$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 24 & 28 & 32 \\ 40 & 48 & 56 & 64 \\ 60 & 72 & 84 & 96 \\ 80 & 96 & 112 & 128 \end{bmatrix}$$

The next few slides show how divide & conquer works for 4x4 matrices

(using the example above, so that you can easily check the arithmetic)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 24 & 28 & 32 \\ 40 & 48 & 56 & 64 \\ 60 & 72 & 84 & 96 \\ 80 & 96 & 112 & 128 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 12 \\ 20 & 24 \end{bmatrix} \qquad \begin{bmatrix} 10 & 12 \\ 20 & 24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 24 & 28 & 32 \\ 40 & 48 & 56 & 64 \\ 60 & 72 & 84 & 96 \\ 80 & 96 & 112 & 128 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 12 \\ 20 & 24 \end{bmatrix} + \begin{bmatrix} 10 & 12 \\ 20 & 24 \end{bmatrix} = \begin{bmatrix} 10 + 10 & 12 + 12 \\ 20 + 20 & 24 + 24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 24 & 28 & 32 \\ 40 & 48 & 56 & 64 \\ 60 & 72 & 84 & 96 \\ 80 & 96 & 112 & 128 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 10 + 10 & 12 + 12 \\ 20 + 20 & 24 + 24 \end{bmatrix}$$

$$- - -$$

$$\begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 7 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 42 + 42 & 48 + 48 \\ 56 + 56 & 64 + 64 \end{bmatrix}$$

$$
\begin{array}{cccc|cccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8
\end{array}
\quad \mathrm{X} \quad
\begin{array}{cccc|cccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8
\end{array}
$$

Similarly, if the matrices were 8 x 8 the recursion would compute the four 4x4 products (and each 4x4 product would be done as in the previous slide, i.e. by recursive calls to its four 2x2 matrices). Etc for larger matrices.

Note: if the input matrices are not $2^p x 2^p$ (ie not 2x2 or 4x4 or 8x8 or …) add rows and columns filled with zeroes to make them so.