

Comments on exercise 2

This is some additional comment on exercise 2

Task 2.1-2.4

In this part of the exercise the uncertainty parameters are, beside of the state parameters, distance driven, Δd , and heading change, $\Delta\theta$. The model for position update will look like:

$$X_k = f(X_{k-1}, U_k) = X_{k-1} + \begin{bmatrix} \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x_{k-1} + \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ y_{k-1} + \Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \theta_{k-1} + \Delta\theta \end{bmatrix}$$

To calculate the variance of the position update the *law of error propagation* is used, i.e. the partial derivatives in square multiplied with the corresponding variances:

$$\Sigma_{X_k} = J_{X_{k-1}} \Sigma_{X_{k-1}} J_{X_{k-1}}^T + J_{\Delta d \Delta\theta} \Sigma_{\Delta d \Delta\theta} J_{\Delta d \Delta\theta}^T$$

There

$$J_{X_{k-1}} = \begin{bmatrix} 1 & 0 & -\Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ 0 & 1 & \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ 0 & 0 & 1 \end{bmatrix}$$
$$J_{\Delta d \Delta\theta} = \begin{bmatrix} \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) & -\frac{\Delta d}{2} \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) & \frac{\Delta d}{2} \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ 0 & 1 \end{bmatrix}$$
$$\Sigma_{\Delta d \Delta\theta} = \begin{bmatrix} \sigma_{\Delta d}^2 & 0 \\ 0 & \sigma_{\Delta\theta}^2 \end{bmatrix}$$

From Wang paper (section 4) we got (assume no correlation between wheels) that :

$$\sigma_{\Delta d}^2 = \frac{\sigma_r^2 + \sigma_l^2}{4}$$
$$\sigma_{\Delta\theta}^2 = \frac{\sigma_r^2 + \sigma_l^2}{L^2}$$

Task 2.5-2.6

Here we got new uncertainty parameters, distance driven by the left wheel, Δl , and distance driven by the right wheel, Δr , and the wheel base. This change our model for position update to:

$$X_k = f(X_{k-1}, U_k) = X_{k-1} + \begin{bmatrix} \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x_{k-1} + \frac{\Delta R + \Delta L}{2} \cos\left(\theta_{k-1} + \frac{\Delta R - \Delta L}{2b}\right) \\ y_{k-1} + \frac{\Delta R + \Delta L}{2} \sin\left(\theta_{k-1} + \frac{\Delta R - \Delta L}{2b}\right) \\ \theta_{k-1} + \frac{\Delta R - \Delta L}{b} \end{bmatrix}$$

To calculate the variance of the position update the *law of error propagation* is used, i.e. the partial derivatives in square multiplied with the corresponding variances:

$$\Sigma_{X_k} = J_{X_{k-1}} \Sigma_{X_{k-1}} J_{X_{k-1}}^T + J_{\Delta r \Delta l} \Sigma_{\Delta r \Delta l} J_{\Delta r \Delta l}^T + J_b \Sigma_b J_b^T$$

There

$$J_{X_{k-1}} = \begin{bmatrix} 1 & 0 & -\Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ 0 & 1 & \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ 0 & 0 & 1 \end{bmatrix}$$

$J_{\Delta r \Delta l}$ = See textbook page 189 (equ 5.11) or the slides from lecture 5

$\Sigma_{\Delta r \Delta l}$ = See textbook page 189 (equ 5.8) or the slides from lecture 5

J_b = See Exercise 3 and 4 in the Error propagation material sent out

Σ_b = See Exercise 3 and 4 in the Error propagation material sent out

Task 3

In this part of the exercise we use a steer-drive robot instead of a differential drive robot. This means that our model change for position update. The new uncertainty parameters are the steering angle, α , and the speed, V , and sampling time T .

$$X_k = f(X_{k-1}, U_k) = X_{k-1} + \begin{bmatrix} \Delta d \cos\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta d \sin\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x_{k-1} + v \cos(\alpha) T \cos\left(\theta_{k-1} + \frac{v \sin(\alpha) T}{2L}\right) \\ y_{k-1} + v \cos(\alpha) T \sin\left(\theta_{k-1} + \frac{v \sin(\alpha) T}{2L}\right) \\ \theta_{k-1} + \frac{v \sin(\alpha) T}{L} \end{bmatrix}$$

To calculate the variance of the position update the *law of error propagation* is used, i.e. the partial derivatives in square multiplied with the corresponding variances:

$$\Sigma_{X_k} = J_{X_{k-1}} \Sigma_{X_{k-1}} J_{X_{k-1}}^T + J_{v\alpha T} \Sigma_{v\alpha T} J_{v\alpha T}^T$$

For guidance, see Exercise 4 in *Calculate Jacobian matrixes* material sent out.