Global Positioning System (GPS)

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This is a report covering the first exercise in the course Intelligent Vehicles (DT8020) at Halmstad University. The exercise gives the student a chance to experiment with GPS-data and different ways to handle the data using functions in the MATLAB-enviroment. The data investigated is comprised of one dataset with positions gathered from one location and another dataset with data gathered from a drive through Halmstad. The data is in NMEA-0183 format will therefore be converted to meters using methods described in this paper. The Mean, variance and error in the data is computed and from these values questions provided from the exercise are answered. The method and results gives the reader a understanding of how to calculate the path and heading of a vehicle using GPS-data, aswell as how to calculate the speed.

I. INTRODUCTION

A. Overview of report

The Global Positioning System(GPS) is often used for navigation for different kinds of vehicles such as cars, boats or aircrafts. This exercise will explore the practical usage of a Global Positioning Sensor(GPS) and some of the challenges it involves. The exercise will introduce the theory for converting longitude and latitude angles to meters. It will also introduce how to calculate and evaluate errors in measured positions and velocity. The exercise will be contucted in MATLAB, where we as students will use already implemented functions to calculate parameters the variance, error, mean and uncertainty for the data provided [1]. The examined data for the exercise is collected during a drive through Halmstads' major streets, including Laholmsvägen and Wrangelsgatan.

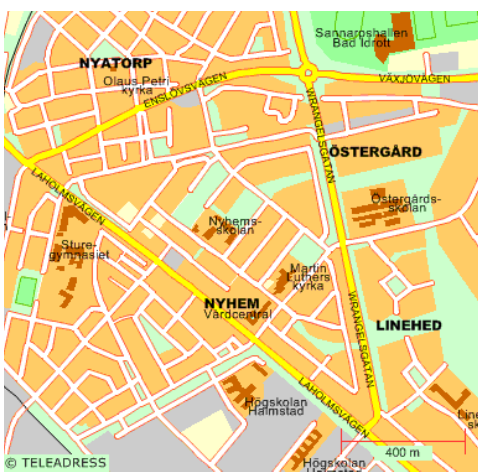

Figure 1. Map over Halmstad where the data had been collected.

B. Preparatory work

The exercise is divided in two parts. The first part consists of tasks to improve the students ability to work with the GPS as a sensor for navigation. For these preparatory tasks the student is handed a data-set which consists of time-stamps and status-flags together with longitude and latitude values. The data is provided in NMEA-0183[2] format which is a common format for modern GPS systems.

The data in NMEA-0183 format will look like this:

```
Status(1)  Timestamp(1)  Latitude(1)  Longitude(1)
Status(2)  Timestamp(2)  Latitude(2)  Longitude(2)
...
Status(N)  Timestamp(N)  Latitude(N)  Longitude(N)
```

All datapoints are provided as vectors containing the information above. We were handed a compendium which consisted of many of the operations needed to correctly work with longitude and latitude values [3]. Section II will go more in to detail what these operations are. The tasks consists of converting the latitude and longitude values to degress and from degrees to meters. The second task for the preparatory part of the exercise was to compute the variance and mean for the positions. In the last part of the preparatory exercise, an analysis of the position error and its correlation is performed.

C. The mobile GPS receiver

In the second part of the exercise the introduced analysis methods from part one is applied on a practic example. Here, a dataset with coordinates collected by a car driving on the streets of Halmstad is analysed. The data is comprised in the same manner as in the preparatory work, ie. NMEA-0183 format. From this, we were to derive what path the car had driven when collecting the data as well as the maximum speed of the vehicle.

The final task of the exercise was to estimate the heading of the vehicle and plot it in relation to time. The heading of a vehicle is in what direction the "nose" of the vehicle is pointing.

D. Continuation of the report

In the next section different methods and the theory behind these methods will be explained. Section III will present the achieved results and in section IV we will discuss what the result means and also mention some possible errors and improvements. We will also try to answer some of the questions posed in section II there.

II. THEORY AND METHOD

A. Method for the preparatory work

To be able to convert longitude and latitude to meters, there's a need for a conversion factor. These factors we will call F_{lon} for longitude and F_{lat} for latitude.

To calculate F_{lon} the following formula is used:

$$F_{lon} = \frac{\pi}{180^\circ}\left(\frac{a^2}{\sqrt{a^2cos^2\varphi + b^2sin^2\varphi}} + h\right)cos\varphi \quad (1)$$

For F_{lat}:

$$F_{lat} = \frac{\pi}{180^\circ}\frac{a^2b^2}{(a^2cos^2\varphi + b^2sin^2\varphi)^{\frac{3}{2}}} + h \quad (2)$$

Where:

- $a = 6378137$ m - semi-major axis of WGS-84 ellipsoid
- $b = 6356752.3142$ m - semi-minor axis of WGS-84 ellipsoid
- φ = geographical latitude($^\circ$)
- h = height over the WGS-84 ellipsoid (m)

WGS-84 is the reference system used by global satellites. Coordinates are often represented with longitude and latitude [4].

For this task we were also provided a conversion table containing the F_{lat}- and F_{lon}-values. This was used to confirm we did the correct calulations. This table could be found at the end of the compendium [3]. Now that F_{lon} and F_{lat} is calculated, the longitude = λ and latitude = φ can be converted to degrees using the following equation:

$$LongDeg = \frac{\lambda}{100} + (\lambda - \frac{\lambda}{100} * 100)/60 \quad (3)$$

$$LatDeg = \frac{\varphi}{100} + (\varphi - \frac{\varphi}{100} * 100)/60 \quad (4)$$

To turn our values from degrees to meters we can use the earlier derived values F_{lon} and F_{lat} to perform the following calculations:

$$X = F_{lon} * LongDeg \quad (5)$$

$$Y = F_{lat} * LatDeg \quad (6)$$

For the next task we were to plot the mean and variance for our derived values. We were also to plot the uncertainty of the data. To do this we needed to calculate the co-variance matrix and the 2D Mahalanobis Ellipse for that co-variance matrix. This ellipse will be the visual representation of the uncertainty. Results for this will be shown in section III.

The third and last task of the preparatory part was to plot the position error and verify its correlation with respect to time. For this part we were also to plot a random signal for comparisons with the errors.

Problems posed for the preparatory work

a) Are the errors Gaussian distributed?

b) Was there anything interesting with the errors?

c) What's the difference between the random Gaussian signal and the GPS-error?

d) Is the GPS error correlated or not?

e) How does this affect your position measurements?

f) What does it mean that the peak in the correlation plot is wider compared to the random signal?

B. Method for the mobile GPS receiver

This part of the exercise could mostly be completed using the same approach as for the preparatory work. The first task was to plot the path taken by the car in x- and y-coordinates. This was done using equation 1 through 6 and then the different plot-methods from MATLABs' library.
The second task was to calculate the speed on the path taken and also distinguish where the vehicle reached maximum speed. Due to the sample-time being 1 second we can use the following formula to calculate the speed:

$$V = \sqrt{\Delta\hat{x}^2 + \Delta\hat{y}^2} \tag{7}$$

Where $V=$ velocity, $\Delta\hat{x}$ and $\Delta\hat{y}$ representing the change in x and y with corresponding errors. Due to correlation the errors counter-act eachother.

The third and final task was to calculate the heading for the vehicle. The heading was then plotted along the path with respect to time. To calculate the heading this equations was used:

$$\theta = arctan\frac{\Delta\hat{y}}{\Delta\hat{x}} \tag{8}$$

Problems posed for the mobile GPS receiver

g) Did the vehicle ever break the speed-limit?

h) How come the estimate in speed is so accurate, while the estimate in position is not?

i) Can the error in headings be calculated?

j) Can the variance in heading be estimated if only a straight line path is considered?

III. RESULTS

A. Results from part 1

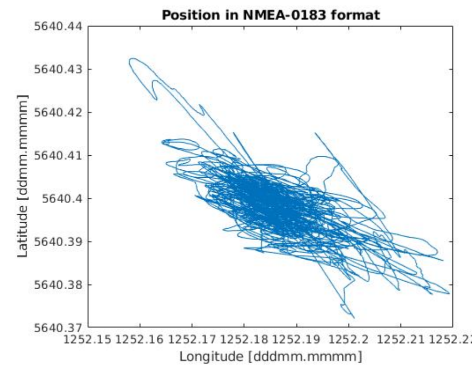

Figure 2. Position in NMEA-0183

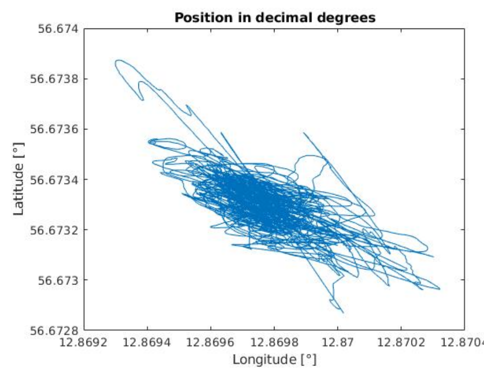

Figure 3. Position in decimal degrees

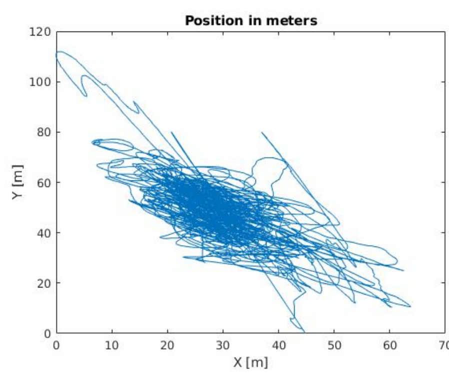

Figure 4. Position in meters

In figure 2, 3 and 4 data given in part 1 is presented first in the NMEA-0183 format and then after being converted to degrees and last in meters. The correlation is easily visible although the units on the axes changes. Note that in figure 4, the axes of the graph are adjusted to scope the positions in which the data is collected. In figure 5 the plot of the position error is shown. Here the uncertainty is displayed as a ellipse centered in x,y = (0,0). The errors of two intervals $I_1 = [1, 10]$ and

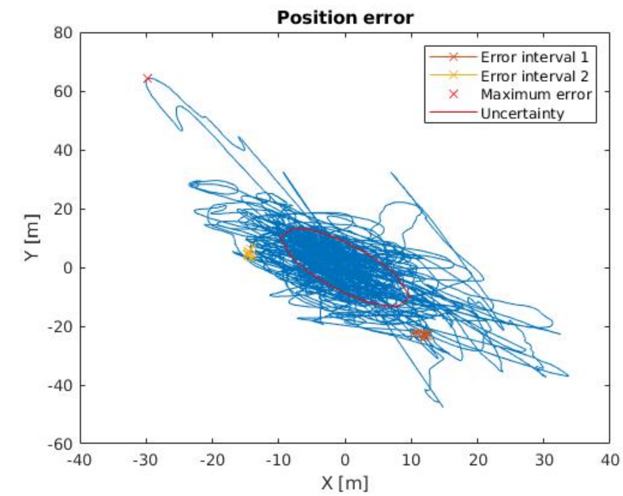

Figure 5. Position error. Yellow and orange parts represent errors correlated in time, while the ellipse represent the uncertainty.

$I_2 = [100, 110]$ are plotted to show correspondense in the errors. Lastly the maximum error, $e_{pMAX} = 70.79$ is shown. In figure 6 and 7 the error in x and y is plotted as a histogram(using 30 bins) to present the relation between the Gaussian distribution and the errors. The relation is almost obvious, answering the question a) in subsection II A.

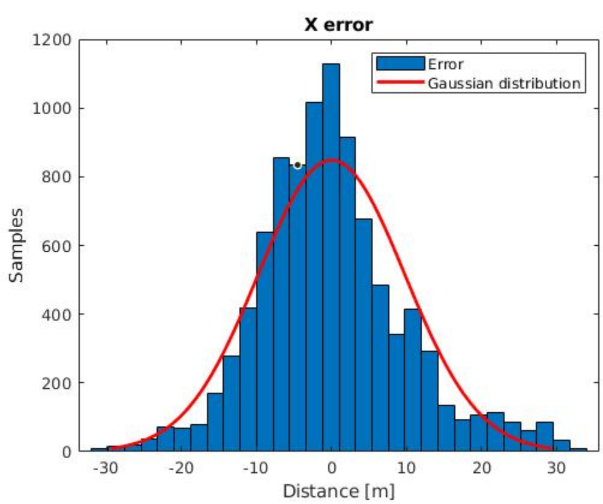

Figure 6. Error in X value in relation to the Gaussian distribution

In figure 8 the error is plotted in relation to time as well as the correlation between the errors compared to a random distribution. The correlation is computed using MATLAB's autocorrelation function. The resemblance in the errors can easily be viewed in figure 9 when compared to the random distribution. The wider peak in the error suggests there is a correlation in the error after shifting it some samples to the left and right. This

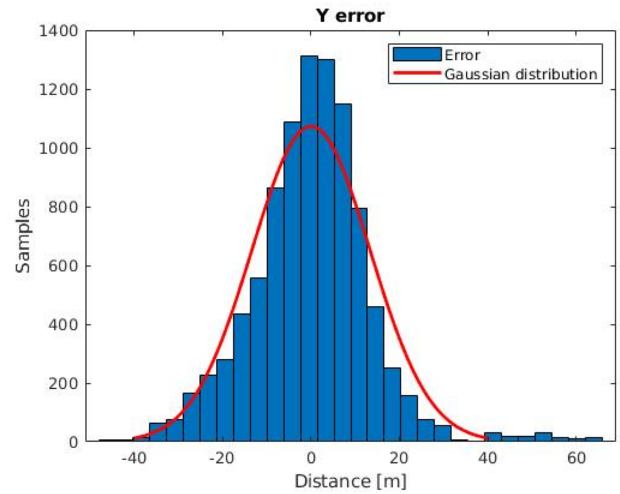

Figure 7. Error in Y value in relation to the Gaussian distribution

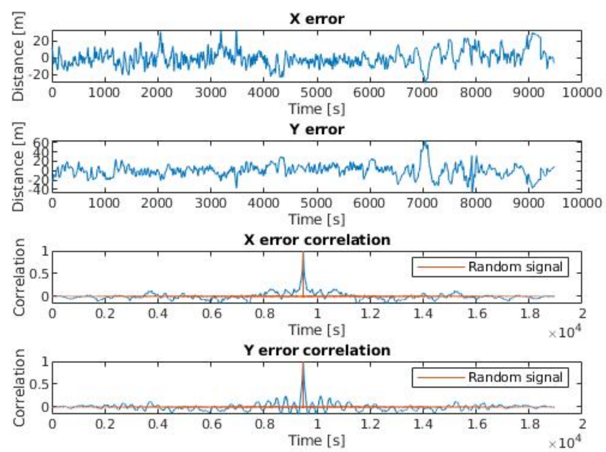

Figure 8. Correlation of the errors in relation to a random distribution

answers question d) and f) in subsection II A.

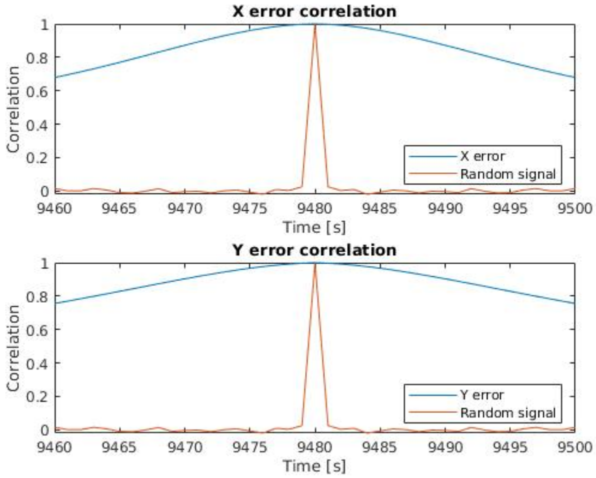

Figure 9. Error correlation, scoped on 20 samples shifted in both directions

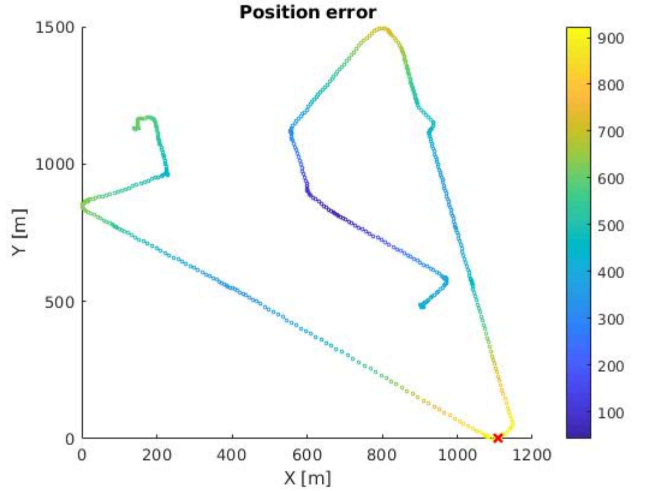

Figure 11. The position error, the maximum error, $e_{pMAX} = 923$ m is marked with a red cross

B. Results from part 2

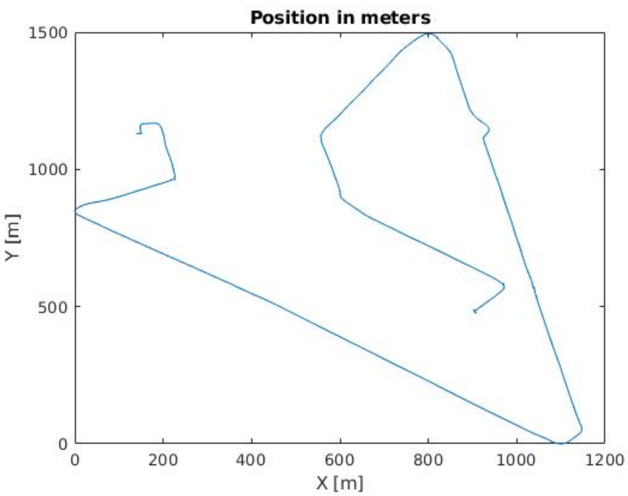

Figure 10. Route in cartesian coordinates, measured in meters

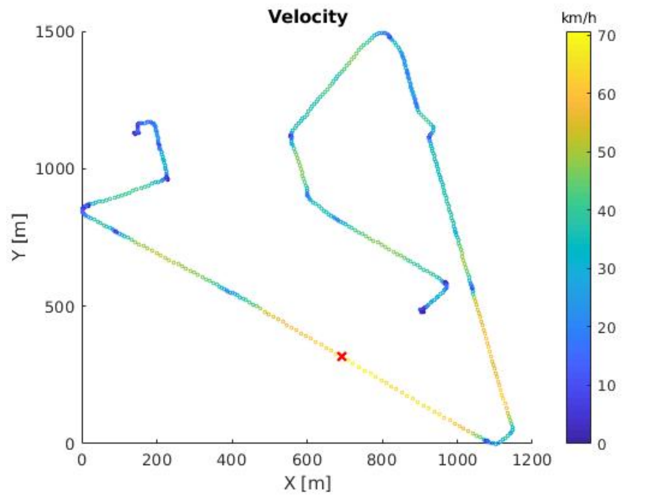

Figure 12. The velocity of the vehicle, the maximum velocity $v_{max} = 70.60$ km/h marked with a red cross

Figure 10 presents the data after conversion from NMEA-0183 format to meters. Same as in figure 4 the converted map is adjusted to only scope the maximum displacement in both axes. To get a overview of the driven path one can compare the route to the map shown in figure 1.

Figure 12 shows the velocity of all the positions along the route. Since the value $v_{max} > 70$ km/h, the vehicle did technically break the speed limit. This answers question g).

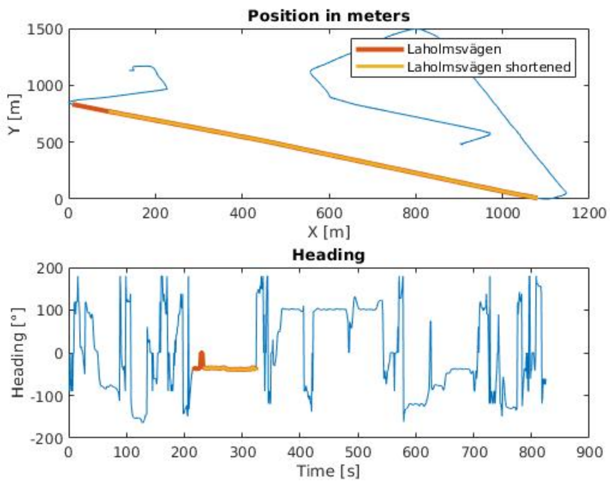

Figure 13. The entire distance driven on Laholmsvägen and a shortened version of the distance. The graph presents the heading while driving these distances

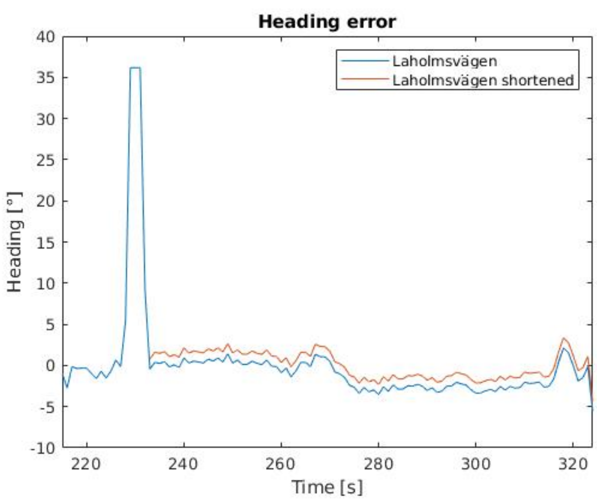

Figure 14. The heading error along the mentioned distances

Estimating the heading error and its variance an abnormal error spike was discovered in the interval [225, 233] s. This abnormality can be viewed in the headings graph in figure 13. For this reason, we chose to also measure a shortened distance when computing the heading error and variance. The error and variance achieved proves very small in the shortened distance but considerably larger in the entire distance due to the mentioned abnormal error. The variance of the two distances were calculated to $\sigma_L^2 = 40.72\ °$ and $\sigma_{Ls}^2 = 2.67\ °$. This answers questions i) and j).

IV. DISCUSSION

In the following section we will answer the questions that require more evaluation and touch upon some future improvements.
Starting with question b) and c) from subsection II A. Question b) asked if there was anything interesting to be found in the errors. After the different results and plots in section III we can see that all the errors have a relation in time, they are all correlated. Question c) asked for the difference between the random Gaussian signal and the position error. What we can see in the Gaussian signal in figure 8 is that there is no correlation in time. Therefore the difference is that the Gaussian isn't correlated while the position error is. To answer question e), how this affects the position measurements, we need to look at the calculation for the position. For each position there is a error corresponding to each value in x and y. So when the position is calculated the error is integrated in to the position and therefor greatly affects the position. Let's take one point as an example:

$$P_1 = \sqrt{(x_1 + e_x)^2 + (y_1 + e_y)^2}$$

From this you can see that for each estimation of a point the error will affect the outcome. This question leads in to question h) in subsection II B. The inaccuracy in position is down to what is stated above but when looking at the speed we can look at equation 8. The corresponding errors in this case will counter-act eachother and therefor more or less cancel each other out. What that means in practice is that the errors does not affect the outcome in speed to an extent that is visibly noticable.

This exercise gave clear examples on how correlating errors can effect different parts of a GPS-system. It was also a good introduction to GPS-systems as a whole. For future improvements of the exercise we think that the size of the exercise could have been smaller. Due to how much there is to do in the course at this moment in time we think that the exercise could have been comprised to only feature the second part, that is the mobile GPS receiver part.

[1] MATLAB. *version 9.5.0 (R2018b)*. The MathWorks Inc., Natick, Massachusetts, 2018.

[2] NMEA. Nmea-0183. `https://www.nmea.org/content/nmea_standards/v411.asp`. Accessed: 2019-02-04.

[3] Viacheslav Adamchuk. Global positioning system data processing. `https://adamchukpa.mcgill.ca/web_ssm/web_GPS.html`. Accessed: 2019-02-04.

[4] Lantmäteriet. Wgs-84. `https://www.lantmateriet.se/sv/Kartor-och-geografisk-information/GPS-och-geodetisk-matning/Referenssystem/Tredimensionella-system/WGS-84/`. Accessed: 2019-02-04.