Problem Chosen	2022	Team Control Number
B	MCM/ICM Summary Sheet	2203422

Wise, Practical Water Allocation Model of Lake Powell and Lake Mead

Summary

Over decades, dams have been constructed to block water resources for people to allocate them wisely. However, as climate change is becoming more and more fierce, the Coronado River is experiencing an unprecedented drought, forcing officials to come up with wiser plans to allocate water resources to satisfy five states' water and electricity needs. The aim of this report is to give our team's best mathematical solution for the allocation of water in this region. We are expected to collect current data and provide some strategies for dam operators so that they can allocate water directly. Therefore, three models have been established: Model 1: Reservoir's Physical Model; Model 2: Water Allocation Model; Model 3: Best Plan Filtering Model.

For Model 1, data like the lake's water level, the total amount of a lake, and other related information are collected. Using this information, we have established equations describing the reservoir's power conversion rate, the relationship between the water level and the content, and a formula describing the upstream and downstream water relationship. Establishing these formulas has not used some highly advanced algorithms. Still, these equations are of great importance since when knowing some reservoir variables, other data can be automatically calculated. This has also laid a strong foundation for our mathematical modeling.

For Model 2, we have used our own experience to list objective functions and constraint conditions. Then we organized our data and used our physical data to calculate some unknown variables. When we need to solve our model, we have used a **multi-objective optimization model NSGA-II** to select the top 50 solutions for a specific problem. Pareto optimal frontier has been drawn and it is shown in Figure 8(a).

For Model 3, since we have acquired the top 50 solutions, we need to select the best one out of them. In this part, we have applied **the TOPSIS algorithm** to help us make the decision. We have calculated the distance from the optimal to the inferior solution and generated a score. Then we sort those scores out, and an exemplary result is shown in Figure 8(b). Scores calculated above have directly demonstrated the quality of a plan. If the quality of a plan is quite satisfying, the generated score is correspondingly high. After calculating the scores for each solution, we can arrange them in descending order and graph the results.

In addition, **robustness and sensitivity analysis** of the model are tested. Instead of following the traditional steps, we have creatively merged this with one of the requirements. By adjusting the input value, the model can still give practical suggestions of water allocation, which means that this model can be used in multiple situations. Moreover, our model will not exhaust the Colorado River, and there is still water to be allowed to flow into the Gulf of California from the Colorado River, which guarantees other country's rights.

Keywords: Reservoir water allocation; Multi-objective optimization;NSGA-II;TOPSIS

Contents

1 Introduction

1.1 Problem Background

As we all know, electricity and water are two main factors necessary in our lives. However, without a wise decision of allocation, a constant and sufficient amount of water and electricity supply would be impossible. Therefore, dams are constructed and put into use to store water and generate electricity using turbines as they convert the potential energy of water into mechanical energy.

With an impact caused by over 20 years of drought and demands from an increasing population, the Colorado River, serving as a critical source of water for people and agricultural land in neighboring seven states, has been hugely impacted. Lake Mead and Lake Powell, which were formed by the construction of the Glen Canyon Dam and the Hoover Dam, have experienced historically low water levels. To still ensure a sufficient usage of water and electricity in those areas, a well-designed allocation plan should be published.

(a) (b)

Figure 1: Two dams constructed on Colorado River: (a) **Glen Canyon Dam**: Glen Canyon Dam is a concrete arch-gravity dam on the Colorado River in northern Arizona, United States, near the town of Page. The 710-foot (220 m) high dam was built by the U.S. Bureau of Reclamation (USBR) from 1956 to 1966 and forms Lake Powell; (b) **Hoover Dam**: Hoover Dam is a concrete arch-gravity dam in the Black Canyon of the Colorado River, on the border between the U.S. states of Nevada and Arizona. It was constructed between 1931 and 1936 during the Great Depression.

1.2 Restatement of the Problem

- Develop and analyze a mathematical model that will assist negotiators to respond to a fixed set of water supply and demand conditions.
- Use your model to recommend the best means to resolve the competing interests of water availability for general usage and electricity production.
- Use your model to address what should be done if there is not enough water to meet all water and electricity demands.
- Use your model to address how your methods are affected if some extra conditions are taken into consideration.

1.3 Our Approach

The topic requires us to give a practical plan to operate two dams to satisfy the use of water and electricity in five states based on current data available on government websites and other databases, even under extreme conditions such as long-term drought. We are also required to give a long-term plan if there is a growth in population, agriculture, and other aspects. These requirements ask us to establish a model whose stability and applicability are ensured. Our work mainly includes the following:

- Many databases have been looked through, and all the data we have collected have been carefully filtered to ensure their accuracy and timeliness.
- Based on the historical hydrologic data and the structural data of those two reservoirs, physical models of two dams are established.
- Regarding some realistic conditions, some objective functions and constraint conditions are established to construct a mathematical water allocation model.
- Using a multi-objective optimization algorithm and Matlab, we found solutions to that model

and then applied them to real problems.

2 General Assumptions and Model Overview

2.1 Assumptions

First and foremost, we make some basic assumptions and explain their rationales:

- *The last five years of drought are representative of the current climate conditions on the Colorado River.*
  This assumption is the prerequisite of our model. With this assumption, we can make the decision based on the existing knowledge.
- *There is no suitable alternative source for water and power under current conditions, and the Colorado River should try to meet the current needs.*
  In our model, we prioritize ensuring Americans' requirements for electricity and water. At the same time, we believe there are no alternative solutions available and make decisions in the worst case scenario.
- *During the time scale we discuss in this paper, the annual demand for electricity and water in the western U.S. does not change substantially.*
  In our model, we don't consider the incidents that may cause great demand changes, since their probabilities are small.
- *The amount of power generated by the dam in previous years is roughly the number of required power generation, and the state of the dam remains unchanged.*
  We consider the condition of the dam to be stable. Its past power generation represents the demand for it, while its mechanical efficiency is almost constant.

2.2 Model Overview

Our modeling process has mainly included three parts. The first one is to establish a physical model of a dam to analyze the relationship between several physical factors (Elevation and amount of water, for example). Secondly, we have built a mathematical model to predict the best method of allocating water resources and satisfying the demands of neighboring five states. Lastly, we have applied the NSGA-II algorithm and TOPSIS algorithm to calculate and select the best methods among our options.

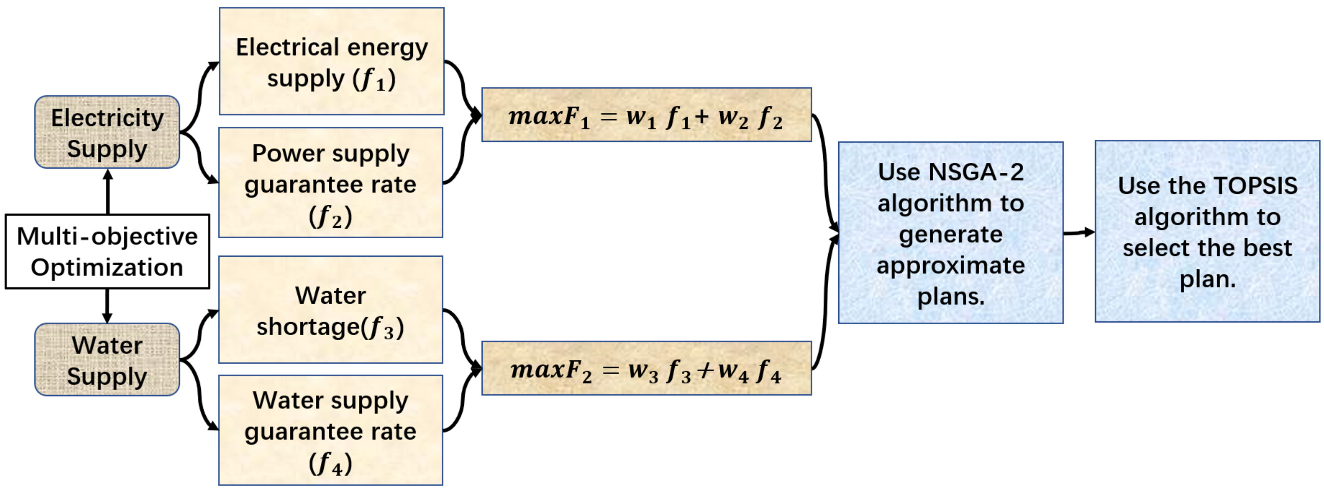


Figure 2: Model Overview of Mathematical Model

Our physical model has also played an essential part in providing unknown data to the mathematical model. It has helped us immensely in the following aspects:

- Estimate the evaporation rate
- Estimate the water consumption rate (the amount of water consumed to generate per unit of electricity)
- Estimate the relationship between water level and the amount of water
- Estimate the relationship between inflow water and the outflow water

3 Model Preparation

3.1 Notations

Symbols	Description	Unit
T	Time during which water supply and demand conditions are fixed	$month$
$\{s_{j,t}\}_{j=1}^5$	Water demand in each state	m^3
$\{d_{j,t}\}_{j=1}^5$	Electricity demand in each state	$kW \cdot h$
$D_{i,t}$	Total electricity demand for a dam	$kW \cdot h$
$S_{i,t}$	Total water demand for a dam	m^3
f_1	Total generated electricity within a year	$kW \cdot h$
f_2	Value of an indicator function for electricity supply	1
f_3	Value of an indicator function for water supply	1
f_4	Water supply shortage	1

F_i	The objective function for electricity or water	1
$N_{i,t}$	Electricity provided by the dam during in a month	$kW \cdot h$
$L_{i,t}$	Water provided by the dam during in a month	m^3
$I_{i,t}$	An indicator function for electricity supply	1
$R_{i,t}$	An indicator function for water supply	1
w_i	Given weight in an expression to determine the best method	1
P	Height of total water in Powell Lake	m
M	Height of total water in Mead Lake	m
ΔP_t	Height of total water added to Powell Lake in a month	m
ΔM_t	Height of total water added to Mead Lake in a month	m
$Q_{i,t}^{in}$	Water flow into a lake in a month	m^3
$Q_{i,t}^{out}$	Water flow out of a lake in a month	m^3
$q_{i,t}$	Water Evaporation in a month	m^3
$L_{i,t}$	Water used in a month	m^3
k_i	Coefficient used to convert total amount of water in cubic to height in meters	m^{-2}
α_i	A ratio describing actual water provided divided by outflow from a lake	m^3

Table 1: Notations

3.2 The Data

3.2.1 Data Collection

The data we used mainly includes historical water data from government databases, electricity usage data, and water usage data from government websites. To ensure the quality of those data, we have compared data from multiple websites and filtered out the best data among all the websites that we have encountered. Those data sources have been summarized in the table below.

Database Names	Database Websites	Data Type
USEIA	https://www.eia.gov/electricity/	Industry Report
Lake Powell Water Database	https://lakepowell.water-data.com/	Geography
Lake Mead Water Database	http://lakemead.water-data.com/	Geography
Google Scholar	http://scholar.google.com	Academic paper
Wikipedia	https://en.wikipedia.org/	Academic Information
Colorado River District	https://www.coloradoriverdistrict.org/	Geography

Table 2: Data source collection

3.2.2 Data Cleaning

Data cleaning that our team performed mainly includes the following steps.Firstly, we discretized our collected data; Secondly, we removed duplicate or irrelevant observations;Thirdly, we fixed some structural errors; Lastly, we have filtered unwanted outliers.

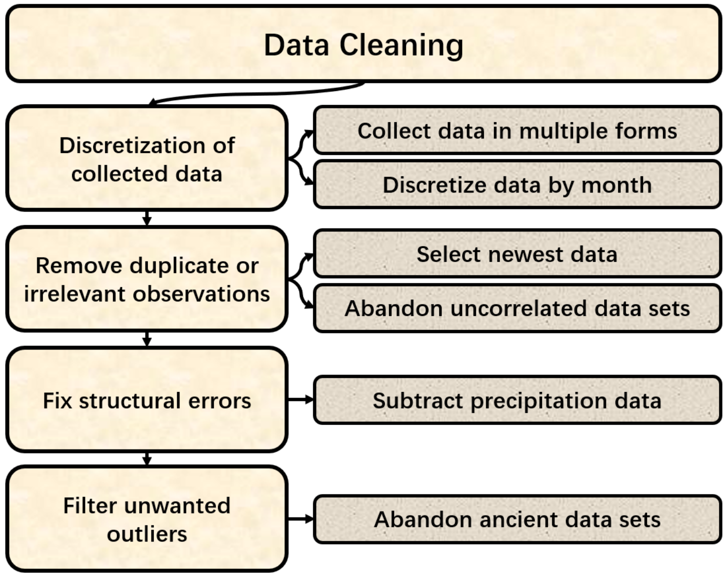


Figure 3: Steps of data cleaning

4 Model 1:Reservoir's Physical Model

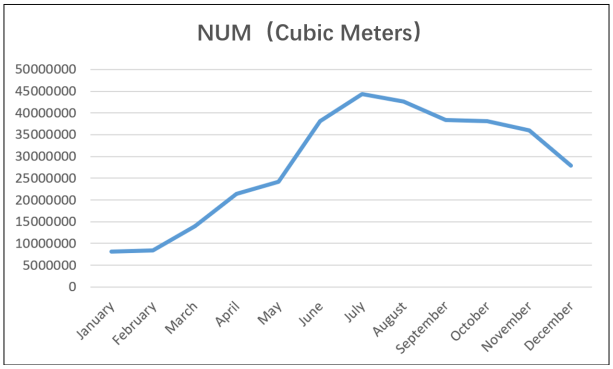
4.1 Reservoir evaporation calculation

The main factors affecting the speed of evaporation are temperature, humidity, air pressure, the surface area of the liquid, water quality, liquid surface, and the airflow on the surface of the liquid (wind speed will affect the diffusion of water vapor) and so on.

After the reservoir storage, the water area will increase, the contact area between the water surface and the air will become more significant, causing water evaporation to rise. At the same time, the fluidity of the river is stronger than the fluidity of the reservoir, so the heat absorption and warming intensity are weaker than that of the reservoir, and the evaporation is smaller than that of the reservoir. Therefore, the evaporation amount is smaller than that of reservoirs.

As we have mentioned previously, the amount of water evaporated from a reservoir is quite dramatic, which means that it is necessary for us to consider this proportion of water. However, since the information we have found is quite limited, we did not directly acquire this data. Instead, we have calculated it using the change of height times the open water area.

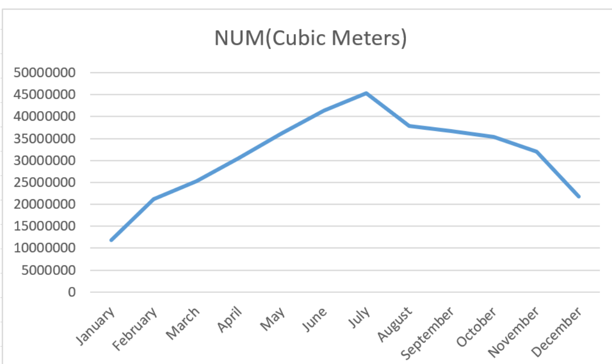
For Lake Powell, the change of water evaporation amount is shown in the line chart below, and the precise data is also listed aside.



MONTH	NUM(Cubic Meters)
January	8141028.974
February	8387726.822
March	13938428.39
April	21339363.83
May	24176389.07
June	38114817.47
July	44282263.66
August	42555378.73
September	38361515.32
October	38114817.47
November	36017885.76
December	27876856.79

Table 3: Yearly Evaporation Data of Lake Powell

For Lake Mead, the same method has been taken and the change of water evaporation amount is shown in the line chart below, and the precise data is also listed aside.



MONTH	NUM(Cubic Meters)
January	11841260.54
February	21166253.22
March	25335363.71
April	30570804.37
May	36056638.36
June	41444411.9
July	45348327.49
August	37879699.09
September	36633899.81
October	35314092.64
November	31931932.6
December	21736113.89

Table 4: Yearly Evaporation Data of Lake Mead

4.2 Reservoir Power Conversion Calculation

A hydro power plant's overall water consumption rate is an important indicator to measure its economic performance. Water consumption rate μ stands for the amount of water consumed per unit of electricity. it's unit is $m^3/(kW \cdot h)$.It can be mathematically calculated by:

$$\mu = \frac{W}{E} = \frac{QT}{NT} = \frac{C}{H\eta}$$

Where variables and coefficients stand for:

Symbols	Descriptions	Units
W	Amount of generated water	m^3
E	Amount of generated electricity	$kW \cdot h$
Q	Velocity of water used to generate electricity	m^3/s
N	Power of the hydroplant	kW
T	Time duration	s
H	The difference between upstream and downstream water levels	m
η	Efficiency of hydropower station	1
C	Unit conversion constant	1

Table 5: Description of variables and coefficients

The formula tells us that the water consumption rate is closely related to the difference between a hydropower plant's upstream and downstream water levels. By determining the difference in water level, we can tell the amount of water consumed to produce per unit of electricity.

Regarding that formula, we have calculated a chart demonstrating the relationship between the difference of height in water levels between upstream and downstream and the water consumption rate. The chart is drawn as follows.

H/m	μ/m$^3 \cdot (kW \cdot h)^{-1}$	H/m	μ/m$^3 \cdot (kW \cdot h)^{-1}$
143	2.95	164	2.47
146	2.88	167	2.43
149	2.81	170	2.40
152	2.74	176	2.33
155	2.67	179	2.30
158	2.60	182	2.26
161	2.53	184	2.24

Table 6: Relationship between H and μ

According to the data collected from Lake Powell and Lake Mead databases, we can tell that the rated head of Hoover Powerplant is 576 feet (176m), and the rated head of Glen Canyon Powerplant is 510 feet (155m). Regarding the chart calculated previously, we can tell that μ of Glen Canyon Powerplant is 2.67 $m^3 \cdot (kW \cdot h)^{-1}$ while μ of Hoover Powerplant is 2.33 $m^3 \cdot (kW \cdot h)^{-1}$.

4.3 The Relationship between Water Level and the Amount of Water

The relationship between the water level and the content is extremely important. It is pretty essential to establish an equation describing these two factors. However, because the underwater topography is exceptionally complicated and hard to measure, and we cannot get the details of the lake,so we dicide to use the data to build the model.

Neural networks can capture details in the data that other models cannot. Our method is to model the reservoir level-storage directly from data from previous years.

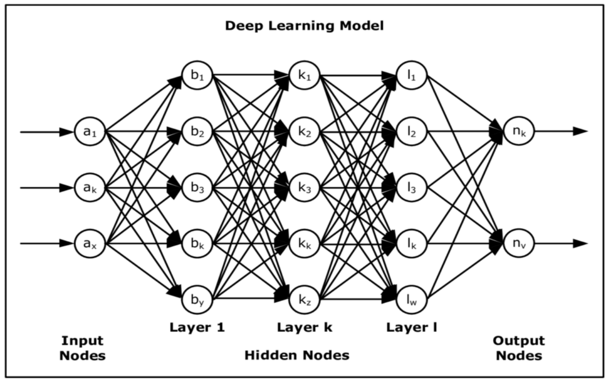


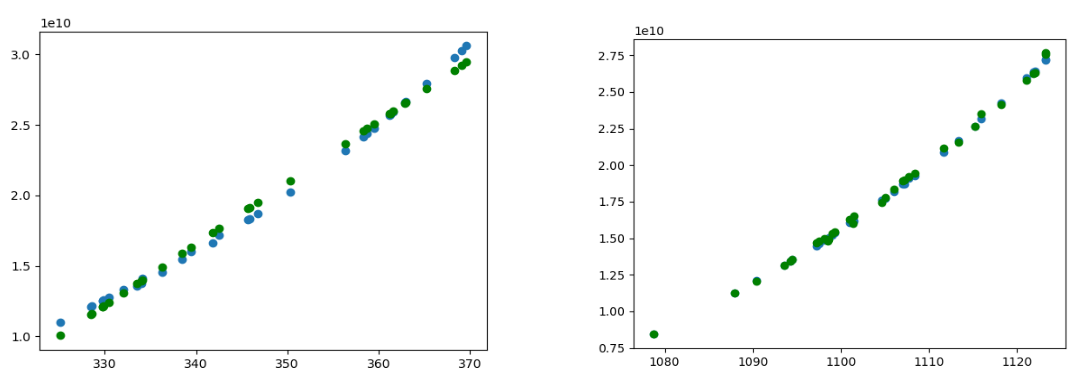
Figure 4: Basic framework of a neural network

We have designed neural networks with 20 hidden layers and used the ReLU function to introduce nonlinear relations to enhance the compatibility of the model. It makes it easier to converge and improve the prediction performance when training multi-layer neural networks. For the training process, we use a dynamically tuned Adam optimizer to accelerate the training process. And the loss function we used is MSE:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - y_{predict})^2$$

We use the data of the water level of the Lake Mead and Lake Powell from January 1, 2001 to December 31, 2021 to do training task. We train 10,000 epochs of data for each lake.

To test our model,we randomly selected 20 time points and plotted their water level versus storage on a scatter plot. Also, we plotted the model-predicted storage on the scatter plot. We find that the water level-storage model approximates a linear model and the neural network achieves a good fit.



(a) Lake Mead (b) Lake Powell

Figure 5: The actual value and predicted value of each dot. The blue dots represent the actual values, the green dots represent the predicted values.

After this, we save the parameters of the neural network as a file and encapsulate them into a function. After that we input the water level to get the corresponding predicted amount of water.

4.4 Upstream and downstream water relationship modeling

Regarding the relationship between the amount of water at Glen Canyon Dam and Hoover Dam, we assumed that a **linear model** is more appropriate. On the one hand, factors such as evaporation during the flowing process will inevitably affect the outflow of water from the Glen Canyon Dam. This diminished amount of water is called the rate of loss. On the other hand, water from other tributaries will also flow into the mainstream in the watershed between the two dams. Therefore, we have concluded that it is easy and practical to use a linear model for the relationship between the flows between dams.

Due to the specificity of linear regression, we have got a relatively satisfactory result using a small amount of data. We used the outflow of the upstream dams and the inflow of the downstream dams over the last decade to build a linear regression model. We use the stochastic gradient descent method for optimization. Finally, we get the relationship between the outflow of Glen Canyon Dam and the inflow, which can be mathematically written as:

$$V_{inflow} = 0.805085755891494 * V_{outflow} + 4057055.049982047$$

5 Model 2:Water Allocation Model

5.1 Brief Explanation of Train of Thoughts

According to the statement of the problem and the assumption mentioned above, we assume that we have a fixed amount of demand for water and electricity in those five states. Centering on those two dams, what we need to ensure is that the water level of each lake is maintained within a rational given section. During a given time, the change of water level is determined by three factors: 1)The inflow of water from upstream; 2)The outflow of water to downstream; 3)The usage of water along the stream. Using those three factors and a constraint condition, a model can be established to describe associations among several factors and moreover, to give some rational, practical methods to operate those dams.

5.2 Determination of Objective Function for Electricity Generation

To clarify, our purpose is to make the overall amount of generated electricity as large as possible. To achieve that goal, several functions are developed.

Firstly, we need to determine a function to calculate the total generated electricity in a given amount of time T. The fundamental logic is to add electricity generated from different dams to get the total amount of power in a given time. The mathematical expression can be written as:

$$f_1 = \sum_{t=1}^{T} \sum_{i=1}^{2} N_{i,t} \Delta T$$

To better demonstrate to which degree the demands are met, we introduce a function f_2. The fundamental logic is that when electricity demand is completed, we assign value 1 to the variable, while when the need is not met, we assign value 0 to the variable. Mathematically, it can be written as:

$$f_2 = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{2} I_{i,t}$$

Where $I_{i,t}$ is determined by a function written as following:

$$I_{i,t} = \begin{cases} 1, N_{i,t} \geq D_{i,t} \\ 0, N_{i,t} < D_{i,t} \end{cases}$$

In the function mentioned right above, D_i stands for the demand for electricity for different dams. Glen Canyon Dam's electricity demand comes from state WY, CO, and NM, and Hoover Dam's electricity demand comes from state CA and AZ. Its mathematical expression can be written as:

$$D_{1,t} = \sum_{j=1}^{3} d_{j,t}, D_{2,t} = \sum_{j=4}^{5} d_{j,t}$$

In the end, assigning different weights to f_1 and f_2, our objective function for electricity generation can be described mathematically as:

$$\max F_1 = w_1 f_1 + w_2 f_2$$

The method used to solve the maximum value of this function will be described in the following section.

5.3 Determination of Objective Function for Water Supply

To measure the quality of a plan, we decide to consider two factors:water supply guarantee rate (f_3) and water shortage (f_4) . We combine these two factors into one function F_2 and make it the objective function of water supply.

For water supply, what we need to take into consideration is to which degree do we meet the demand in states. Under this kind of circumstances, an indicator function is constructed to show whether the demand is sufficiently completed or not, which can be written as following:

$$f_3 = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{2} R_{i,t}$$

Where variable $R_{i,t}$ is used to determine whether the water demand is met or not. When the demand is completed, we assign value 1 to the variable, while when the need is not met, we assign value 0 to the variable. Mathematically, it can be written as:

$$R_{i,t} = \begin{cases} 1, L_{i,t} \geq S_{i,t} \\ 0, L_{i,t} < S_{i,t} \end{cases}$$

In the function mentioned right above, S_i stands for the demand for water from different dams. Glen Canyon Dam's water demand comes from state WY, CO, and NM, and Hoover Dam's water demand comes from state CA and AZ. Its mathematical expression can be written as:

$$S_1 = \sum_{j=1}^{3} s_{j,t}, S_2 = \sum_{j=4}^{5} s_{j,t}$$

Also, we calculate the total water shortage during the year to measure the water supply performance of the dam.To testify to which degree the demand is met, we need to consider the value of f_4.

$$f_4 = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{2} L'_{i,t}, \quad L'_{i,t} = \begin{cases} L_{i,t} - S_{i,t}, L_{i,t} < S_{i,t} \\ 0, L_{i,t} \geq S_{i,t} \end{cases}$$

$$\max F_2 = w_3 f_3 + w_4 f_4$$

5.4 Determination of Constraint Conditions

As we all know, the change in water level is determined by several factors listed below: 1) The inflow of water; 2) The outflow of water; 3)Water used directly from the dam. Considering the difference of units, we introduce a coefficient k_i to convert the quantity of water in cubic meters into height in meters. The association of those factors can be written mathematically as:

$$\Delta P_t = (Q_{1,t}^{in} - Q_{1,t}^{out} - L_{1,t} - q_{1,t})k_1$$

$$\Delta M_t = (Q_{2,t}^{in} - Q_{2,t}^{out} - L_{2,t} - q_{2,t})k_2$$

Where ΔP_t stands for the change of height in Powell Lake within the given time while ΔM_t stands for the change of height in Mead Lake within the given time. Considering that the height of water level must be maintained within a specific section, we can construct two inequalities to show their constraint association:

$$P_{min} \leq P + \Delta P_t \leq P_{max}$$

$$M_{min} \leq M + \Delta M_t \leq M_{max}$$

It's worth noting that the value of $Q_{i,t}^{out}$, which stands for the outflow of water from a certain dam or lake is not necessarily equals to the value of $N_{i,t}$, which stands for the amount of water provided by the same dam or lake. We introduce a coefficient α_i to describe the deviation between the amount of water supplied by the lake and the amount of water outflow. The mathematical expression can be written as:

$$N_{1,t} = \alpha_i Q_{1,t}^{out}$$

$$N_{2,t} = \alpha_i Q_{2,t}^{out}$$

6 Model Solution

There are two main ways to solve the problem in the field of reservoir scheduling: the first one is to transform multi-objective optimization into single-objective optimization by methods such as constraint method and weight method, and then solve it using traditional mathematical planning methods; the other one is to solve it by classical multi-objective evolutionary algorithms (MOEAs) based on Pareto dominance theory such as NSGA-II[1] and MOPSO[2]. Foued B.A.[3] and other experts, aiming to solve the multi-objective scheduling problem of a reservoir complex using applied stochastic objective programming, transformed the multi-objective into a single objective, and verified the method's effectiveness using a multi-reservoir system in northern Tunisia as an example. At the same time, Ahmadi M. and his team[4] used the NSGA-II algorithm to study the multi-objective generation optimal scheduling of the Karoon4 hydropower plant and extract its optimal operation rules.

The two-objective optimal scheduling model established in this paper is nonlinear, highly coupled, etc. To solve the problems such as dimension disaster, the NSGA-II algorithm is directly used to analyze and solve it. Compared with the traditional genetic algorithm, it improves the individual elitism through the calculation and ranking of the congestion degree, and improves the computational efficiency while reducing the complexity through the elite strategy.

The steps of model solution are listed as follows:

- Initialize the model parameters listed below:

Parameters	Descriptions
$\{Q_{1,t}^{in}\}_{t=1}^{12}$	Amount of water coming from Lake Powell in typical months
P_{min}	The lower limit of the warning water level in Lake Powell and Lake Mead
P_{max}	The higher limit of the warning water level in Lake Powell and Lake Mead
$\{s_{j,t}\}_{j=1}^{5}$	Fixed water demand in five states
$\{d_{j,t}\}_{j=1}^{5}$	Fixed electricity demand in five states
$\{N_{i,t}^{min}\}_{t=1}^{12}$	The least amount of generated electricity in typical months
$\{s_{j,min}\}_{j=1}^{5}$	The least amount of water demand in five states
$\{Q_{i,t}^{in,min}\}_{t=1}^{12}$	The least amount of water discharged from those two dams in typical months
$\{Q_{i,t}^{in,max}\}_{t=1}^{12}$	The greatest amount of water discharged from those two dams in typical months

Table 7: Description of variables and coefficients

- Calculate the discharge flow, power generation, etc., of the two reservoirs, and use the constraints to eliminate the population individuals that do not satisfy the constraint conditions. Crossover and mutation will generate new offspring populations that satisfy the constraints. Then we merge them with the parent population. Then we perform non-dominated sorting based on the objective function value to calculate the crowding degree of individuals and select from the lowest to the highest.
- The elite strategy is used to retain the best individuals, which means that the parents are also included in the offspring for ranking analysis.
- Repeat the above steps until the number of iterations is satisfied, and output the non-inferior solution set after optimal scheduling of the model.

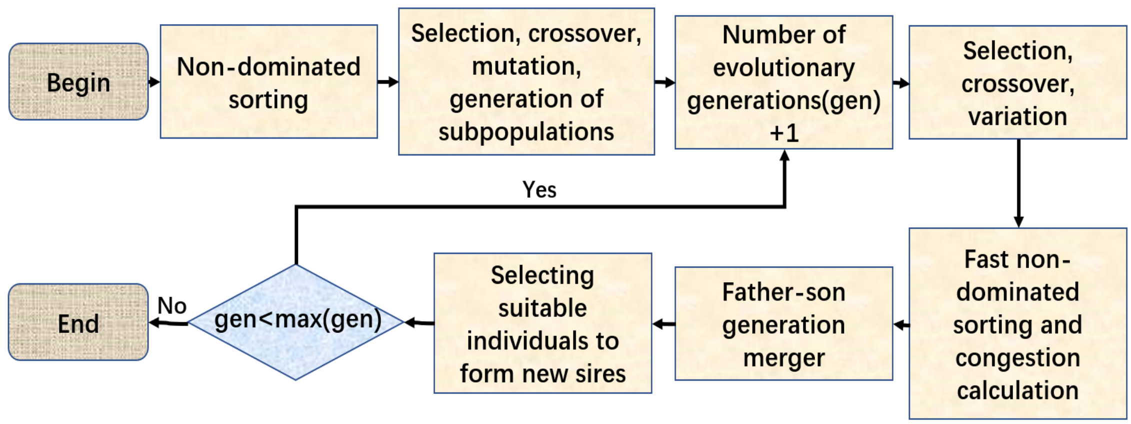
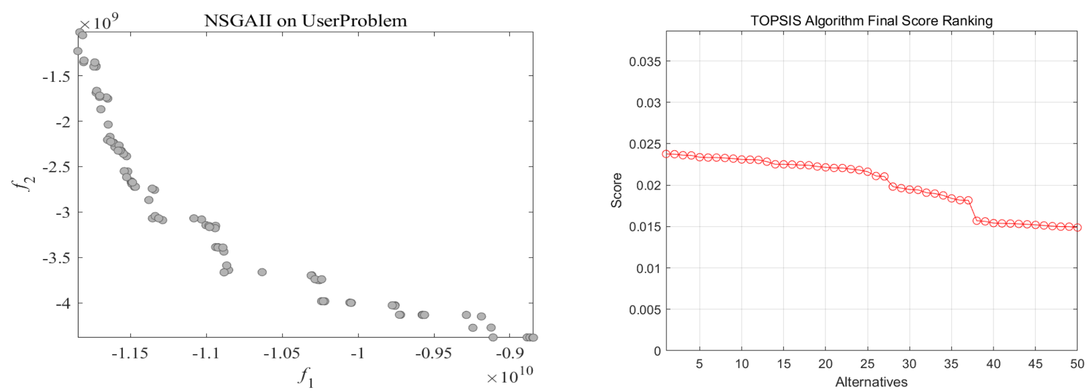


Figure 6: Solution Summary



(a) NSGA-II Result

(b) Topsis Result

Figure 7: NSGA-II & TOPSIS Result

The NSGA-II algorithm yields a non-inferior set of solutions to the multi-objective optimization problem. Here we set the size of the solution set to 50 and draw the Pareto optimal frontier according to the resulting solutions.

7 Model 3: Best Plan Filtering Model

The complete optimization problem of reservoir scheduling should contain two parts. The first one is to obtain a high-quality non-inferior solution set by the algorithm of optimal scheduling. The second one is to perform multi-attribute decision-making for the other non-inferior solution set in order to obtain the best equilibrium solution. Classic multi-objective decision-making methods include the simple weighting method, ELECTRE[5], TOPSIS[6], etc.

The TOPSIS method is a commonly used comprehensive evaluation method that makes full use of the information from the raw data, and its results accurately reflect the gaps between the evaluation options. The steps of performing this method include the following:

- Data enlargement process, which means that all 48 allocation schemes should be as large as possible.
- Normalization is performed.
- By default, the amount of water used for power generation is weighted the same as the amount of water supplied for domestic, industrial, and agricultural use.
- For each scenario, we calculate the distance from the optimal to the inferior solution and calculate a score.
- We sort those scores out.

The result of the algorithm is shown in the following figure.

Scores calculated above have directly shown the quality of a plan. If the quality of a plan is quite satisfying, the generated score is correspondingly high. After calculating the scores for each solution, we can arrange them in descending order and graph the results. The vertical axis represents the specific scores.

8 The Model Results and Solutions to Requirements

8.1 Solution to Requirement 1

8.1.1 Solution to Question 1

The first question of the first requirement asks us to calculate how much water should be drawn from each lake to meet stated demands. Since the original formula is stated above (in the model construction section), a restatement of the equation is entirely redundant. Because we have obtained the data of the current water level (elevation) of both lakes, we have initialized these two variables as:

$$M = 325.03872m, P = 1088.523096m$$

According to the math theories and algorithms we have used, we have generated the best method. Considering that Lake Mead provides water to Arizona and California and Lake Powell provides water to New Mexico, Wyoming and Colorado, our allocation strategy is listed below (Unit:m^3):

Lake Mead	Lake Powell
12031195560	19831738950

Table 8: Solution to Problem 1.1

8.1.2 Solution to Question 2

Problem 1.2 has told us that if no additional water is supplied, and considering the demands as fixed, and asked us about how long it will take before the demands are not met. Our solution is using annual average data and our basic logic is:

$$T = \frac{R}{V}$$

R: The rest of water; V: Water consumption rate

And our answer to this question is, if this kind of aridity condition still lasts, the time before the demands are not satisfied is listed below (unit:day):

Lake Mead	Lake Powell
200.3140	637.4880

Table 9: Solution to Problem 1.2

8.1.3 Solution to Question 3

Problem 1.3 has asked how much additional water must be supplied over time to meet these fixed demands. We have ensured that the surrounding environment stays relatively the same. Using our model stated previously and combining our answer to Problem 1.1, we have calculated the least amount of water that should be added to this hydrological cycle to satisfy all the demands. Our solution is listed below (unit: m^3):

Lake Mead	Lake Powell
13814661297.3	6334377917.6

Table 10: Solution to Problem 1.3

8.2 Solution to Requirement 2

Power generation and water supply present a competitive and synergistic relationship. On the one hand, the realization of the power generation target requires reservoirs to continuously release water from their stock, while the demand of agriculture and industry also requires a certain amount of water, thus making power generation and water supply two contradictory factors; on the other hand, for reservoirs with water supply requirements downstream, the water used for power generation can simultaneously meet the demand for domestic and industrial and agricultural water in downstream towns. At this time, the two present a synergistic relationship.

There is a certain degree of reciprocal feedback among the objectives. The increase of benefits of one objective may cause the decrease of benefits of other objectives, which means that there are contradictions among multiple objectives. Moreover, with the change of time and external environment, the reciprocal feedbacks are not constant. Therefore, if the optimal scheduling is carried out only with the maximum benefit of a single objective, the coordinated development of each objective of the reservoir is ignored.

We utilize a multi-objective optimal scheduling method, where the objective function for power generation includes maximizing power generation and power generation guarantee rate. Water supply includes minimizing water shortage and water supply guarantee rate. The two objectives of power generation and water supply interact and compete with each other, presenting a dynamic game process under changing external conditions. The quantification of the multi-objective feeder relationship is an important guideline for resource allocation at Glen and Hoover Dams.

MONTH	$Q_{1,t}^{out}$	$Q_{2,t}^{out}$	$L_{1,t}$	$L_{2,t}$
January	827537530	356656700	944965240	474337410
February	723602990	493663510	840658440	496267340
March	716005030	475675330	940733720	550646210
April	708815980	348227090	969512730	516867000
May	784462650	531368960	935134720	503061080
June	836998400	344515340	798341730	452679040
July	515035930	536510100	960199700	487430280
August	634310460	498730060	1016268000	481862530
September	720587470	457215460	938538140	461441010
October	802612400	449120180	985764370	470170100
November	562598300	588291840	941957280	452883900
December	750875400	497018970	947975750	532344270

Table 11: Solution to Problem 2

8.3 Solution to Requirement 3

Lack of water is something that would constantly happen, especially during aridity seasons. Our team tried to simulate this kind of situation by diminishing the total amount of inflow to 80% of the original data. Under this kind of circumstances, we have used our model to re-generate a set of plans and then used the TOPSIS algorithm to select the best data. Our best method is placed below in the form of charts (unit:m^3):

MONTH	$Q_{1,t}^{out}$	$Q_{2,t}^{out}$	$L_{1,t}$	$L_{2,t}$
January	869076740	422552550	734090670	325868440
February	611698400	456089920	785379420	388515440
March	708107160	439642630	751275070	476068880
April	575390820	300025170	722464650	519363710
May	591022570	307399780	702075740	490608320
June	742016860	421471640	959775910	452901630
July	550879850	318953570	730733510	535639500
August	624157870	432159060	778695140	329718820
September	503845150	334491200	784153650	388402450
October	563393990	342221700	740854400	454642200
November	719583830	341261410	708571180	483219700
December	827867420	534809010	765665100	451168600

Table 12: Solution to Problem 3

By analyzing the strategy provided by the model under conditions of water scarcity (80%), we find that the strategy given by the model sharply reduces the amount of water used by the two dams to generate electricity each month. It also reduces to some extent the amount of water they use to supply, reducing the drop in water levels as much as possible while ensuring basic demands.

However, in this case, it is still difficult to maintain a continuous supply of energy. Based on our model, we have found that if we want to solve the problem of water shortage, we need to either reduce the water demand and the demand for power generation accordingly or try to compensate for the water loss. Our stratagies are listed below:

- **Increase the water supply:** Bring in enough water from other sources to guarantee the water supply.
- **Reduce the demand for dams to generate electricity:** Build more power plants to reduce the need for energy from dams.

8.4 Solution to Requirement 4

The following experiments were made based on our model to test its sensibility and robustness.

8.4.1 Population, Agricultural, and Industrial Growth

In our test model, water demand has been timed 1.2 and electricity demand has been timed 1.3 for a single dam.

In this case, comparing the change in the strategy, we find that its upstream part appropriately grows each month to meet the need for electricity and grows the water supply simultaneously. While the downstream part properly reduces the previous excess downstream electricity generation and increases the water supply.The change rate in several factors is listed below.

$\Delta N_{upstream}$	$\Delta N_{downstream}$	$\Delta L_{upstream}$	$\Delta L_{downstream}$
0.08095	0.14663	-0.19448	-0.21808

Table 13: Sensitivity Analysis 1

Since the change rate is in an acceptable range, the robustness and sensitivity of our model is high.

8.4.2 Population, Agricultural, and Industrial Diminish

In our test model, water demand has been timed 0.8 and electricity demand has been timed 0.7 for a single dam.

In this case, comparing the changes in the strategy, we find that both the upstream and downstream parts reduce their allocation of water supply as the demand for water supply becomes smaller. At the same time, both upstream and downstream increase the amount of water generated as our optimization objective is to increase the power energy as much as possible while providing more water to the downstream cities and Mexico.The change rate in several factors is listed below.

$\Delta N_{upstream}$	$\Delta N_{downstream}$	$\Delta L_{upstream}$	$\Delta L_{downstream}$
0.04017	0.02216	-0.16358	-0.1362

Table 14: Sensitivity Analysis 2

Since the change rate is in an acceptable range, the robustness and sensitivity of our model is high.

8.4.3 Increase in Renewable Energy Source

In our test model, water demand has been timed 1.2, since a single dam should carry more responsibility to produce energy.

In this case, comparing the change in strategy we find that as the dam will take on more responsibility for power generation, the demand for water supply becomes greater and both the upstream and downstream sections appropriately reduce their allocation of water supply. At the same time, as we normally try to generate as much electricity as possible, we find that there is no significant change in the amount of water used for power generation after the increase in the proportion of renewable energy.The change rate in several factors is listed below.

$\Delta N_{upstream}$	$\Delta N_{downstream}$	$\Delta L_{upstream}$	$\Delta L_{downstream}$
0.04017	0.02216	-0.16358	-0.1362

Table 15: Sensitivity Analysis 3

Since the change rate is in an acceptable range, the robustness and sensitivity of our model is high.

8.4.4 Decrease in Water and Electricity Demands

In our test model, water demand has been timed 0.9 and electricity demand has been timed 0.9 for a single dam.

In this case, comparing the changes in the strategy, we find that the dam will take on less power and water demand due to the water and energy conservation measures been put into practice, so that both the upstream and downstream reduce their allocation of water supply. At the same time, both upstream and downstream increase their water generation by almost the same percentage due to our optimization goal of increasing the power energy as much as possible while providing more water to the downstream cities as well as to Mexico.The change rate in several factors is listed below.

$\Delta N_{upstream}$	$\Delta N_{downstream}$	$\Delta L_{upstream}$	$\Delta L_{downstream}$
0.10808	0.09676	-0.21527	-0.12512

Table 16: Sensitivity Analysis 4

Since the change rate is in an acceptable range, the robustness and sensitivity of our model is high.

9 Strengths and Possible Improvements

9.1 Strengths

- **Applies widely**
  This model can be applied to multiple situations. For instance, we have considered water use from various aspects, including residential use, agricultural use and industrial use, which

means that an official or an expert from one of these fields can use our model to predict the future allocation of water. Even if in the recent future, this area experiences a decrease in precipitation or explosion of population, for two simple examples, this model can still be put into use quickly to help local governments to come up with practical plans.

- **Can save lives**
  This model can be used to save lives. During drought periods, some people would die from the lake of water. However, with this model in hand, we can allocate water in reservoirs wisely to ensure everyone can have a specific water quota. This can undoubtedly save lives under urgent circumstances.

- **Allocates water resources wisely**
  This model can balance the water allocation between electricity and water supply. Sometimes these two factors will collide with one another. Using this model, we can allocate water to these two aspects based on real-time demands. This can surely diminish the time to negotiate with officials from other states or departments.

9.2 Possible Improvements

- The physical model of the dam could be more accurate if we had the original structural data. Since we only have data measured by non-governmental organizations, the two dams' physical model still has spaces to improve.
- Some natural factors are not carefully calculated and considered. For example, the amount of water evaporated from rives and subtracted from the hydrological cycle by permeating into the ground.

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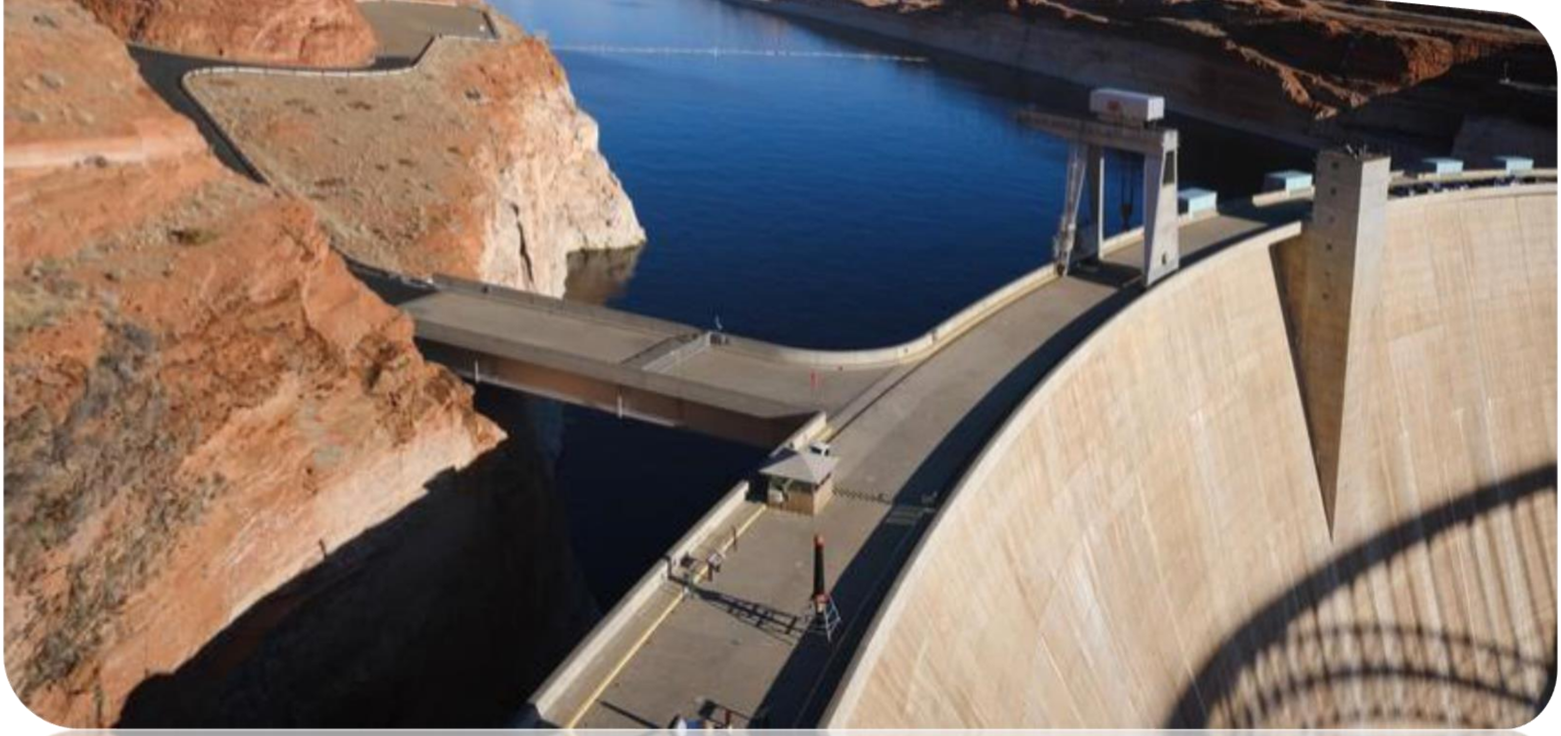
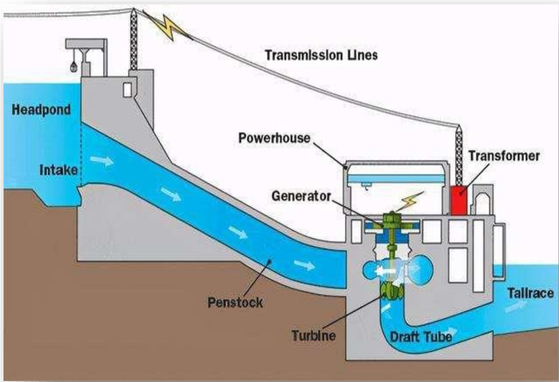
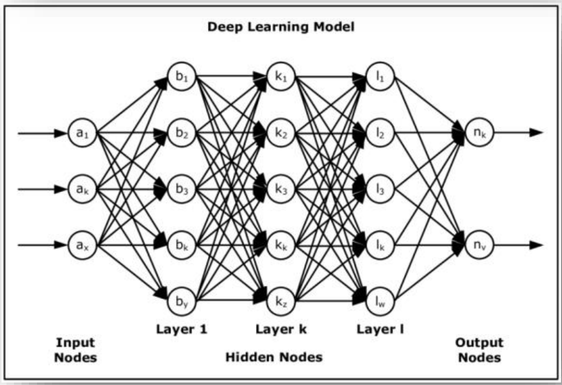


The Wiser Use of Water
The Better Our Future

Aridity is coming. Where lies our solution?

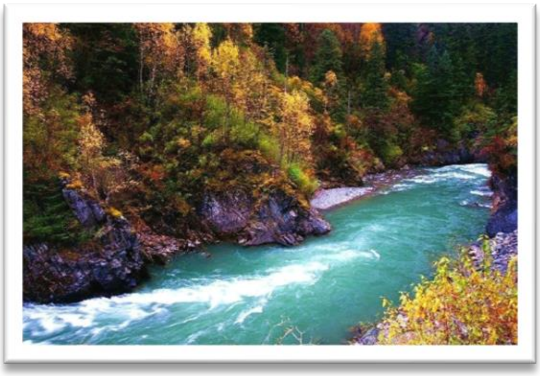
As we all know, electricity and water are two main factors necessary in our lives. However, without a wise allocation decision, a constant and sufficient amount of water and electricity supply would be impossible, even with the help of dams. When aridity is coming, what should we do in order to solve this problem and make full use of the tools we have today? In this article, we will take the Colorado River drainage basin as an example, on which two famous dams are constructed----Glen Canyon Dam and Hoover Dam. The construction of these two dams has formed Powell Lake and Mead Lake. How can we make full use of the water resources stored in those two lakes? How can we ensure everyone can have electricity as well as water even during drought periods as much as possible? Let's find out!





Researchers from MCM have established several models to describe a single dam and associations of multiple dams. These models include a multi-objective optimization model and a neural network matching model. These physical and mathematical models have greatly helped our researchers to achieve accurate results.

With our careful calculation, a set of wise and practical operating suggestions has been made. In the following section, these data will be presented to you in a form of charts.



If the weather relatively remains the same, our team has provided a strategy for you to allocate your water. You just need to manage your dam by this chart:

MONTH	$Q_{1,t}^{out}$	$Q_{2,t}^{out}$	$L_{1,t}$	$L_{2,t}$
January	827537530	356656700	944965240	474337410
February	723602990	493663510	840658440	496267340
March	716005030	475675330	940733720	550646210
April	708815980	348227090	969512730	516867000
May	784462650	531368960	935134720	503061080
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October	802612400	449120180	985764370	470170100
November	562598300	588291840	941957280	452883900
December	750875400	497018970	947975750	532344270

However, as we all know, the Colorado River is experiencing a period of drought, which means that the water input to a hydrological cycle is not enough. But no worries! We still have an alternative for you guys. You just need to manage your dam by this chart:

MONTH	$Q_{1,t}^{out}$	$Q_{2,t}^{out}$	$L_{1,t}$	$L_{2,t}$
January	869076740	422552550	734090670	325868440
February	611698400	456089920	785379420	388515440
March	708107160	439642630	751275070	476068880
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July	550879850	318953570	730733510	535639500
August	624157870	432159060	778695140	329718820
September	503845150	334491200	784153650	388402450
October	563393990	342221700	740854400	454642200
November	719583830	341261410	708571180	483219700
December	827867420	534809010	765665100	451168600

Since water resources are extremely variable, a lot of activities of human beings depend on them. Wiser use of the water resources in the Colorado River can help our planet to get better, and our world will thank you for that.

MCM

22^{th} Feb 2022

Appendices

Appendix A First appendix

A.1 Planning solutions for dams

	Jan	Feb	March	Apr	May	Jun
Glen Canyon	831985000	723605470	715572040	708366320	779874820	836850640
Hoover	355840880	593285550	513851530	598600300	517560540	599287390
	Jul	Aug	Sep	Oct	Nov	Dec
Glen Canyon	703241790	996521130	703208280	941858130	716434880	752353290
Hoover	534667370	495756590	451475910	449500060	599709700	497158160

Table 17: Amount of water used to generate electricity per month(unit:m^3)

	Jan	Feb	March	Apr	May	Jun
Glen Canyon	990047820	731868080	940545030	938663040	937065710	767981890
Hoover	494371580	499239190	490088690	518156200	502784780	482089050
	Jul	Aug	Sep	Oct	Nov	Dec
Glen Canyon	959683920	816721950	948574200	730935760	935108080	726350200
Hoover	461722720	474164250	460099020	471149320	451520050	515617260

Table 18: Amount of water used to supply water per month(unit:m^3)

A.2 Distribution planning for five states

	Jan	Feb	March	Apr	May	Jun
Arizona	162011763	163606940	160608206	169806281	164768874	157986624
California	332359816	335632250	329480483	348349918	338015905	324102425
Wyoming	443901598	328143150	421706340	420862523	420146338	344335275
New Mexico	68604197	50713936	65173959	65043549	64932864	53216400
Colorado	477542023	353010993	453664730	452756966	451986506	370430214
	Jul	Aug	Sep	Oct	Nov	Dec
Arizona	151312322	155389567	150780215	154401537	147968779	168974238
California	310410397	318774682	309318804	316747782	303551270	346643021
Wyoming	430287525	366188553	425306329	327725132	419268608	325669132
New Mexico	66500166	56593785	65730331	50649332	64797212	50331581
Colorado	462896227	393939610	457537539	352561295	451042258	350349485

Table 19: Water supply allocation strategies for the five states(unit:m^3)

	Jan	Feb	March	Apr	May	Jun
Arizona	88126	76646	75795	75032	82606	88641
California	268948	233914	231317	228987	252103	270521
Wyoming	21190	35330	30600	35646	30821	35687
New Mexico	34251	57106	49460	57618	49817	57684
Colorado	77831	129767	112392	130929	113204	131080
	Jul	Aug	Sep	Oct	Nov	Dec
Arizona	74489	105554	74485	99764	75886	79691
California	227331	322137	227320	304466	231596	243207
Wyoming	31839	29522	26885	26767	35713	29606
New Mexico	51464	47718	43456	43266	57724	47853
Colorado	116945	108435	98749	98317	131172	108741

Table 20: Electricity distribution in five states(unit:MWh)

A.3 Other materials related to the basis of distribution

	Agriculture	Industrial	Residents	Total
Arizona	3536585662	995582.52	1880544.76	3539461789
California	7089930296	145520978.3	25594767.29	7261046042
Wyoming	10026276491	5005567.67	1852889.69	10033134948
New Mexico	1549859260	525446.33	218328.9	1550603035
Colorado	10637232297	156126697.7	122234.1	10793481229

Table 21: Surface freshwater water demand in five states(unit:m^3)

	Total
Arizona	81,960,074
California	250,174,672
Wyoming	15,331,018
New Mexico	24,777,155
Colorado	56,050,264

Table 22: Electricity consumption of the five states in 2020(unit:MWh)

Plant	Primary energy source	Operating company	Generation (MWh)
Glen Canyon Dam	Hydroelectric	U S Bureau of Reclamation	3,598,780
Hoover Dam	Hydroelectric	U S Bureau of Reclamation	1,867,070

Table 23: Power generation capacity of the two dams in 2020(unit:MWh)