m} - e^{(i+1)\lambda/m}$$

Therefore, you can see by simple calculation that

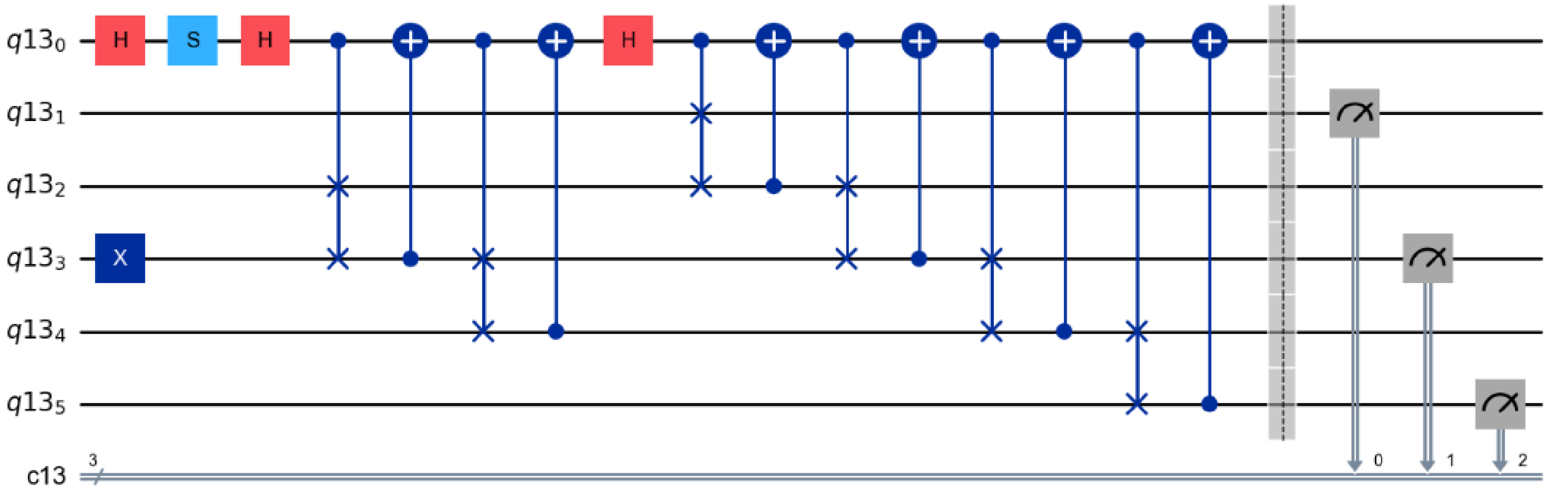
$p = 1 - \exp(-\lambda)$

$p = 0$

with the arrangement on the right, satisfies the expected probability for each bucket.

Hadamard Quantum Walk

n-layer board



By removing the reset, and adding additional CNOTs in their place. The above circuit performs the transformation.

$$U = S\left(\mathbb{1} \otimes H\right) \qquad S = \sum_j |j+1\rangle\langle j| \otimes |0\rangle\langle 0| + \sum_j |j-1\rangle\langle j| \otimes |1\rangle\langle 1|$$

Why is exactly the Quantum Hadamard Random Walk (Quantum walk | Cirq | Google Quantum AI)